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## Dynamic Analysis and Optimization of a Vibration Isolation System for Knapsack Spray Duster

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### Abstract

As a small farm implement, the spray duster has been widely used in China. This paper aims at the engineering research problem of vibration isolation design and structure optimization of a knapsack spray duster. The theoretical modeling calculation and vibration modal test are used to analyze the dynamic characteristics, and the parameters of the system's force transmission rate are optimized for the spray duster. Vibration and noise reduction design provides a reference. First, consult the literature and carry out on-site investigations to understand the research status and development trend at home and abroad. Use 3D mapping software to draw the 3D solid model of the whole machine. According to its actual characteristics, establish and simplify the vibration isolation model, and list the vibration equations of the vibration isolation system. Find the first six natural frequencies. Using the admittance transfer matrix method and the generalized four-terminal parameter method to establish the relationship between the parameters of the knapsack sprayer vibration isolation system lay the foundation for the following[1]. Secondly, based on the dynamic model, the power flow analysis is performed to analyze the influence of the parameters of the vibration isolation system on the basic transmission power flow. Based on the stiffness and damping of the vibration isolation system as optimization variables, optimization was carried out using the MATLAB optimization toolbox. After optimization, the power flow transmitted to the foundation was significantly reduced, the resonance peak was also significantly reduced, and the amplitude at the power frequency was reduced from -108.2 dB to -115.3dB decreased by 6.5%, which shows that the optimized vibration isolation system has greatly improved its energy absorption capability.[2] Thirdly, The finite element modal analysis of the whole machine was performed by ANSYS Workbench, and the whole machine experimental modal analysis was performed using LMS Test.Lab. The relative errors of the first two natural frequencies obtained by the theoretical modeling calculations and the modal tests are less than 5% within the allowable range, which proves the reliability of the theoretical modeling and also proves the validity of the parameter optimization performed in this way; The error of the first six natural frequencies of the two modal analyses is less than 5% within the allowable range, and the first six modes are consistent, which proves the accuracy of the finite element model and provides the results for the next harmonic response analysis and verification optimization. A good foundation. In addition, the fourth-order natural frequency is close to the operating frequency of 142Hz and should be reinforced at the engine cover.[3]. Finally, harmonic response analysis was used to verify the effect of vibration reduction and noise reduction after optimization. The vibration curve after optimization was significantly reduced compared with the whole before optimization. The amplitude at the frequency of 142Hz was reduced from 4.1056e-4m to 1.0233e-4m, a decrease of 75%, and the damping effect was obvious, indicating that the parameter optimization result based on the theoretical model was effective[4]. The optimized noise curve is significantly lower than before optimization. After optimization at 142Hz, the sound pressure level is reduced from

86.5dB to 62.1dB, and the sound pressure level is reduced by 28.2%. This shows that the noise reduction effect is obvious and the optimization result is effective.

## Keywords

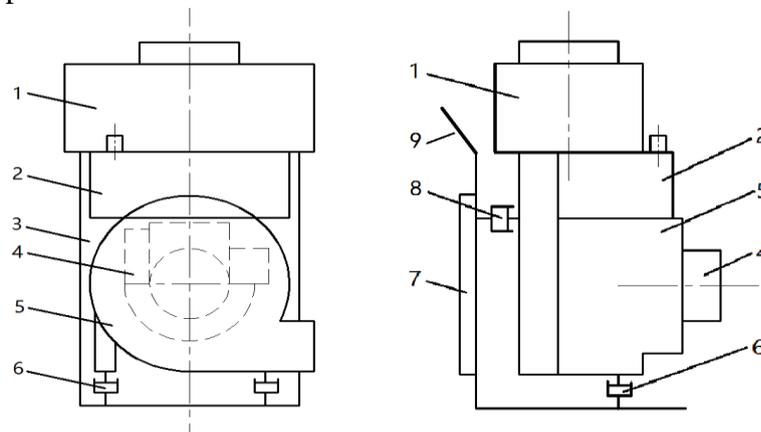
Spray duster, dynamic analysis, optimization, MATLAB, mode.

## 1. Rigid Body Dynamic Modeling of a Backpack Spray Powder Blower

### 1.1 Vibration Isolation Model of Back Spray Powder Spray Machine

A certain type of knapsack spray duster is used as the research object to understand the composition and working principle of the mechanism. It mainly consists of a single cylinder engine, a fan, a medicine box and an oil tank. The principle of it is as follows: the engine drives the fan to spin the high pressure air, and the high pressure air passes through the pipeline to spray out the liquid and powder. When working, the engine is the main source of vibration, and the fan pulsation is the secondary vibration source. [5]The fan and the engine are connected to the frame through the vibration isolator, the other side of the frame is attached to the back contact with the operator, and the vibration source and frame, and the corresponding vibration isolator and back cushion form a complex vibration isolation system. According to its working principle and vibration source analysis, the next step is to simplify the complex structure of the knapsack spray duster to a more simple and straightforward kinetic model.

Figure 1 is a simplified model of the structure of a backpack sprayer. The belt is connected to the frame. The fan and the engine are connected to the frame by three isolators, including two horizontal vibration isolators and two vertical directional isolators. The frame is in a horizontal direction with a large area back contact with the operator's back.



1-Medicine chest,2-Tank,3-Frame,4-Engine,5-Fan,6-Vibration isolators A,7-Back pad,8- Vibration isolators B,9-Strap

Fig. 1 The simplified model of knapsack sprayer duster

According to the actual working state of the mechanism, the back type spray powder sprayer is equivalent to a mechanical model. The intermediate mechanism consisting of a tank, a medicine box, an engine and a fan can be considered as a rigid body. The mass is concentrated at the center of mass O without considering its elasticity. The rigid body passes the elastic column in the horizontal Y direction and the vertical Z direction. The vibration isolator is connected with the frame. The frame is connected in the horizontal Y direction and the vertical Z direction through the elastic column type isolator and the elastic foundation (human body). Figure 2 is the dynamic model of the vibration isolation system of

the back mounted spray powder machine, in which figure 2 (a) is a positive picture, and figure 2 (b) is left view.

The vibration isolation system of the back type spray blower is subjected to the effect of simple harmonic exciting force and simple harmonic exciting force in the direction of X axis and Z axis respectively. L1 and L2 are the distance between the mass center and the two spring along the Z axis respectively. The spring stiffness of the two springs along the Z axis is the same as K1, the spring damping is C1, the back belt is elastic and the spring two spring is the same spring stiffness. For K5, the spring damping is C5, the spring along the Y axis, its spring stiffness is K2, and the spring damping is C2. The backing pad is regarded as three springs with stiffness of K3 and K4, and the spring damping is C3 and C4.

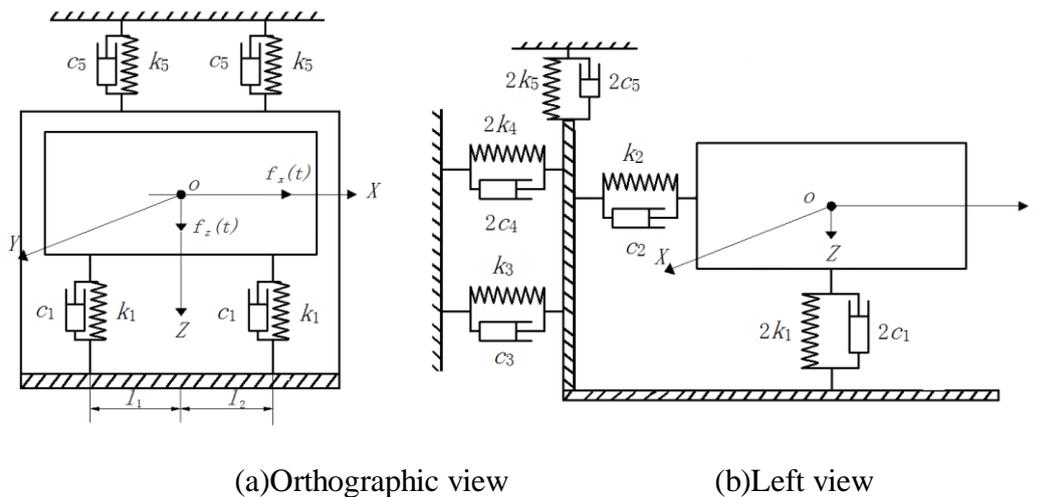


Fig. 2 The dynamics model of knapsack sprayer duster

### 1.2 Simplification of Vibration Isolation Model of Back Spray Powder Spray Machine

The analysis of the structure and working principle of the backpack spray blower can be seen that the movement of the back type spray powder sprayer in space mainly includes the linear motion in the direction of the intermediate mechanism along the X axis, the Y axis and the Z axis, and the linear motion of the frame along the direction of the X axis, the Y axis and the Z axis. Through the study of the motion characteristics of the mechanism, it is found that the linear motion along the X axis and the linear motion along the direction of the Y axis and the linear motion along the direction of the Z axis are relatively small, and the coupling degree is relatively low. The correlation and coupling degree of the line motion is very large. The spring in the direction of the Y axis is mainly connected to the support function, and the spring damping effect is small. Therefore, the dynamic model of the vibration isolation system can be simplified as a simplified model without the spring in the direction of the Y axis, and the vibration along the direction of the X axis can not be felt for the user. Obviously, the straight line movement of the first unit of the backpack sprayer is not considered in the X axis direction. To sum up, only the vertical straight line motion of the intermediate mechanism and the frame in the direction of the Z axis is considered. As the frame is a whole, the C5 and K5 are connected to the chassis floor. The dynamic model of the knapsack spray duster is simplified to the two degree of freedom vibration isolation system model shown in Figure 3.

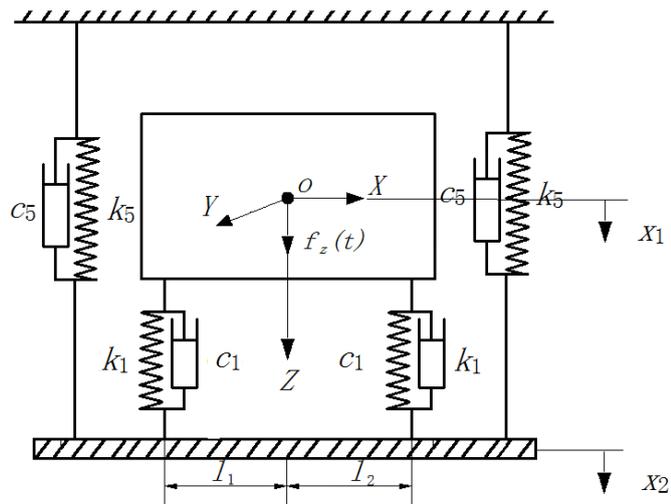


Fig. 3 The simplified dynamics model of knapsack sprayer duster

**1.3 Vibration Equation and Natural Frequency of Vibration Isolation System**

Based on Newton's second law, the equation of vibration is established.

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 - c_1\dot{x}_2 + k_1x_1 - k_1x_2 = f_z(t) \\ m_2\ddot{x}_2 - c_1\dot{x}_1 + (c_1 + c_5)\dot{x}_2 - k_1x_1 + (k_1 + k_5)x_2 = 0 \end{cases} \quad (1)$$

The parameters of the vibration isolation system for knapsack sprayer duster are shown in table 1.

Table 1. Knapsack sprayer vibration isolation system component data

Component name	Quality (kg)	Rigidity(N/m)	Damping( $\eta$ )
Intermediate mechanism	4.8	$+\infty$	0
First layer vibration isolator	$-\infty$	3400	0.1
Frame	2.9	$+\infty$	0
Second layer vibration isolator	$-\infty$	3400	0.1

$\eta$  is a damper C is a damping factor

Formula (1) can be written as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f(t)\} \quad (2)$$

$[M], [C], [K]$  are the mass matrix, damping matrix and stiffness matrix of the structure, which represent the external excitation force of the system.  $\{x\}, \{\dot{x}\}, \{\ddot{x}\}$  represent the column vectors of the displacement, velocity and acceleration of the system respectively.

When the system has no damping and external forces, the original system is reduced to a two degree of freedom undamped free vibration system, and its motion differential equation is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (3)$$

In order to write out the general equations of motion for vibration systems, the formula (3) of  $K_{11}=k_1, K_{12}=K_{21}=-k_1, K_{22}=k_1+k_5, M_{11}=m_1$  and  $M_{22}=m_2$  is rewritten.

$$\begin{bmatrix} M_{11} & 0 \\ 0 & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (4)$$

The formula (4) is a homogeneous constant coefficient linear differential equation set. The displacement X1 and X2 have the same form solution. It is assumed that the displacement of the plastid of the equation varies according to the sine law, and the frequency is, the initial phase angle is, the amplitude is A and B respectively, then the displacement is

$$\begin{cases} x_1 = A \sin(\omega_n t + \varphi_0) \\ x_2 = B \sin(\omega_n t + \varphi_0) \end{cases} \tag{5}$$

The acceleration is replaced by the two derivative of the displacement, and the substitution is replaced by (4) and the  $\sin(\omega_n t + \varphi_0)$  is eliminated.

$$-\omega_n^2 \begin{bmatrix} M_{11} & 0 \\ 0 & M_{22} \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{6}$$

The upper form becomes the characteristic matrix equation, where the amplitude array composed of displacement amplitude is called the amplitude vector. Since the amplitude is not 0, if the equation is to be established, the determinant of the coefficient of the amplitude vector must be equal to 0.

$$\Delta(\omega_n^2) = \begin{vmatrix} k_{11} - \omega_n^2 m_1 & k_{12} \\ k_{21} & k_{22} - \omega_n^2 m_2 \end{vmatrix} = 0 \tag{7}$$

The upper form is the frequency equation, whose root is the eigenvalue of the system, which is only related to the mass and stiffness. It depends on the physical characteristics of the system and is called the natural frequency of the system. Resonance occurs when the excitation frequency is close to the natural frequency. Knapsack spray duster operates at a frequency of  $f = 142\text{Hz}$  and an angular frequency of  $\omega = 2\pi f = 892\text{rad/s}$ . Each parameter is replaced by a formula to obtain natural frequencies as shown in table 2, and  $\frac{\omega}{\omega_0}$  is greater than  $\sqrt{2}$  to meet design criteria.

Table 2. Knapsack spray duster vibration isolation system natural frequency

order	1	2
$\omega /(\text{rad/s})$	190.14	453.102
$f /\text{Hz}$	30.277	72.015

## 2. Dynamic Modeling of Flexible Foundation Vibration Isolation System for Knapsack Spray Duster

The dynamic modeling process of the vibration isolator of the back-type spray powder spray machine is essentially a process of establishing the dynamic relationship of the system with the important physical parameters (mass, position, stiffness and damping, etc.) in the vibration isolation device. This also explores the course of the basic problems of the vibration isolation device, and the mathematical equation is the connection. A bridge between the input and output of the system.

At present, there are many ways to create dynamic models. Among them, the four end parameter method, the admittance synthesis method, the multi rigid body dynamics, the transfer wave method, the modal impedance synthesis method and the finite element dynamic method are widely used. This paper mainly uses the admittance transfer matrix method and the generalized four terminal parameter method to establish the dynamic model of the vibration isolation device of the backpack spray duster.2.3.1

### Admittance synthesis method

The admittance synthesis method is one of the main means to analyze the dynamic characteristics of composite components at present. This method is also a special case of the dynamic substructure method. In this method, a complete system is dismantled and divided into multiple subsystems or

substructures, then the theoretical analysis and experimental measurement of the mechanical impedance / admittance method and the dynamic equations of each subsystem are created. Secondly, the constraint conditions of the coupling interface are determined according to the actual situation of the coupling interface between the systems; finally, the constraints of the coupling surface are determined. Finally, the constraints of the coupling interface are determined. Through these constraint equations, the equations of motion of each sub structure are coupled, and the motion equations and dynamic characteristics of the whole system are obtained.

The following concepts are briefly introduced: mechanical admittance, mechanical transfer admittance and mechanical impedance.

#### (1) mechanical admittance

The mechanical admittance is the ratio of the speed exerted on the whole device to the excitation force causing the velocity. If the measuring point is the action point of the exciting force, the ratio of the velocity to the force is called the mechanical action point admittance  $M$ , which can be expressed as:

$$M = \frac{V}{F} \quad (8)$$

In the formula,  $F$  is the force, and  $V$  is the velocity caused by the force direction. The quantity  $F$  and  $V$  are given in the plural form.

#### (2) Mechanical transfer admittance

The speed of the other points of the device (which can be different from the direction of the applied force) and the ratio of the force to it are called mechanical transfer admittance.

$$M_{12} = \frac{V_2}{F_1} \quad (9)$$

In the formula,  $F_1$  is the excitation force exerted at a certain point, and  $V_2$  is the velocity caused by the force at another point.

#### (3) Mechanical impedance

The reciprocal of mechanical admittance is called mechanical impedance. The impedance of the action point can be expressed as:

$$Z = \frac{F}{V} \quad (10)$$

### 2.1 Four End Parameter Method

The four terminal parameter method is a commonly used research method in the modeling of mechanical systems. This method is based on the basic definition of network theory in electrical systems. The four end parameters of a mechanical system depend only on the dynamic characteristics of the mechanical system and are independent of the whole structure of the mechanical system. Therefore, this method is very suitable for the analysis and research of the composite components.

An elastic system can be represented by a generalized linear mechanical system with input  $a$  and output  $B$ . Fig. 4 is a schematic diagram of the system. Among them, the mechanical system is composed of a number of ideal components ( $m$ ,  $K$ ,  $c$ ), or some linear parameter distribution systems. The input force and input speed of input  $a$  are  $F_a$  and  $V_a$  respectively. The two part is the former part of the system, which is connected by the connection point  $a$ . The output power and output speed of the output  $B$  are  $F_b$  and  $V_b$  respectively. They are [23] due to the role of  $F_a$  and  $V_a$  and the reaction of the subsequent parts of the system. The direction of the arrow in the diagram is the direction of the energy flow generated by the vibration source. In general, the mechanical system in Fig. 4 can be expressed by a set of linear equations as follows:

$$\begin{cases} F_a = \alpha_{11}F_b + \alpha_{12}v_b \\ v_a = \alpha_{21}F_b + \alpha_{22}v_b \end{cases} \tag{11}$$

The coefficient  $\alpha_{ij}$  is called the four end parameter, and can be obtained:

$$\begin{aligned} \alpha_{11} &= \left. \frac{F_a}{F_b} \right|_{v_b = 0}; & \alpha_{12} &= \left. \frac{F_a}{v_b} \right|_{F_b = 0}; \\ \alpha_{21} &= \left. \frac{v_a}{F_b} \right|_{v_b = 0}; & \alpha_{22} &= \left. \frac{v_a}{v_b} \right|_{F_b = 0}; \end{aligned} \tag{12}$$

The  $v_b=0$  in the formula indicates that the output B is fixed, and the  $F_b=0$  indicates that the output terminal is free.  $\alpha_{ij}$  is actually the impedance parameter of the mechanical system. The  $\alpha_{ij}$  value can be calculated by theory and can be obtained by experiment.

The four end parameters of the ideal original are easily obtained.

quality

$$[\alpha_{ij}] = \begin{bmatrix} 1 & im\omega \\ 0 & 1 \end{bmatrix} \tag{13}$$

Spring

$$[\alpha_{ij}] = \begin{bmatrix} 1 & 0 \\ \frac{i\omega}{k} & 1 \end{bmatrix} \tag{14}$$

damper

$$[\alpha_{ij}] = \begin{bmatrix} 1 & 0 \\ \frac{1}{C} & 1 \end{bmatrix} \tag{15}$$

For the complex stiffness matrix,  $k^* = k(1 + i\eta)$  has:

$$[\alpha_{ij}] = \begin{bmatrix} 1 & 0 \\ \frac{i\omega}{k^*} & 1 \end{bmatrix} \tag{16}$$

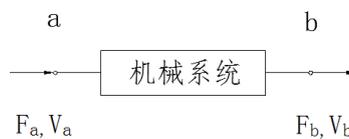


Fig. 4 Generalized linear mechanical system

### 3. System Transfer Matrix

The four terminal parameter model of the knapsack spray duster isolation system is shown in Figure 4. There are five substructures in the system. The force and velocity of each coupling interface of the vibration isolation system can be obtained from the admittance matrix of the substructure.

From the above, we can know that the generalized four terminal parameter method is a common and simple method for solving the dynamic parameters of series or parallel combination systems.

The generalized four terminal parameter method is applied to the intermediate mechanism, and is expressed by the relation between its force and speed response.

$$\begin{bmatrix} \mathbf{V}_I \\ \mathbf{V}_A^b \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}_I \\ \mathbf{F}_A^b \end{bmatrix} \tag{17}$$

The generalized four terminal parameter method is applied to the first level isolator, and it is expressed by the relation between force and velocity response.

$$\begin{bmatrix} \mathbf{F}_B^t \\ \mathbf{V}_B^t \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{G}_B & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}_B^b \\ \mathbf{V}_B^b \end{bmatrix} \tag{18}$$

Among them,  $\mathbf{I}$  and  $\mathbf{O}$  are unit matrix and zero matrix, respectively.

The generalized four terminal parameter method is applied to the frame, and is expressed by the relation between its force and velocity response.

$$\begin{bmatrix} \mathbf{V}_C^t \\ \mathbf{V}_C^b \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{11} & \mathbf{C}_{12} \\ \mathbf{C}_{21} & \mathbf{C}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}_C^t \\ \mathbf{F}_C^b \end{bmatrix} \tag{19}$$

The generalized four terminal parameter method is applied to the second level vibration isolator, and is expressed by the relation between its force and velocity response.

$$\begin{bmatrix} \mathbf{F}_D^t \\ \mathbf{V}_D^t \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{G}_D & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{F}_D^b \\ \mathbf{V}_D^b \end{bmatrix} \tag{20}$$

The relationship between force and speed response is expressed as follows:

$$\mathbf{V}_E^t = \mathbf{E} \cdot \mathbf{F}_E^t \tag{21}$$

In the form:

the subscript  $t$  is input to the substructure;

The next table  $b$  is output to the substructure;

The transfer matrix of  $\mathbf{A}$  intermediate mechanism;

$\mathbf{G}_B$  and  $\mathbf{G}_D$  the total stiffness matrix of the first level vibration isolator and the second level isolator.

The admittance transfer matrix of the  $\mathbf{C}$  frame;

The admittance transfer matrix of the  $\mathbf{E}$  base;

The boundary conditions of each substructure satisfy the continuity of force and velocity on the boundary surface show in next figure :

$$\begin{bmatrix} \mathbf{F}_A^b \\ \mathbf{V}_A^b \end{bmatrix} = \begin{bmatrix} \mathbf{F}_B^t \\ \mathbf{V}_B^t \end{bmatrix} = \begin{bmatrix} \mathbf{F}_A \\ \mathbf{V}_A \end{bmatrix}, \begin{bmatrix} \mathbf{F}_B^b \\ \mathbf{V}_B^b \end{bmatrix} = \begin{bmatrix} \mathbf{F}_C^t \\ \mathbf{V}_C^t \end{bmatrix} = \begin{bmatrix} \mathbf{F}_B \\ \mathbf{V}_B \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{F}_C^b \\ \mathbf{V}_C^b \end{bmatrix} = \begin{bmatrix} \mathbf{F}_D^t \\ \mathbf{V}_D^t \end{bmatrix} = \begin{bmatrix} \mathbf{F}_C \\ \mathbf{V}_C \end{bmatrix}, \begin{bmatrix} \mathbf{F}_D^b \\ \mathbf{V}_D^b \end{bmatrix} = \begin{bmatrix} \mathbf{F}_E^t \\ \mathbf{V}_E^t \end{bmatrix} = \begin{bmatrix} \mathbf{F}_D \\ \mathbf{V}_D \end{bmatrix} \tag{22}$$

Substituting formula (18), (19) and (20) into boundary conditions (22), we can find out:

$$\mathbf{V}_C^b = \mathbf{C}_{21}\mathbf{F}_A + \mathbf{C}_{22}\mathbf{F}_D \tag{23}$$

Substituting formula (19), (20) and (21) into boundary conditions (22), we can find out:

$$\mathbf{V}_C^b = (\mathbf{E} + \mathbf{G}_D)\mathbf{F}_D \tag{24}$$

By formula (23) and formula (24), we can find out:

$$\mathbf{F}_D = (\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21} \mathbf{F}_A \tag{25}$$

Simultaneous (17), (18), (19), (20), (22) and (25) can be obtained:

$$\mathbf{F}_A = \mathbf{F}_B = [(\mathbf{C}_{11} + \mathbf{G}_B - \mathbf{A}_{22}) + \mathbf{C}_{12}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21}]^{-1} \mathbf{A}_{21} \mathbf{F}_I \tag{26}$$

By formula (25) and (26), we can find out:

$$\mathbf{F}_D = (\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21} [(\mathbf{C}_{11} + \mathbf{G}_B - \mathbf{A}_{22}) + \mathbf{C}_{12}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21}]^{-1} \mathbf{A}_{21} \mathbf{F}_I \tag{27}$$

In the formula (26) substituting (17), we can find out:

$$\mathbf{V}_I = \{\mathbf{A}_{11} + \mathbf{A}_{12} [(\mathbf{C}_{11} + \mathbf{G}_B - \mathbf{A}_{22}) + \mathbf{C}_{12}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21}]^{-1} \mathbf{A}_{21}\} \mathbf{F}_I \tag{28}$$

The formula (27) is substituted (21) and combined (22).

$$\mathbf{V}_D = \mathbf{E}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21} [(\mathbf{C}_{11} + \mathbf{G}_B - \mathbf{A}_{22}) + \mathbf{C}_{12}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21}]^{-1} \mathbf{A}_{21} \mathbf{F}_I \tag{29}$$

The formula (22), (26) and (27) are substituted in (19).

$$\mathbf{V}_B = [\mathbf{C}_{11} + \mathbf{C}_{12}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21}] [(\mathbf{C}_{11} + \mathbf{G}_B - \mathbf{A}_{22}) + \mathbf{C}_{12}(\mathbf{E} + \mathbf{G}_D - \mathbf{C}_{22})^{-1} \mathbf{C}_{21}]^{-1} \mathbf{A}_{21} \mathbf{F}_I \tag{30}$$

Through the above deduction, we get the expression of the dynamic input parameters of the intermediate mechanism, the frame and the base, that is the expression of the excitation force and the excitation speed of each input surface (from the next chapter, it is known that these parameters are the dynamic parameters of the power flow transfer equation). Therefore, these derivations can lay a foundation for the derivation of the power flow transfer equation. At the same time, we know that when the admittance matrix at the interface is obtained, these admittance matrices can be replaced in the formula (19) to (26), and the expressions of the specific forces and velocities of the coupling interfaces can be obtained.

#### 4. Admittance Matrix of Substructure

In the dynamic model of the vibration isolator of the back spray powder spray machine, it is assumed that the intermediate mechanism and the frame are rigid body, the foundation is elastic, and the vibration isolator has structural damping characteristics, and the quality of the isolator is ignored. For the admittance matrix of the ideal element (mass, spring and damper), it is easy to find the [24-25] by the four end parameter method, and the specific derivation can be referred to the relevant literature. The admittance transfer matrix of the five molecular structures can be deduced on the basis of ideal components. For non ideal elements, the admittance matrix can be derived from the simplified idea of series and parallel connection.

##### (1) Admittance matrix of intermediate mechanism

For the intermediate mechanism, the kinetic equation of the intermediate mechanism can be derived from the rigid body motion theory.

$$\begin{bmatrix} \mathbf{V}_I \\ \mathbf{V}_A^b \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{F}_I \\ \mathbf{F}_A^b \end{bmatrix} \tag{31}$$

Among them,  $\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$  is the intermediate admittance matrix, which is:

$$\mathbf{A}_{11} = - \begin{bmatrix} 1/\bar{M} + L_{11}^2/\bar{J} & 1/\bar{M} + L_{11}L_{12}/\bar{J} & \cdots & 1/\bar{M} + L_{11}L_{1N}/\bar{J} \\ & 1/\bar{M} + L_{12}^2/\bar{J} & \cdots & 1/\bar{M} + L_{12}L_{1N}/\bar{J} \\ & & \ddots & \vdots \\ & sym. & & 1/\bar{M} + L_{1N}^2/\bar{J} \end{bmatrix}_{N \times N}$$

$$\mathbf{A}_{12} = \begin{bmatrix} 1/\bar{M} \\ 1/\bar{M} \\ \cdots \\ 1/\bar{M} \end{bmatrix}_{N \times 1}, \quad \mathbf{A}_{22} = 1/\bar{M}, \quad \bar{M} = j\omega M, \quad \bar{J} = j\omega J$$

In the formula, M and J are the mass and moment of inertia of the intermediate mechanism, A is the excitation frequency, and  $L_{ij}$  ( $i=1,2,\dots,n, j=1,2,\dots,N$ ) is the local coordinate of the isolator.

(2) Admittance matrix of first layer vibration isolator

Because the quality of vibration isolator is smaller than that of intermediate mechanism and rack, it can be ignored. Therefore, it can be calculated without considering the self mass of the isolator, and can be regarded as a spring damping system.

For isolators, which are parallel to springs and dampers, it can be regarded as a spring with complex stiffness.

$$k^* = k(1 + i\eta) \tag{32}$$

According to the formula (16) the first floor single vibration isolator complex stiffness matrix is expressed as:

$$G_1 = \begin{bmatrix} 1 & 0 \\ \frac{i\omega}{k_1(1+i\eta)} & 1 \end{bmatrix} \tag{33}$$

Because the first level isolator has 2, the total complex stiffness matrix of the first layer isolator is:

$$G_B = \begin{bmatrix} G_1 & 0 \\ 0 & G_1 \end{bmatrix}$$

(3) Admittance matrix of the frame

The rack is a connecting link, which plays a key role in the study of the vibration isolation device of the whole knapsack spray duster.

The relationship between input and output power and speed of the rack is shown in formula (19):

$$\mathbf{C}_{11} = \begin{bmatrix} L_{t1}^2/\bar{J} + 1/\bar{M} & L_{t1}L_{t2}/\bar{J} + 1/\bar{M} & \cdots & L_{t1}L_{tr}/\bar{J} + 1/\bar{M} \\ & L_{t2}^2/\bar{J} + 1/\bar{M} & \cdots & L_{t2}L_{tr}/\bar{J} + 1/\bar{M} \\ & & \ddots & \vdots \\ & sym. & & L_{tr}^2/\bar{J} + 1/\bar{M} \end{bmatrix}$$

$$\mathbf{C}_{21} = \begin{bmatrix} L_{t1}L_{b1}/\bar{J} + 1/\bar{M} & L_{t2}L_{b1}/\bar{J} + 1/\bar{M} & \cdots & L_{tn}L_{b1}/\bar{J} + 1/\bar{M} \\ & L_{t2}L_{b2}/\bar{J} + 1/\bar{M} & \cdots & L_{tn}L_{b2}/\bar{J} + 1/\bar{M} \\ & & \ddots & \vdots \\ & sym. & & L_{tn}L_{bN}/\bar{J} + 1/\bar{M} \end{bmatrix}$$

$$\mathbf{C}_{12} = -\mathbf{C}_{21}^T$$

$$\mathbf{C}_{22} = \begin{bmatrix} L_{b1}^2/\bar{J} + 1/\bar{M} & L_{b1}L_{b2}/\bar{J} + 1/\bar{M} & \dots & L_{b1}L_{br}/\bar{J} + 1/\bar{M} \\ & L_{b2}^2/\bar{J} + 1/\bar{M} & \dots & L_{b2}L_{br}/\bar{J} + 1/\bar{M} \\ & & \ddots & \vdots \\ \text{sym.} & & & L_{br}^2/\bar{J} + 1/\bar{M} \end{bmatrix}$$

$M$  and  $J$  are in turn the quality and moment of inertia of the rack, and  $L_{i1}$  and  $L_{i2}$  are the local coordinates of the first and second layers of isolators in the frame respectively.

## 5. Conclusion

This chapter establishes and simplifies the structural model of the vibration isolation device of the knapsack spray duster, lists the vibration equation and solves the natural frequency. A variety of vibration isolation modeling methods are introduced. The mathematical relationship between the parameters of the vibration isolation system of the back type spray powder jet machine is established by the admittance transfer matrix method and the generalized four end parameter method, which lays the foundation for the study of the power flow in the third chapter. The conclusion of this chapter is that the vibration isolation model is created and simplified, and the rigid body dynamic model is established by the actual structure of the vibration isolator. The two natural frequencies of the vibration isolation system are 30.277Hz and 72.05Hz, respectively.

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