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# Stability of Torsional and Axial Vibrations in Drilling

Shuai Wang <sup>a</sup>, Xin Zhong and Baoyi Wang

School of Shandong University of Science and Technology, Shandong 266590, China.

<sup>a</sup> 17853272105@qq.com

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## Abstract

In this work, dynamic models for chatter in drilling are developed that deal with the axial and torsional vibration. In the first part, a dynamic model is developed to obtain the limit of stability for axial and torsional vibration mode. The equations of motion are formulated based on a lumped representation of the drill, and axial viscous damping coefficient in deeping rotary drilling is included. It is shown that, axial viscous damping coefficient has a profound effect on the resulting stability lobes, it makes the stability lobes lower as the decrease of the axial viscous damping coefficient.

## Keywords

Chatter; Stability lobes; Axial viscous damping coefficient.

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## 1. Introduction

Deep drilling system play an important role in the exploration of natural resources, it is widely used to machine precise hole on the workpiece of the rock using conventional or micro drilling tools, deep drilling system can be understood as rotary drilling, dynamics of drilling tools plays a significant role in cutting force, chatter stability and hole quality in the deeping rotary drilling process. In Fig 1, we have presented the model of the drilling systems with the assembly of the drill, the fixture, the fixture base. In this system, the appearance of failure of the drilling system owes to the self-excited vibrations, self-excited vibrations of this assembly in axial, torsional directions are one of the possible reasons for their failure and damages to the hole walls, so it is essential for the workers to understand the dynamics and behavior of the drilling system when the vibration occurs, in this work, we will research the improvement of the stability of capturing and dealing with cutting dynamic characters through comprehensive consideration with chatter behaviors of machining in the drilling of the rock with the purposes of avoiding chatter-induced damages of cutting tools and scrap of work-pieces and enhancing productivity. In broad sense, drilling system vibrations can be divided into three categories-axial, torsional, lateral vibrations, these vibrations can be complicated, and in this work the coupled axial and torsional vibrations of the rotary drilling will be considered. The pre-twisted geometry of the flute section contributes to the coupling of torsional and axial vibrations of the drilling tool. When the torsional load is applied on the drilling tool, wrapping deformation of the tools cross section occurs, and results into displacements in axial direction. Natural frequency and mode shapes corresponding to the coupled torsional-axial dynamics need to be determined to avoid unnecessary tool vibrations in the operations.

During the coupled axial and torsional vibrations, the roundness, dimensional accuracy, and surface quality of the rock holes is greatly affected by the vibration of the drill bit, two modes of the vibration eventuate during deep rotary drilling- axial vibrations and torsional vibration, when vibration develops at the frequencies close to natural frequency of the drill, resonance occurs, whirling instability result in multiside holes, torsional and axial vibration lead to dimensional tolerance and wear to the tools, so it is essential to predict the stable cutting parameters to reduce losses.

In rotary drilling processes, the vibration instability due to the interactions between the drill and the rock is regarded as self-excited or regenerative chatter which is a common form of instability during drilling, it is due to the generation of surface waviness which modulates the cutting force, Bayly et al. developed a single degree of freedom model to study the torsional-axial chatter in drilling. In his work, torsional-axial stability lobes were developed and verified experimentally, they simulated the drill bit with a pre-twisted beam in the torsional-axial mode. In most cases, the rock is essentially rigid in comparison with the drill, and so the prediction of the chatter requires a measurement of the drilling-point frequency response function (FRF). To obtain the tool tip frequency response data, the measurement is taken to make with a modal hammer and collocated accelerometer, the drill is mounted on the fixture, and this approach yields good predictions of chatter stability.

In this work, the author cites the background theory together to indicate how drill stability and micro-stability could be predicted from the experimental data, and the author changes the axial damping in the drilling and micro-drilling to make a series of stability lobes. Following the research of the work, some conclusions are drawn to demonstrate technique.

## 2. Image Edge Processing Based on Canny Algorithm

### 2.1 Stability Lobes

#### 2.1.1 Dynamic Model of Drilling

In this work, we present a coupled two degrees-of-freedom model for axial and torsional vibration for analysis, Figure 1 shows the physical model as a two degrees-of-freedom system (one axial, and one torsional) to model the axial and torsional vibrations of deeping rotary drilling system. The mathematical description for the deeping rotary drilling system is following to demonstrate the interaction between the drill bit and the rock. In the drilling process, the depth of cutting is  $b$ . In the micro-drilling process, while the drill diameter and cutting width are in the same magnitude, the feed rate is expressed as  $f$ . In the axial direction, the deeping rotary drilling system is modeled as spring-mass-damper system with with spring stiffness ( $k_a$ ), viscous damping coefficient ( $c_a$ ), the combined mass ( $M$ ). For torsional direction, the deeping rotary drilling system with torsional spring stiffness ( $k_t$ ), torsional viscous damping coefficient ( $c_t$ ), Reactions from rock,  $F$ , and the reactive torque,  $T$ , are acting on the drilling bit, The rotary deeping system at the top is driven at constant angular velocity  $\Omega$ , the chip thickness is  $h$ , the mean chip thickness is  $h_m$ . Figure 1 shows the physical model as a two degrees system (one axial, and one torsional) to model the axial and torsional vibrations of the deeping rotary drilling system.

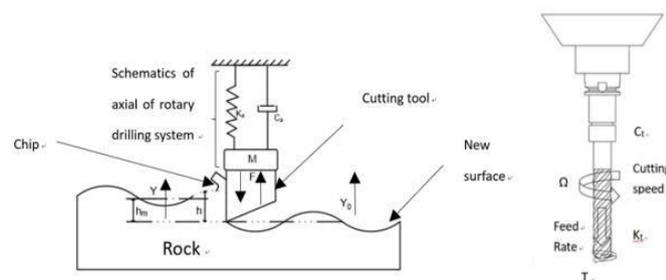


Fig 1

#### 2.1.2 Mathematical Model

To predicted the boundaries of stability for the torsional -axial chatter, the cutting and dynamics systems parameters are needed. The cutting parameters are obtained from the cutting model, and the dynamics parameters are obtained from model testing. In Fig1, a chip is removed from the surface of the rock, the surface of the rock is conceived to be rigid, while as usual, in the progress of the axial feed, The drill bit needs to overcome the resistance force from the rock due to the operation is related to the chip geometry

$$F = k_a bh$$

Where b is radial width of cut. The general equations of motion for the dynamic rotary drilling system can be formulated in the stationary frame as follows:

$$My''(t) + C_a y'(t) + k_a y(t) = M \cdot F$$

$$J\theta''(t) + C_t \theta'(t) + K_t \theta(t) = T$$

Where  $y(t)$  is axial vibration displacement,  $\theta(t)$  is torsional vibration displacement, M and J are the combined mass and rotary inertia about the rotational system

$$J = K_t / \omega_n^2$$

Note that we have ignored any wear flat on the drill-bit and consequently there are no frictional forces and torques. While we don't take the wear of the drill-bit into consideration, consequently, frictional forces and torques are ignored. The cutting force and torque on the drill-bit are related to the system and operational parameters as:

$$F_c = \epsilon \epsilon_r h H(\dot{\phi}) H(h)$$

Where  $H(\dot{\phi})$  indicate the transfer function,  $\epsilon$  is the cutter inclination coefficient,  $\epsilon_r$  is the rock specific strength, r is the radius of the drill-bit, and h is the instantaneous depth of cut per revolution of the drill-bit. Assuming that the rock is homogeneous and the drill-bit has n identical cutters, the total depth of cut per revolution can be written as

$$h = nh_n$$

$$h = d_c - d$$

where  $d_c$  is pilot hole diameter, d is tool diameter,  $h_n$  is the depth of cut per cutter

$$h_n = Y - Y_0 = y(t) - y(t - t_n)$$

Y is the current vibration of the tool from its mean position,  $Y_0$  is the vibration from the mean position for the previous pass.  $t_n$  is the time taken by the drill-bit to rotate by an angle of  $2\pi/n$  which can be computed through

$$\phi(t) - \phi(t - t_n) = 2\pi/n$$

Where  $\phi(t)$  is Spindle rotation angle

Generally in the deeping drilling process, To find the stability boundary, a frequency is chosen which is around the natural frequency,  $w = w_n$ , where r is some ratio,  $w_n$  is the natural frequency of the drill, w is the chatter frequency. For this specific frequency, the delay time between the tool passages,  $t_n$ , is found so that the imaginary part of b becomes zero. Subsequently, the speed at the limit of stability is determined from  $\Omega = \frac{2\pi w}{z(2\pi j + wt_n)}$  where z is the number of cutting edges engaged in the cut and j is the integer number of full waves. This procedure for plotting the boundary of stability can be summarized for  $z=2$  in the diagram shown in Fig.2

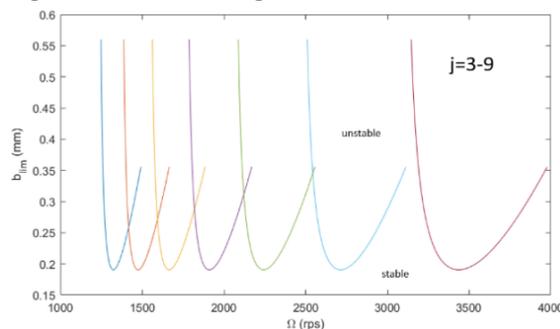


Fig.2 Chatter stability lobes due to torsional-axial vibration

This results in the stability lobe diagram, which illustrates the relationship between stable depth of cut and spindle speed, the stability lobe diagram has enormous significance in practice. The inner part of the leaflet type is unstable area, at any speed, the area below the stability lobe diagram lines are stable area.

**2.2 Modal Analysis**

A two degree of freedom system has two natural frequencies, Vibration in either of the two degree of freedom system’s natural frequencies is associated with a characteristic deformation pattern, The magnitude and phase parts of the frequency response function emphasize key features of forced vibration, the forced vibration occurs at the frequency of the exciting force; A standard procedure using a modal impact hammer and an accelerometer were used to determine the frequency response function (FRF). Modal analysis was performed on the deeping non-rotary drilling system, hammer impacts were performed directly and crossed with the accelerometer to create a modal matrix that shows tool coupling .The accelerometer was attached as close to the end of the cutting edge as possible. Removing the hammer impacts up the tool in the axial diection provided information to produce single-degree-of freedom mode shape .The torsional natural frequency was found by placing the accelerometer on one of the cutting edge ,and impacting the other cutting edge. The extensive impact tests were performed on the tool ,instrumentation, and machine structure. These test results show that the chatter frequency is related to the natural frequency of a non-rotating tool embedded into the workpiece. Fig3 shows coupled frequencies at 500 and 770HZ.

$$\text{Re}\left(\frac{Y}{F}\right) = \frac{1}{K} \left( \frac{1-r^2}{(1-r^2)^2 + (2\epsilon r)^2} \right)$$

$$\text{Im}\left(\frac{Y}{F}\right) = \frac{1}{K} \left( \frac{-2\epsilon r}{(1-r^2)^2 + (2\epsilon r)^2} \right)$$

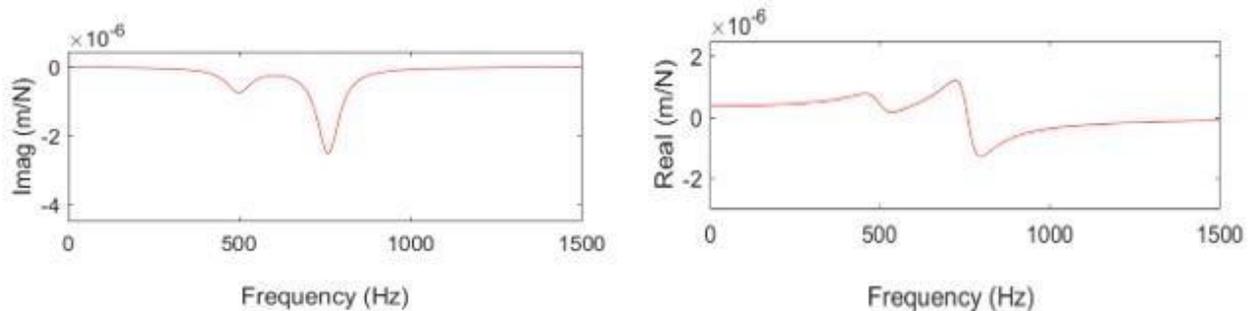


Fig 3

**2.2.1 Analytical Lobes Due to Axial Viscous Damping Coefficient in Deeping Rotary Drilling System**

One of the widely known assertion in stability analysis is that retarding damping force is proportional to the viscous damping coefficient, You may have experienced this phenomenon if you’ve attempted to make comparisons to force a body through a fluid and a gas, such as pulling your hand through water or sticking your hand out the window of a moving vehicle. The chatter occurs when the tip of the drill is supported, while the interactions between the rock and the drill-bit leads to occurrence of exciting force, The vibration speed itself will generate feedback control to the system, this theory gives better insight into the analysis of the stability lobes. If the system is between steady state and unstable state, this critical state can be called equal amplitude vibration.

$$c_a + \frac{WK_a \sin 60w/j}{w} = 0$$

The critical cutting width can be calculated

$$w_{min} = \frac{c_a}{k_a} \frac{w}{\sin 60w/j}$$

$w > w_{min}$  The sum of damping of the deeping rotary system in axial direction is less than zero, then self-excited vibration occurs. It can be concluded that  $w_{min}$  is the stability Critical value.

$$m\ddot{y}(t) + (k + wk_a B)y(t) = 0$$

The equation above becomes the equation of motion for the free vibration of an undamped system, from which the natural frequency is

$$w = \frac{\sqrt{K + WK_a B}}{m}, w^2 = \frac{k + wk_a B}{m} = w_n^2 + \frac{wk_a B}{m}$$

$w$  is the frequency of self-excited vibration,  $w_n$  is natural frequency of the system,  $k$  is the stiffness of the non-rotary drilling system. It can be seen that  $w > w_n$ .

$$c_a + \frac{wk_a C}{w} = 0$$

$$w_n^2 + \frac{wk_a B}{m} = w^2$$

Assuming that  $\frac{w}{\sin 60w/j} = -1$ , then the minimum cutting width can be expressed as

$$w_{min} = \frac{c_a w}{k_a}$$

the minimum cutting width is proportional to the axial viscous damps, changing the axial damping coefficient affects the cutting stability lobes. The minimum cutting width increases with the increase of the axial viscous damping coefficient. Fig.4 shows the stability lobes with axial viscous damping coefficient of 0.3, 0.4, 0.5.

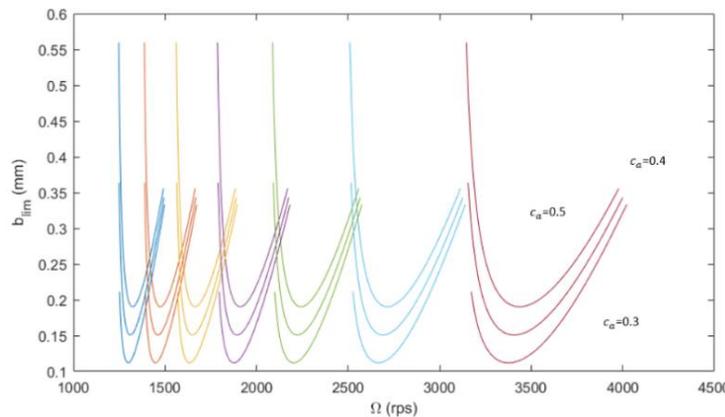


Fig.4 Comparison of Chatter stability lobes due to the changes of the axial viscous coefficient in drillnig

### 3. Conclusion

In this paper, We have established a two-degree-of-freedom drilling model(axial and torsional) , presenting modal analysis of non-rotating parts of the rotary drilling , and we obtain the two natural coupled frequencies at 500 and 770Hz via the modal testing we obtained the Machine dynamics of deeping rotary drilling has been studied , This paper focuses on the prediction of drilling stability using the method available in the earlier literature We have established stability lobes for macro drilling and of rocks, presented a detailed analysis of the deeping rotary drilling system, the influence of changing the axial damping coefficient on the critical rotating speed during drilling is analyzed in macro drilling .Overall we have studied fairly rich machining dynamics for deeping rotary drilling system .In this study, the axial viscous progress damping between the drilling tools and the rock is obtained.This rubbing mechanism tends to dampen the vibration and it plays an important role in the stability of the tools.And also, another parameter play a significant role in damping vibration is the chisel edge, but in

the present study ,the piloted holes were used and the impact of the chisel edge was excluded. Therefore, the parameters of this paper will have great research value in the future.

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