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# Optimal Production Planning with Personnel Allocation

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## Abstract

In this paper, we study, solve and summarize the problems related to the optimal production planning of the combination of personnel and time allocation, using model construction and integer programming.

## Keywords

Pipeline work, Integer programming, 0-1 planning.

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## 1. Introduction

The core of this question is the optimal production plan for pipeline operations and non-pipeline operations. Therefore, it is necessary to distinguish the difference between the two. The assembly line operation is that at least one person on each production line completes one process and the next is completed by another person. That is, each production line can produce at least one product, and for non-pipeline operations, each product can be produced on one production line.

First of all, the problem of "pipeline work" is divided into two types: personnel transfer and staff transfer. According to the characteristics of pipeline work, an easy-to-understand, convenient and practical integer planning model is established. Then, using Lingo software programming, the maximum profit of the first problem is 155873 yuan and the first question is the optimal distribution scheme. For considering the problem of personnel transfer between production lines, on the basis of the first model, a 0-1 programming part was added, and the established model was solved using variable substitution and Lingo software. The production arrangement scheme was obtained in the case of pipeline operation and personnel transfer at the same time, and the maximum profit was 170,266 yuan.

Secondly, for non-running problems, only the decision variables in the first two questions are changed to non-negative integers. On this basis, a new integer programming model is established to use LINGO software programming to obtain the optimal solutions of 155942 yuan and 170333 yuan for two questions and the optimal distribution and transfer strategy for the third question.

Key words: Pipeline work, Integer programming, 0-1 planning

## 2. Restatement of issues

A company plans the production of four products(P1, P2, P3, P4) on five production lines(L1 to L5). The unit net profit of products P1 and P4 is 7 yuan, the unit net profit of product P2 is 8 yuan, and the unit net profit of product P3 is 9 yuan. The length of time that each of the five production lines can produce during the planning period varies. The maximum available production times for L1 to

1. Assuming that production is a pipeline operation, how much should P1 to P4 be produced to maximize the total profit?

2. What is the maximum profit if the transfer of personnel between production lines is allowed during the production process (and thus the corresponding transfer of working hours), as shown in table 2? How many hours should be transferred and how?

3. If production is not a pipeline operation, how should the model be modified?

Table 1 Unit production time

product	production line				
	L1	L2	L3	L4	L5
P1	1.3	0.9	2.0	0.3	0.9
P2	1.8	1.7	1.4	0.6	1.1
P3	1.3	1.2	1.3	1.0	1.4
P4	0.9	1.1	1.0	0.9	1.0

Table 2 Possible personnel transfers

Source	destination					Maximum transferable Total hours worked
	L1	L2	L3	L4	L5	
L1	-	yes	yes	yes	no	400
L2	no	-	yes	no	yes	800
L3	yes	yes	-	yes	no	500
L4	no	no	no	-	yes	200
L5	yes	yes	yes	no	-	300

### 3. Problem analysis

This question requires four products (P1, P2, P3, P4) in five production lines (L1 to L5). The production planning on the premise of giving the unit net profit and unit production time, in order to obtain the maximum total profit, according to the problem requires that the problem be studied in the case of running water operation and non-running water operation respectively.

Question 1: This problem is a simple problem of running water, provided that personnel remain fixed and can not be transferred. Taking into account the characteristics of running water, it is necessary to ensure that products can be distributed on each production line. A typical integer programming model is established to solve the problem.

Question 2: This problem is based on the first question and ensures that personnel can be transferred. Considering that the personnel transfer between any two production lines is not bidirectional, a 0-1 variable is introduced, and a 0-1 integer programming model can be established to solve the problem.

Problem 3: This problem requires solving on the premise of non-running water operations. At this time, it is not necessary to ensure that each production line can be assigned to the product. It is only necessary to limit the decision variable to non-negative based on the original model, and to solve it with LINGO.

**4. Symbol Description**

symbol	instructions
$N_{ij}$	Assigned production capacity of product J on line I.
$t_{ij}$	Article I Time required for production line to produce unit J
$S_j$	Net profit per unit of product J
$b_i$	Maximum available production time on line I.
$b_p$	Article P Maximum available production time on production line
$L_{pq}$	If the working hours can be moved from the P production line to the Q production line, the value is 1; Others are 0
$M_p$	Maximum possible transfer hours for line P
$T_{pq}$	Transfer of work hours from line P to line Q
$h_p$	Maximum available production time for the transfer of line P
$h_i$	Maximum available production time for transferred line I

**5. Model assumptions**

In response to the above, the following assumptions are made:

- (1) Assuming that there is no loss of personnel during production;
- (2) The total profit is only related to the unit net profit of the product and the production quantity, and has nothing to do with other things;
- (3) The production volume of each production line is an integer;
- (4) During the production period, the company stopped production for no other reason and was normal production;

**6. Model development and solution**

**6.1 Pipeline work doesn't allow personnel to move.**

6.1.1 Model one establishment:

As mentioned in the question, assuming that production is assembly line production, each production line from L1 to L5 should be divided into products for production, which represents the total profit. Based on the symbol description, the objective function can be obtained as follows:

$$Z_1 = \sum_{i=1}^4 \sum_{j=1}^5 S_j N_{ij}$$

The maximum available production time for each production line needs to be guaranteed, and the constraints are obtained  $\sum_{i=1}^4 N_{ij} t_{ij} \leq b_i$

In summary, the available integer programming model is as follows:

$$\text{Max } Z_1 = \sum_{i=1}^4 \sum_{j=1}^5 S_j N_{ij}$$

$$N_{ij} \in N^*$$

$$\sum_{i=1}^4 N_{ij} t_{ij} \leq b_i$$

Then use LINGO programming to solve it.

5.1.2 Model 1 solution

If there is a unit net profit for each product

$$S=(7 \ 8 \ 9 \ 7);$$

The time required to produce each product unit on each production line is;

$$T = \begin{bmatrix} 1.3 & 0.9 & 2.0 & 0.3 & 0.9 \\ 1.8 & 1.7 & 1.4 & 0.6 & 1.1 \\ 1.3 & 1.2 & 1.3 & 1.0 & 1.4 \\ 0.9 & 1.1 & 1.0 & 0.9 & 1.0 \end{bmatrix}$$

The maximum available production time for L1 to L5 is  $B = (4500 \ 5000 \ 4500 \ 1500 \ 2500)$

The maximum profit from programming solutions is:  $Z_1 = 155873$

And get the optimal distribution plan as shown in the table below.

Table 3. optimal distribution plan

product \ product line	L1	L2	L3	L4	L5
P1	1	5551	1	4991	2217
P2	1	1	1	1	2
P3	1	1	2	1	1
P4	4995	1	4494	1	1
Total profit	155873				

6.2 Pipeline operations allow personnel to move.

6.2.1 Model 2 establishes:

As mentioned in the question, there is a second table:

Transfer of production lines	L1	L2	L3	L4	L5	Maximum transferable work hours
L1		Yes	Yes	Yes	No	400
L2	No		Yes	No	Yes	800
L3	Yes	Yes		Yes	No	500
L4	No	No	No		Yes	200
L5	Yes	Yes	Yes	No		300

In order to facilitate the calculation, the data in the table is preprocessed and the 0-1 variable is introduced as follows:

$$L_{pq} = \begin{cases} 1 & \text{Ability to move working hours from line P to line} \\ 0 & \text{Other circumstances} \end{cases}$$

Handle the table as follows

Transfer of production lines	L1	L2	L3	L4	L5	Maximum transferable work hours
L1	0	1	1	1	0	400
L2	0	0	1	0	1	800
L3	1	1	0	1	0	500
L4	0	0	0	0	1	200
L5	1	1	1	0	0	300

On the basis of the first question, the decision variable should also be guaranteed to be a non-negative integer. The number of transfer hours from the p-line to the qline is set to ensure that it can correspond to the maximum transferable total hours of each production line. The constraints are as follows:

$$\sum_{q=1}^5 T_{pq} L_{pq} \leq M_p$$

For each production line, it is possible to transfer from and to each production line. Once the 0-1 variable is defined, the maximum transferable total work hours after each production line is transferred can be obtained. The transfer and transfer of the production line of the p-line are respectively. There is the above analysis:

$$Y_{RP} = \sum_{p=1}^5 T_{pq} L_{pq}$$

The maximum available production time for the p-line after transfer is:

$$h_p = b_p + \sum_{p=1}^5 T_{pq} L_{pq} - \sum_{q=1}^5 T_{pq} L_{pq}$$

In order to correspond to the maximum available production time after transfer, the constraints are as follows:

$$\sum_{i=1}^4 t_{ij} N_{ij} \leq h_i$$

Summarize the above problems that can be established as a 0-1 integer programming model as follows

$$\text{Max } Z_2 = \sum_{i=1}^4 \sum_{j=1}^5 S_j N_{ij}$$

$$N_{ij} \geq 1$$

$$\sum_{q=1}^5 T_{pq} L_{pq} \leq M_p$$

$$h_p = b_p + \sum_{p=1}^5 T_{pq} L_{pq} - \sum_{q=1}^5 T_{pq} L_{pq}$$

$$\sum_{i=1}^4 t_{ij} N_{ij} \leq h_i$$

There is also a need to ensure that X is an integer.

6.2.2 Model II solution:

Based on the data of the second problem, the maximum profit is obtained by means of LINGO programming.

$$Z_2 = 170266$$

The mode of transfer of hours and the amount of transfer carried out are shown in the table below:

Transfer of production lines	L1	L2	L3	L4	L5
L1		0.1	0	399.9	0
L2	0		0	0	800
L3	0.1	0		499.9	0
L4	0	0	0		0
L5	1.1	0	0	0	

The distribution of products on each production line is shown in the table below

production line \ product	L1	L2	L3	L4	L5
P1	1	4661	1	7991	3106
P2	1	1	1	1	1
P3	1	2	2	1	1
P4	4552	1	3994	1	1
Total profit	170266				

### 6.3 Non-pipeline work

#### 6.3.1 No personnel transfers allowed for non-pipeline operations

##### Model III Establishment

From the difference between pipeline operations and non-pipeline operations, it is only necessary to limit the decision variables slightly on the basis of the first two questions. Since non-pipeline operations are used, the production volume allocated on any production line may be zero, so it can be set. Non-negative numbers can be Therefore, the following model is obtained:

$$Max Z_3 = \sum_{i=1}^4 \sum_{j=1}^5 S_j N_{ij}$$

$$N_{ij} \in N \quad N_{ij} \geq 1$$

$$\sum_{i=1}^4 N_{ij} t_{ij} \leq b_i$$

Model III solution:

The same maximum profit solved using LINGO programming i

$$Z_3 = 155942$$

The production of each product for each production line is shown in the table below:

production line \ product	L1	L2	L3	L4	L5
P1	0	5554	0	5000	2221
P2	0	0	0	0	1
P3	0	1	0	0	0
P4	5000	0	4500	0	0
Total profit	155942				

Non-pipeline operations allow for movement of personnel

##### Model 4 Establishment

From the analysis of 5.3.1, it can be set to non-negative numbers, so the following model is obtained:

$$Max Z_4 = \sum_{i=1}^4 \sum_{j=1}^5 S_j N_{ij}$$

$$N_{ij} \geq 1$$

$$\sum_{q=1}^5 T_{pq} L_{pq} \leq M_p$$

$$h_p = b_p + \sum_{p=1}^5 T_{pq} L_{pq} - \sum_{q=1}^5 T_{pq} L_{pq}$$

$$\sum_{j=1}^4 t_{ij} N_{ij} \leq h_i$$

Where IJ must be an integer.

Model 4 solution:

The maximum profit solved using LINGO programming is

$$Z_4 = 170333$$

The specific distribution measures are shown in the table below:

production line product	L1	L2	L3	L4	L5
P1	0	4667	0	8000	3110
P2	0	0	0	0	0
P3	0	0	1	0	0
P4	4556	0	3699	0	0
Total profit	170333				

The transfer programme is shown in the following table:

	L1	L2	L3	L4	L5
L1	-	0	0	400	0
L2	0	-	0.3	0	799.7
L3	0	0	-	500	0
L4	0	0	0	-	0
L5	0.4	0.3	0	0	-

### 7. Model analysis

From the above results, it can be seen that there are personnel transfers and no personnel transfers, and the maximum profits obtained by non-pipeline operations are 69 yuan and 67 yuan more than those obtained by pipeline operations. The use of non-pipeline operations and the use of pipeline operations, the maximum profit obtained in the case of personnel transfer is 14,793 yuan and 14,391 yuan more than the maximum profit obtained in the case of no transfer. Comparing the production arrangement strategy and profit under various conditions, it can be seen that non-pipeline operations and allowing personnel transfer are the most profitable.

From the point of view, the main factors that affect profit are personnel transfer and product production methods. However, from the point of view of results, personnel transfer has a great impact

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on final profit, while product production methods have a small impact. Finally, it is known that the transfer of personnel can maximize the use of company facilities and personnel, and eventually maximize

## 8. Model evaluation and promotion

### (1).Model Advantages

Using the linear relation of objective function and constraint condition to establish the integer programming model, the model is easy to understand and easy to solve.

### (2).Model Disadvantages

Without considering the impact of other factors on total profits, there will be deviations if applied directly to reality.

### (3).Model extension

The model established in this paper is simple and easy to use, and can be widely applied to the distribution of logistics express delivery and other issues.

## References

- [1] A Generalized Inverse Optimization Model of Linear Programming for Enterprise Optimal Production Plan.
- [2] Properties of Optimal Production Function Established by Mathematical Programming Method.