
The Impact of the Bezier Curves Order on the Function of the Turbodrill Blades

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Abstract

The hydraulic performance of the turbodrills mainly depends on the linear structure of the blades. In design, Bezier curves are commonly used because of its unique strength that it can conveniently control the two-dimensional leaf-tape angle. Based on the theories of Bezier curves and the turbomachine, we can get the linear designing method of the turbodrill. Taking the 3 times of Bezier curves for example, we can deduct the parameterization formula of different Bezier curves orders used for the shape of the blades and create the parameterization method if designing the linear types of the turbodrill. Taking the designing of the $\Phi 127$ turbodrill blades for example, we also verify the applicability of this method by testing the linear curvature and the runner of the blades. We conclude that the turbodrill blades with 5-time Bezier curves blade profiles have best comprehensive property by the 3 parts. The first part is the 3 process from the design of turbodrill blade profiles and three-dimensional modeling to the stream field simulation. The second part is that changing regulations of the turbo capability are systematically studied on the condition of 3-time, 4-time and 5-time Bezier curves fitting the blade profiles on the Turbosystem. The third part is that the capability differences of different orders are analysed in the aspects of torsion, pressure drop and the efficiency.

Keywords

Turbo blades/ Bezier curves/ Turbosystem.

1. Introduction

Stratigraphic oil and gas reservoir resources are always the predominant social energy sources. Long-term development have made the mining amount of the conventional oil and gas declined and the exploration of the unconventional oil and gas faced the challenge of different kinds of complicated drilling situation such as high temperature and high pressure. Turbodrill, a kind of downhole motor which can be heat resisted and keep from cross rotation, has already actually showed its strengths of safety and efficiency in complicated situation[1-5]. The linear structure of the turbodrill blades, as the key part, plays an important role in influencing the various functions of it. So the study of blades design is the primary part to develop its capability.

At the beginning of, there were two usual ways to design the blade[6]. One is the arc combination with different line and radius. It is modeling-convenient and simple. But there are turning points in collecting the line and the arc as a result of large curvature and energy loss[7-9]. Another is to confirm the coordinates of the blade suction surface and the pressure surface in plane coordinate systems. And then smooth the blade profiles. These two ways are difficult to achieve parameterization. With the widely usage of the computer aided technique in later design of blade profiles, many kinds of geometrical moldings are used for line description, such as spline curve, higher order polynomial, Bezier curve and so on. Now, we will give some examples. Analysing the relation between the

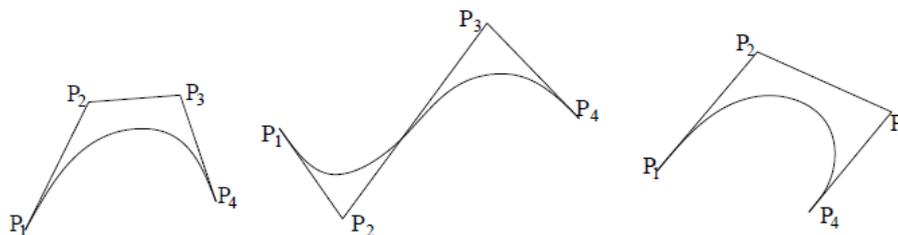
turbodrill blade profiles and the flow parameter of the fluid, Feng Jin[10] and some other people think that discontinuous curvature is the key factor of the turbo capability. They concluded that blade profiles need to be single line and continuous third derivative. And they propose to design the blade profiles with quintic polynomial. Soo-Yong Cho[11] and so on propose to design the blade profiles with the composite curves and the pressure surface of the cubic polynomial and the quintic polynomial. Lin Yuanhua[12] and so on propose to use third order polynomial curves to make up the blade profiles. And they apply PRO/E to make 3D modeling and design a turbodrill blades designing soft for computer aided design. Based on the quintic polynomial curve designing, Li Junhua[13] from Southwest Petroleum University and Zhao Hongbo[14] from China University of Geosciences, optimized the blade shapes with the computer aided soft. Ma Wensheng[15] and so on use 3-time Bezier curves to convey medial arc, and superpose the thickness distribution of the blade shapes to design the 2D blade shape. Gao Kun[16] and so on use 4-time Bezier curves to express medial arc. The standards to choose the way to design blades are great shape-controlling capability of the curves and few controlling parameters. In design, the tangential point of the Bezier curves is very beneficial to control the 2D-shaped leaf-taped angle. It gives expression to the advantages of Bezier curves in the parametric molding[15].

Taking the 3 times of Bezier curves for example, in this paper, we create the paramterization method of 3-time、4-time and 5-time Bezier curves designing the blade profiles. And we create the numerical models to predict the capability of the designing blades with different Bezier curves on Turbosystem. We will analyse the capability differences of different orders in the aspects of torsion, pressure drop and the efficiency.

2. Design of turbodrill blades profiles

2.1 Bezier curves

The shapes of the Bezier curves are only made of vertexes of a group of multilateral folding line. The polygon made of this multilateral folding line is called characteristic polygon. Every vertex is called controlling point. And each controlling point is on line of Bezier curves except from the starting point and the finishing point. That is, the first side and the last side of the characteristic polygon are the tangent line of the starting point and the finishing point. If we change the position of the controlling point, we can get different curve shapes. As the picture 1 shown.



Picture 1. Characteristic polygon and the Bezier curves

In general, Bezier curves are defined by controlling points of characteristic polygon. If we make interpolation function between the starting point and the finishing point, that is, we give space $k+1$ controlling points $P_i(i=0,1,2,\dots,k)$, these controlling points can make the k -time Bezier characteristic polygon. Now, we can express the curves as follows[17]:

$$P(\gamma) = \sum_{i=0}^k P_i B_{i,k}(\gamma) \tag{1}$$

The value range of γ is $[0,1]$. $B_{i,k}$ are the Berstein primary function of k , that is, $B_{i,k}$ are the interpolation function of k

$$B_{i,k}(\gamma) = C_k^i (1-\gamma)^{k-i} \gamma^i = \frac{k!}{i!(k-i)!} (1-\gamma)^{k-i} \gamma^i \tag{2}$$

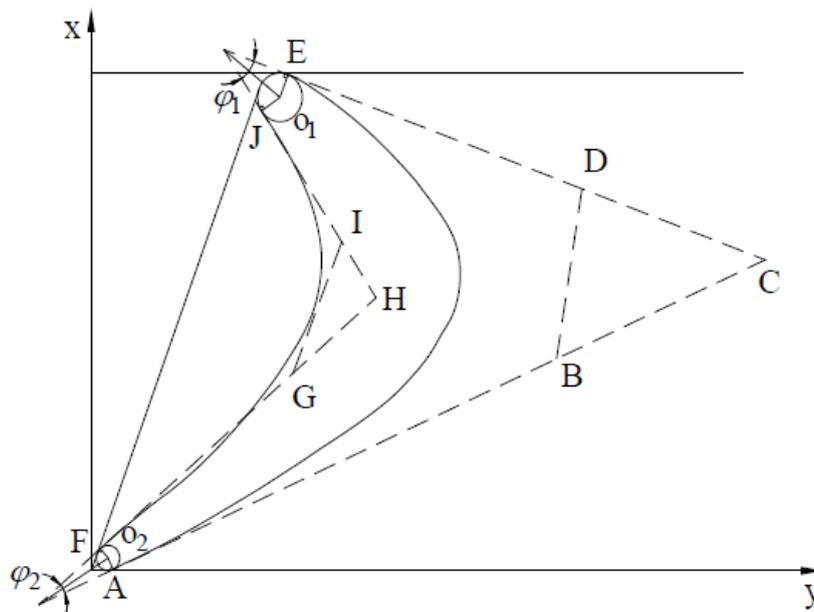
According to equation (1) and (2), we can deduce different Bezier curves to design blade shapes. And we only need to give the coordinates and the orders to confirm the shape and the expression of the curve.

3-time Bezier curves expression is:

$$P(\gamma) = \sum_{i=0}^3 P_i B_{i,3}(\gamma) = \sum_{i=0}^3 P_i C_3^i (1-\gamma)^{3-i} \gamma^i, 0 \leq \gamma \leq 1 \tag{3}$$

2.2 Blade profiles Bezier curves parameterized shape

When we know parameters of blade structure, former radius r_1 , post radius r_2 , axial height b , setting angle β , inlet structure angle β_1 , outlet structure angle β_2 , front wedge angle φ_1 and post wedge angle φ_2 , we use the 3-time Bezier curves to fit the lines. Taking the stator for example, its characteristic points and controlling points and their positions are shown as the picture 2. The former circle is tangent to point E with suction side and to point J with pressure side. The post circle is tangent to point A with suction side and point F with pressure side. That is, E,J,A,F are the first point and the last point of the suction side and the pressure side. The tangent included angles of these tangent points are front wedge angle φ_1 and post wedge angle φ_2 . In suction characteristic polygon, the first side ED and the extension line of the last side AB intersect at point C. In pressure characteristic polygon, the first side JI and the extension line of the last side FG intersect at point H.



Picture 2 Blade profiles and controlling points based on the 3-time Bezier curves

Coordinate of the center of the former circle O1 is:

$$\begin{cases} x_{O_1} = b - r_1 \\ y_{O_1} = b \cot \beta_m + r_1 \tan(\frac{\beta_m}{2}) \end{cases} \tag{4}$$

Coordinate of the center of the former circle O2 is:

$$\begin{cases} x_{O_2} = r_2 \\ y_{O_2} = r_2 / \tan(\frac{\beta_m}{2}) \end{cases} \tag{5}$$

Coordinate of the first point E of the suction side:

$$\begin{cases} x_E = b - r_1 + r_1 \cos(\beta_{1k} - \frac{\varphi_1}{2}) \\ y_E = b \cot \beta_m + r_1 \tan(\frac{\beta_m}{2}) + r_1 \sin(\beta_{1k} - \frac{\varphi_1}{2}) \end{cases} \quad (6)$$

Coordinate of the last point A of the suction side is:

$$\begin{cases} x_A = r_2 - r_2 \cos(\beta_{2k} - \frac{\varphi_2}{2}) \\ y_A = r_2 / \tan(\frac{\beta_m}{2}) + r_2 \sin(\beta_{2k} - \frac{\varphi_2}{2}) \end{cases} \quad (7)$$

Coordinate of the first point J of the pressure side is:

$$\begin{cases} x_J = b - r_1 - r_1 \cos(\beta_{1k} + \frac{\varphi_1}{2}) \\ y_J = b \cot \beta_m + r_1 / \tan(\frac{\beta_m}{2}) - r_1 \sin(\beta_{1k} + \frac{\varphi_1}{2}) \end{cases} \quad (8)$$

Coordinate of the last point F of the pressure side is:

$$\begin{cases} x_F = r_2 + r_2 \cos(\beta_{2k} + \frac{\varphi_2}{2}) \\ y_F = r_2 / \tan(\frac{\beta_m}{2}) - r_2 \sin(\beta_{2k} + \frac{\varphi_2}{2}) \end{cases} \quad (9)$$

According to the geometrical relationship, we can get the slope of the tangent AC. It is:

$$k_{AC} = \cot(\beta_{2k} - \frac{\varphi_2}{2}) \quad (10)$$

The slope of the tangent CE is:

$$k_{CE} = -\cot(\beta_{1k} + \frac{\varphi_1}{2}) \quad (11)$$

The slope of the tangent FH is:

$$k_{FH} = \cot(\beta_{2k} + \frac{\varphi_2}{2}) \quad (12)$$

The slope of the tangent HJ is:

$$k_{HJ} = -\cot(\beta_{1k} - \frac{\varphi_1}{2}) \quad (13)$$

Coordinate of point C is:

$$\begin{cases} x_C = x_A + (y_C - y_A) / k_{AC} \\ y_C = (k_{AC}k_{CE}x_A - k_{AC}k_{CE}x_E - k_{CE}y_A + k_{AC}y_E) / (k_{AC} - k_{CE}) \end{cases} \quad (14)$$

Coordinate of point H is:

$$\begin{cases} x_H = x_F + (y_H - y_F) / k_{FH} \\ y_H = (k_{HJ}k_{FH}x_F - k_{HJ}k_{FH}x_J - k_{HJ}y_F + k_{FH}y_J) / (k_{FH} - k_{HJ}) \end{cases} \quad (15)$$

For fixing controlling points (D,B,I,G), We can use Bezier curves to control parameters. That is, the proportion of the split segment, is used as t^* (i), $i=(1,2,3,4)$. The length ratio of the segment CD and CE is t^* (1).

Coordinate of the controlling point D of the controlling polygon in suction side is:

$$\begin{cases} x_D = x_C - (x_C - x_E)t^* \\ y_D = y_E + (x_D - x_E)k_{CE} \end{cases} \quad (16)$$

Coordinate of the controlling point B of the controlling polygon in suction side is:

$$\begin{cases} x_B = x_C - (x_C - x_A)t^* \\ y_B = y_A + (x_B - x_A)k_{AC} \end{cases} \quad (17)$$

Coordinate of the controlling point I of the controlling polygon in pressure side is:

$$\begin{cases} x_I = x_H - (x_H - x_J)t^* \\ y_I = y_J + (x_I - x_J)k_{HI} \end{cases} \quad (18)$$

Coordinate of the controlling point G of the controlling polygon in pressure side is:

$$\begin{cases} x_G = x_H - (x_H - x_J)t^* \\ y_G = y_F + (x_G - x_F)k_{FH} \end{cases} \quad (19)$$

According to above controlling points, we can get the expression of the stator blade profiles parameterization. The 3-time Bezier curves expression of suction side of the stator blades is:

$$\begin{cases} x_s(\gamma) = (1-\gamma)^3 x_E + 3\gamma(1-\gamma)^2 x_D + 3\gamma^2(1-\gamma)x_B + \gamma^3 x_A \\ y_s(\gamma) = (1-\gamma)^3 y_E + 3\gamma(1-\gamma)^2 y_D + 3\gamma^2(1-\gamma)y_B + \gamma^3 y_A \end{cases} \quad (0 \leq \gamma \leq 1) \quad (20)$$

The 3-time Bezier curves expression of suction side of the stator blades is:

$$\begin{cases} x_p(\gamma) = (1-\gamma)^3 x_J + 3\gamma(1-\gamma)^2 x_I + 3\gamma^2(1-\gamma)x_G + \gamma^3 x_F \\ y_p(\gamma) = (1-\gamma)^3 y_J + 3\gamma(1-\gamma)^2 y_I + 3\gamma^2(1-\gamma)y_G + \gamma^3 y_F \end{cases} \quad (0 \leq \gamma \leq 1) \quad (21)$$

The expression of the leading edge arc curve is:

$$(x - x_{O_1})^2 + (y - y_{O_1})^2 = r_1^2 \quad (22)$$

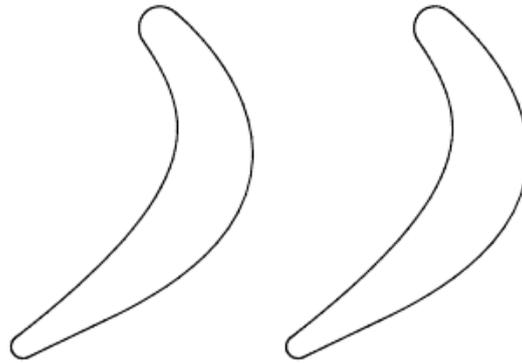
The expression of the trailing edge arc curve is:

$$(x - x_{O_2})^2 + (y - y_{O_2})^2 = r_2^2 \quad (23)$$

Now, we have already created the blade profiles parameterized expressions based on the 3-time Bezier curves. When we know parameters of blade structure, former radius r1, post radius r2, axial height b, setting angle β_m , inlet structure angle β_{1k} , outlet structure angle β_{2k} , front wedge angle φ_1 and post wedge angle φ_2 , the shape of the blade profiles are mainly decided by the curve-controlling parameter t^* (i). We can get different blade profiles from different controlling parameters. As for symmetric turbo, its stator blade and rotor blade are image symmetrical. That is, when we know one of the blade shape, we can get another blade shape by image. In addition, according to above calculating process, we can get the blade profiles about 4-time and 5-time Bezier curves by calculating.

3. Numerical analysis of blade profiles

Taking the 3 times of Bezier curves for example, in this paper, we create the parameterization modeling of 3-time、4-time and 5-time Bezier curves designing the blade profiles. And we analyse the capability differences of the different orders blades with different Bezier curves on Turbosystem. And we get the the capability differences of the different Bezier curves. Based on the available turbo drill blades $\Phi 127$, its related structure parameters are shown in chart 1 and the designed blade shapes are shown as picture 3.



Picture 3 Turbo drill blade shapes

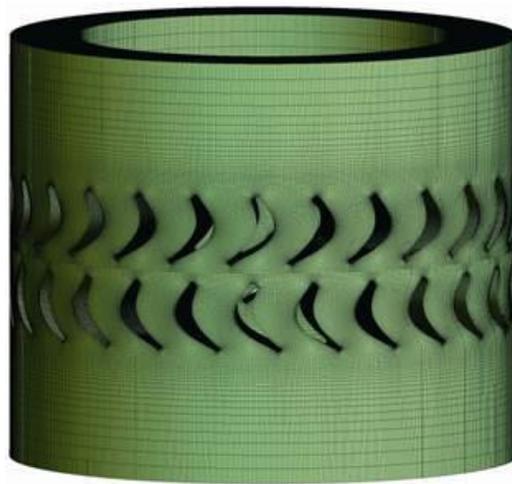
Table 1 Turbo drill blade structure parameters

Name	Numerical value	Name	Numerical value
Former radius r_1 (mm)	0.7	Stator inlet structure angle α_{2k} ($^\circ$)	135.63
Post radius r_2 (mm)	0.4	Stator outlet structure angle α_{1k} ($^\circ$)	32.14
Radial height h (mm)	13	front wedge angle φ_1 ($^\circ$)	20
Axial length b (mm)	13	post wedge angle φ_2 ($^\circ$)	12
Rotor inlet structure angle β_{1k} ($^\circ$)	135.63	Number of rotor blade z	26
Rotor outlet structure angle β_{2k} ($^\circ$)	32.14	Number of stator blade z	26

Firstly, we adopt the PROE 3D modeling software to model the turbo blades. Then import the modeling to BladeGen to make the blade runner 3D modeling to offer the foundation of TurboGrid getting the blade runner high-qualified grid. We import the generated blade number, blades, hubs and rims and other outer data into the TurboGrid. Then divide the grids and create the limited meta grid. Basing on the TurboGrid special grid quality evaluation system, we improve the grid and get the high-qualified turbo blade 3D runner grid modeling. And then in the CFX-Turbo processing module, we make the simulation analysis of the blades' velocity field and pressure field, and get the parameters of the torsion, pressure drop, and efficiency.

3.1 Building of numerical modeling

In order to reduce the difference between the fluid flowing on the entrance and the exit and the actual flowing, and get the approximate stable solution, we specially extend up and down the blade height to 2-time distance on the blade runner entrance and exit when calculating. We import the generated blade number, blades, hubs and rims and other outer data from BladeGen into the TurboGrid. And then revise and improve the grid quality. We get the turbo blade 3D runner grid modeling as picture 4.



Picture 4 Turbo inner runner grid stereogram

3.2 Setting of the boundary condition

For the turbodrill on a certain working condition, its fluid media is incompressible drilling fluid. Its density is a constant. So its mass conservation equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \tag{24}$$

In fact, there are some differences between the theoretical analysis and the actual flowing of the fluid. In order to describe the action of the fluid accurately, based on the turbulence modeling theory, this article adopt the Navier-Stokes momentum conservation equation. Its working condition is[64]:

$$\frac{\partial \rho}{\partial x} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{25}$$

$$\rho \left[\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left[u_j \frac{\partial u_i}{\partial x_j} \right] + \frac{\partial R_{ij}}{\partial x_j} \tag{26}$$

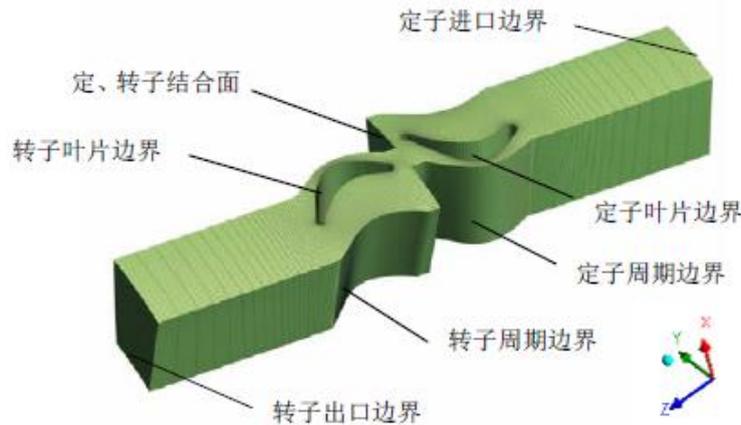
In the equation, ρ stands for fluid density, u stands for velocity, p stands for static pressure, R_{ij} stands for Reynolds stress tensor, its expression is:

$$R_{ij} = -\overline{\rho u_i' u_j'} = 2\mu_i S_{ij} - \frac{2}{3} u_i \frac{\partial u_k}{\partial x_k} \delta_{ij} - \frac{2}{3} \rho k S_{ij} \tag{27}$$

$$S_{ij} = \frac{1}{2} \left| \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right| \tag{28}$$

In the equation, μ_i stands for turbulent viscosity, k stands for turbulent knietic energy, R_{ij} stands for Kronecker’s function, S_{ij} stands for strain rate tensor.

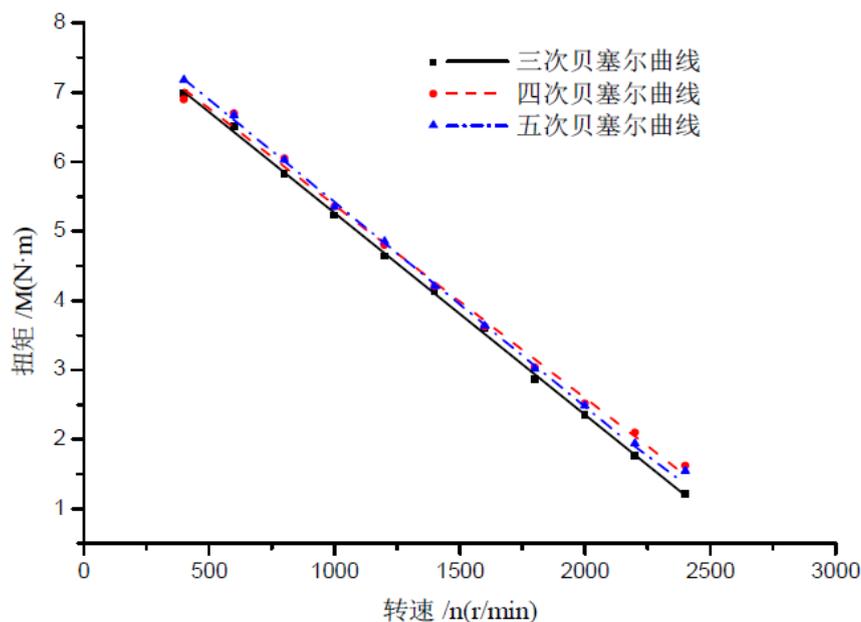
We make the simulation solution using the CFX. The process of the setting of the boundary conditons are: hub, outer cover, the anterior and posterior border of the fixed rotor and the wall surface of the fixed rotor are set as no-slip wall surface; the boundary condition of the fixed rotor is set as periodic cycle; in the actual working process of turbodrill, if there is an interaction between the boundary of fixed rotor, we set the boundary as Frozen rotor; we use clean water to replace the actual drilling fluid. Boundary condition of the entrance is the mass flow, and boundary condition of the exit is the pressure exit. We set it as a normal atmosphere; we adopt the $k-\varepsilon$ modeling in the turbulence modeling, and we set it as medium turbulence intensity.



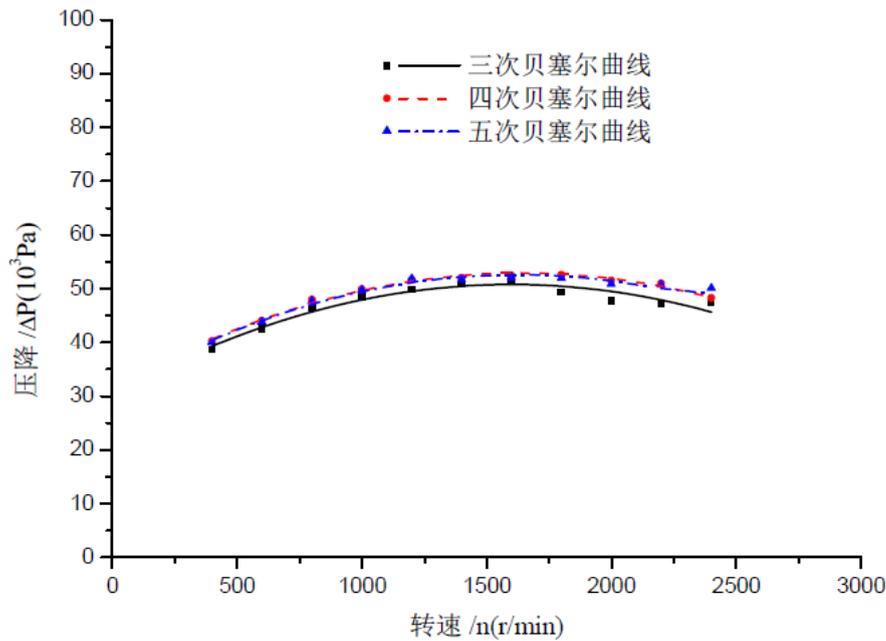
Picture 5 Single cycle cross blade runner boundary condition setting

4. Example analysis

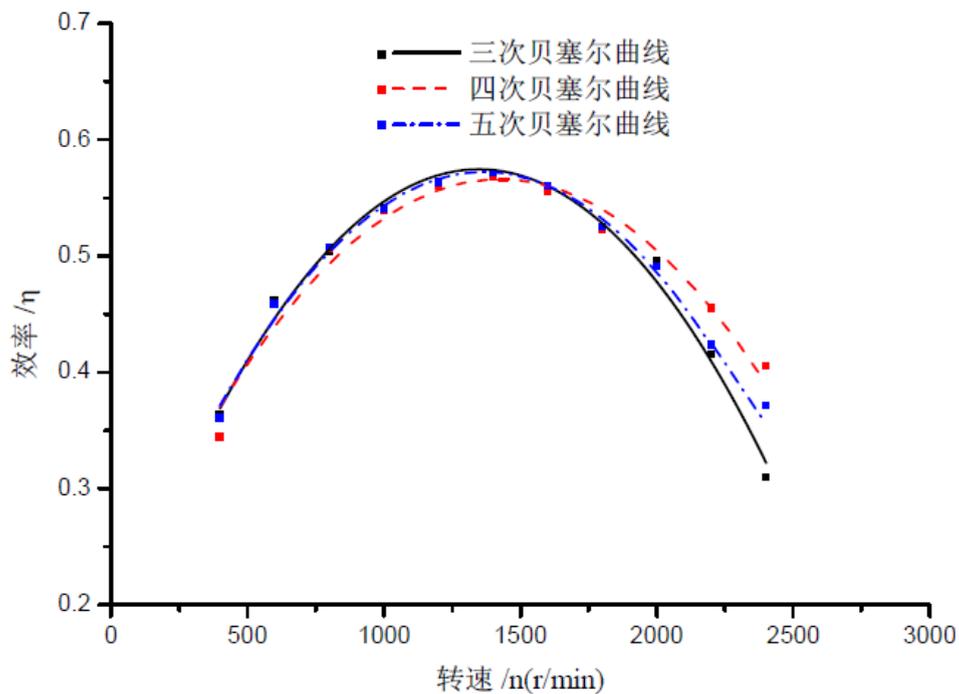
When confirming the working condition, the hydraulic performance of the turbo is related not only to the blade structural parameters, but also to the curves of the fitting blade shapes. When curves is different, the blade profile structure is different. And When the orders of the curves are different, their fitting profiles are different. So, based on the Bezier curves, this article research the Bezier curves fitting the blade profiles in different orders. That is, we make the flow fluid simulation analysis of the 3-time, 4-time and 5-time Bezier curves fitting blade profiles in different speed. And comparing their capability, we choose the most capable turbo blade as the research object. Taking a pair of turbo to analyse, we choose 15L/s as the flow rate. And considering the energy loss caused by the volume loss and the mechanical friction, we choose 0.8 and 0.9. We set the total mass velocity as 15kg/s on the turbo entrance, set the entrance as a normal atmosphere, set the fluid density as 1000kg/m³, set the fluid dynamic viscosity as 8.899x10⁻⁴N·s/m². On these condition, we make the simulation analysis of each group in different velocity, and get the corresponding turbo capability. And then we draw the fitting blade profiles capability curve contrast diagram of 3-time, 4-time and 5-time Bezier curves. As shown in the following picture.



Picture 6 Turbo torsion curve contrast diagram



Picture 7 Turbo pressure drop contrast diagram



Picture 8 Turbo efficiency curve contrast diagram

Picture 6, 7, 8 are the turbo torsion contrast diagram, turbo pressure drop contrast diagram and the turbo efficiency contrast diagram of the 3-time, 4-time and 5-time Bezier curves. According to the pictures, we can know:

The capability of the 4-time Bezier curves fitting blade profiles is not good as another two blades. Although its torsion is larger than 3-time Beizer blade, its pressure drop is larger, too. Its largest torsion is 6.98758N·m, but its largest pressure drop is 0.0552MPa. And when the rotation rate is lower than 1700r/min, its efficiency is poor.

Compared with 5-time Bezier curves fitting blade profiles, although the turbo pressure drop of the 3-time Bezier blade profiles is smaller, its torsion is not good as the 5-time Bezier blade profiles.

In summary, this article choose the 5-time Bezier blade profiles as the analysis and research object.

5. Conclusion

Based on the Bezier curves theory, 3D modeling software and the Turbosystem, we create the turbo blade analysis method from the blade designing, 3D modeling to flow simulation. The result shows that: for a certain flow pump and fluid density, the turbo torsion reduce with the increase of the flow simulation and the efficiency curves show the relation of the 2-time parabola and the pressure drops are basically the same; for a certain working condition, the hydraulic performance is related not only to the blade structure parameters and to the fitting blade profiles. Firstly, we research systemically turbo capability changing rules of the 3-time, 4-time and 5-time Bezier curve fitting blade profiles. The 5-time Bezier curve fitting blade profiles are better than 3-time and 4-time Beizer curve on turbo torsion and efficiency. So we choose them as the research object.

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