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# A New Design of Toll Plaza on Highway Based on Cellular Automation Algorithm

Congwen Jiang, Shiping Wang, Lina Liu<sup>a,\*</sup>

School of Electronic and Information Engineering, Soochow University, Soochow 215006, China,

<sup>\*,a</sup> lln@suda.edu.cn

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## Abstract

With today's increasing number of vehicles, traffic congestion is a troubling problem. Especially on the highway, there are many tollbooths that will easily cause traffic jams. Without removing toll booths, factors such as the shape of the toll plaza and the vehicle's driving rules will also affect traffic efficiency. At the same time, the toll station construction costs are expensive. To reduce costs, the toll station square area should be as small as possible. So, the design of toll plaza is an extremely important issue that draws attention from many countries. In this article, we mainly develop a model base on the cellular automation to simulate the actual situation and determine the size and shape of the plaza, which can maximize the traffic efficiency at the lower cost. With the traffic volume almost unchanged every hour, our plan has resulted in a significant reduction in toll plaza area comparing with the traditional model.

## Keywords

Toll plaza; Cellular automation; Traffic flow; Ways of charging.

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## 1. Introduction

It is well-known that cellular automation is a common method to simulate the traffic system, and here we use this model to study the problem. Before our study, there are many researches about the efficiency of toll plazas. Someone studied the problem Base on Generalized Optimization in [1] and someone studied that base on cellular automation. In [2], they discuss the optimal layout of the toll booths to reach an efficient traffic level. They also consider the charge rules and the energy consumption of the traffic flow. But none of them considered the impact of the shape of the square and the road on traffic efficiency. In our paper, we main design the shape of the square and the road to reach a high efficient traffic. The smaller area of toll plaza will reduce more costs. We simulated the vehicle flow based on the classical NaSch model proposed by Nagel and Schreckenberg in 1992 in [3]. Firstly, we set the movement rules for vehicles under the normal traffic flow. In this rule, we set up three fast lanes and four slow lanes based on actual vehicle movements. Meanwhile we do not give restrictions on the boundary of the toll plaza, then vehicles can travel freely according to the given rules. Through simulation, we calculated frequencies of each cell occupied by cars in a certain period of time. According to this, we obtained two important parameters: the size and shape of toll plaza. Secondly, we limit vehicles to move in the boundary of toll plaza that we have figured out. Modify the program and observe the congestion level of each lane, we found that there will not be much impact. Finally, we determined the boundary of plaza: its edge should be a smooth curve, rather than a straight line as most roads are. Compared with the traditional model, we found from our result that the plaza area was reduced by 15.8%, while the number of vehicles per hour only decreased by 2.2%.

## 2. General Assumptions

- The size and shape of the car are the same.
- There are 7 lanes from toll stations to the main roads. The 1st, 2nd and 3rd lane are main roads.
- We divide the road into many grids. The length of each cell is 5 meters. Each car occupies one grid.
- The speed of the car is discrete. Vehicles on the main roads can speed up to 10 m/s (two grids), while vehicles on other lanes can only move at 5 m/s (one grid).
- The vehicle speed is appropriate, the distance between the car and the car is within a safe distance.

## 3. Notations and Symbol Descriptions

### 3.1 Notations

- Degree of traffic congestion: The sum of all cars' waiting time on 4th lane to 7th lane because of traffic jams (in 10 minutes). When the value approaches 600s, the traffic condition began to become significantly crowded.
- Throughput: Number of vehicles per hour passing the point where the end of the plaza joins three outgoing traffic lanes
- Charging ways: Conventional tollbooths, exact-change tollbooths and electronic toll collection tollbooths.
- Cells: Essential element. One cell is one grid.

## 4. A Simulation Model: Cellular Automaton

### 4.1 Model Overview

We regard the road as a flat space, then divide it equally into grids of the same size. We regard each vehicle as an idealized particle of the same size and shape. Each lattice is either empty, or prepared to occupy a car. If the last step of a car in front of the nearest neighbor grid point is empty, the next step the car forward a grid point, otherwise the car does not move, even if the vehicle in front at this time to leave, the car does not move. The whole system uses periodic boundary conditions to keep the number of vehicles conservation. This is similar to the cellular automata model in [4].



Figure 1. Cellular automaton model diagram

### 4.2 Building and solving of the model

#### 4.2.1 General description

Our model is based on the needs of each driver, that is, they want to be safe and rapid integration into the main road after leaving the tollbooths. However, due to a number of reasons, some can turn around, while others can only continue going straight along the original direction. Therefore, our model mainly consider from two cases: the “straight line” and “change lane”[5].

First of all, we set out the scope of the study. We only study the road in one direction. There are seven lanes, and the distance from the toll to the main road is certain and long enough. Now the road is divided into (35, 7) cells, that is, a total of 35 rows, 7 columns. Define a matrix as follows. Each cell is an element of the matrix and each element has  $(i, j)$  value cells.

Each element in the matrix can take -1, 0, 1, and 2. On the main road, speed can be accelerated (this is more realistic). (As is shown in the fig2 and fig3)

## 4.2.2 Notations:

 $v$ : Velocity of each car $i$ : The number of rows $j$ : The number of columns $v'$ : The velocity of the next step $p$ : The probability of lane change $s$ : The length of a grid

M; Degree of traffic congestion

Q:Traffic flow.

 $v = -1$ : There is no car $v = 0$ : The car is still $v = 1$ : The normal uniform speed, 1grid/s $v = 2$ : Accelerated speed of the car (only on main lanes), 2grids/s $a \rightarrow b$  (such as  $2 \rightarrow 1$ ): From a lane to b lane (From 2nd lane to 1st lane)

## 4.3 Algorithm Descriptions

## 4.3.1 Two cases: the “straight line” and “change lane”.

There are three basic rules in the model.

Acceleration rules: drivers always want to be able to travel at maximum speed, so if a car does not reach the maximum speed, it will increase the speed. This will also lead to other car's speed changes.

Prevent collision: if the speed of a car is more over the number of empty cells in front of it, it will slow down.

Location update: the simulation car goes forward, and its speed will change, and the location also changed. Of course, the original location may be occupied.

If  $v(i, j) \geq 0$ , that means the positio( $i, j$ ) occupied by a car.

## 4.3.1.1 The “straight line”

“Fast straight line rule”:

The car on the 1st, 2nd or 3rd lanes. The car can speed up to 2. ( $\max v = 2$ )

The first situation:

$$v(i, j) \geq 0 \& v(i+1, j) = -1 \& v(i+2, j) = -1$$

$$\Downarrow$$

$$v'(i, j) = -1$$

$$v'(i+v(i, j)) = \min(2, 1+v(i, j))$$

The second situation:

$$v(i, j) \geq 0 \& v(i+1, j) = -1 \& v(i+2, j) \geq 0$$

$$\Downarrow$$

$$v'(i, j) = -1$$

$$v'(i+1, j) = 1$$

The third situation:

$$v(i, j) \geq 0 \& v(i+1, j) \geq 0$$

$$\Downarrow$$

$$v'(i, j) = 0$$

“Slow straight line rule”:

The car on the 4th, 5th, 6th or 7th lanes. The car cannot speed up to 2. ( $\max v = 1$ )

The first situation:

$$v(i, j) \geq 0 \ \& \ v(i+1, j) = -1$$

↓

$$v'(i, j) = -1$$

$$v'(i+1, j) = 1$$

The second situation:

$$v(i, j) \geq 0 \ \& \ v(i+1, j) \geq 0$$

↓

$$v'(i, j) = 0$$

#### 4.3.1.2 The “change lane”

Meet the following rules, the car can change lane. 1st lane can only go straight. (The arrows indicate a lane change)

Conditions of lane change:

Fast lanes’ change condition:

2nd→1st or 3rd→2nd (Figure 2 .(a))

$$\begin{cases} v(2,3) = 2 \\ v(2,2) \neq -1 \\ v(3,2) \neq -1 \\ v(4,2) \neq 0 \end{cases}$$

Fast-slow lanes’ change condition:

4th→3rd (Figure 2. (b))

$$\begin{cases} v(3,4) > 0 \\ v(3,3) = -1 \\ v(2,3) = -1 \\ v(4,3) \neq 0 \end{cases}$$

Slow lanes’ change condition:

5th→4th or 6th→5th or 7th→6th (Figure 2. (c))

$$\begin{cases} v(3,5) > 0 \\ v(3,4) = -1 \\ v(4,4) \neq 0 \end{cases}$$

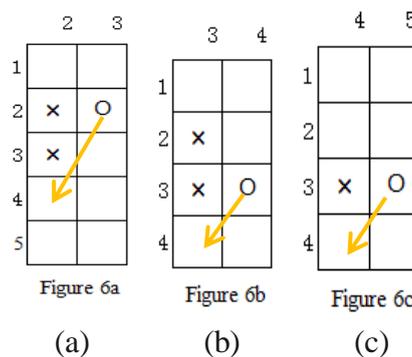


Figure 2 Fast lanes’ change condition

#### 4.4 Results and conclusions

Under the assumptions, we use the cellular automata model to simulate the traffic situation of the exit of the toll station. After the computer program operation, we found that our model is reasonable. The use of the model will not produce a large area of traffic jam.

As is shown in figure 3(a), the ordinate represents the number of lanes and the abscissa represents the distance from the origin (tollbooths) to the main road. Small blue cube is on behalf of the vehicle, we could see clearly every cars' positions at one moment. We can clearly see that there is not gather a large number of cars in some small area, that is, no serious traffic jams happened. Next, we overlap all pictures of every moment, as is shown in figure 3(b). Now the small blue cube is on behalf the area which cars have passed, and the white area is no car arrived. So this picture draws an approximate shape of toll plaza for us.

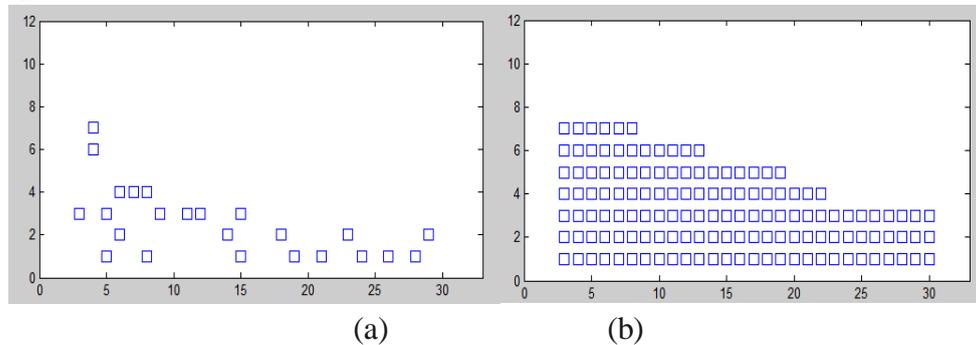


Figure 3. Toll plaza outline

### 5. Improved model

We firstly do not give restriction on the boundary of the toll plaza, so that the vehicles can travel freely according to the rules (the traffic flow is medium). Through simulation, we calculate the frequencies of each cell occupied by cars in a certain period of time. T means the certain period of time.

Here, T=10 (mins).In addition, we take 35 column of cells into account.

We counted about fifty times and take the average. Finally, we acquire the following table.

Table1.Throughout in 10 mins (in each cell)

Serial number of cells of cells	1st lane	2nd lane	3rd lane	4th lane	5th lane	6th lane	7th lane
1	53	53	53	56	57	52	60
2	56	59	66	56	57	52	60
3	59	59	88	56	57	52	59
4	62	59	110	62	59	61	19
5	64	53	135	66	65	37	6
6	66	57	146	72	51	20	2
7	70	60	174	74	37	10	1
8	72	67	172	66	25	6	0
9	75	76	189	61	17	2	0
10	77	83	176	53	12	1	0
11	80	92	189	41	10	0	0
12	83	96	173	33	6	0	0
13	87	103	182	26	5	0	0
14	92	103	166	23	3	0	0
15	96	109	174	16	3	0	0
16	99	108	157	14	2	0	0
17	104	113	163	11	1	0	0
18	107	113	144	9	1	0	0
19	111	120	151	7	1	0	0
20	114	119	133	6	0	0	0

21	118	126	142	5	0	0	0
22	121	121	124	4	0	0	0
23	127	128	132	3	0	0	0
24	131	120	114	2	0	0	0
25	135	130	122	2	0	0	0
26	137	122	106	2	0	0	0
27	143	131	111	1	0	0	0
28	146	120	99	1	0	0	0
29	152	129	105	1	0	0	0
30	154	118	93	1	0	0	0
31	160	126	98	0	0	0	0
32	161	116	87	0	0	0	0
33	167	125	92	0	0	0	0
34	168	116	80	0	0	0	0
35	176	123	86	0	0	0	0

As is shown in the table 1, we can see that the number of the cars is normal in the first two cells, that is, cars will not change lane so early. However, for the 4th lane to 7th lane, the serial number of cells is larger, the number of cars is smaller. According to this, we calculate the length l of 4th lane to the 7th lane.

$$14 = (2+16) \times 5m = 90m;$$

$$15 = (2+9) \times 5m = 55m;$$

$$16 = (2+5) \times 5m = 35m;$$

$$17 = (2+2) \times 5m = 20m.$$

Thus, we plot the figure. The vertical coordinate represents the number of cars of each cell. We can see that the vertical coordinate of 1st lane to 3rd lane is very large, while the 4th lane to 7th lane is small. This is in accordance with the real situation.

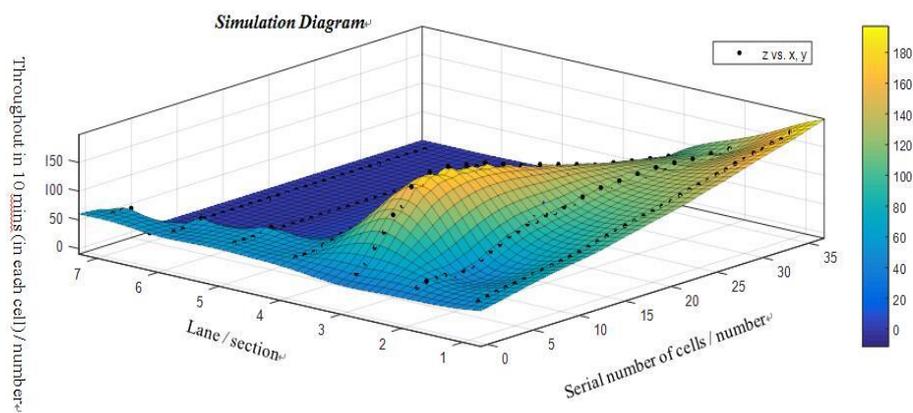


Figure 4. simulation diagram

## 6. Comparison

At present, the shape of most toll plazas is shown as the red part of Figure 5, and its edge is a straight line. Our model, shown as blue part, is a smooth curve. The construction cost of the toll plaza can be expressed by its area. The larger the area of the toll plaza, the more the construction cost.

For the 4th lane to 7th lane:

The area of traditional scheme  $S = (2+17) \times 4 / 2 = 38$

The area of new scheme  $S = 2+5+9+16=32$

The decrease of cost of building plaza  $\Delta D = (38 - 32) / 38 = 15.8\%$

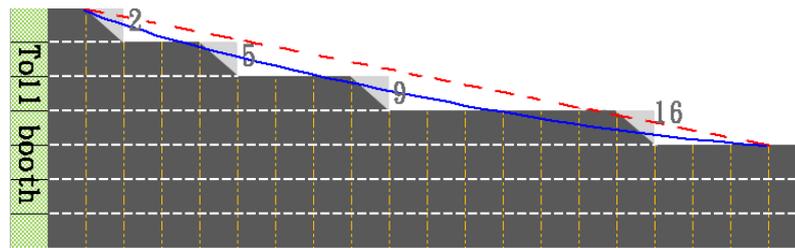


Figure 5. the comparisons between traditional and new scheme

By changing the frequency of vehicles leaving the toll stations, we obtained different values of  $M$  and got a number of points. In the simulation, we used the exponential function to analyze data. The fitting confidence is as high as 95%. According to the actual situation, when  $M > 600s$ , the traffic situation began to become congested. Obviously, traffic flow  $Q$  will reduce.

$$M = 600s$$

$$Q : 4910 \rightarrow 4800$$

$$\Delta Q = (4910 - 4800) / 4910 = 2.2\%$$

Compared with the traditional model, we found that when the plaza area was reduced by 15.8%, the number of vehicles per hour decreased by only 2.2%. That is, we can make full use of saved money to expand area in order to increase traffic flow.

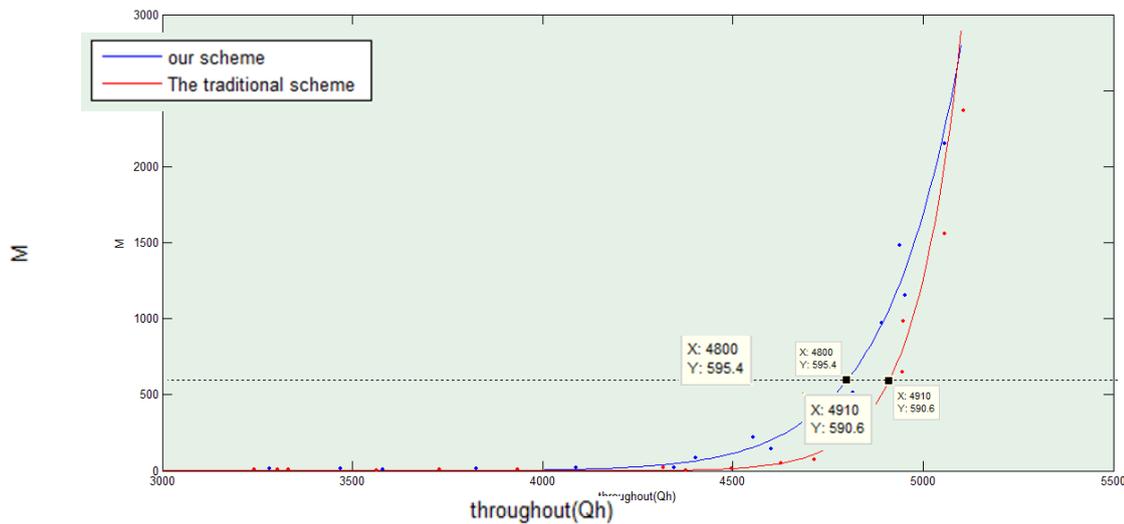


Figure 6. Fitting graph

## 7. Strengths and weaknesses

### 7.1 Strengths

Compared with the common situation, our model can reduce the road construction cost significantly while affecting traffic flow very little, so we can make full use of saved money to expand area in order to increase traffic flow.

It should be illustrated that accidents will not happen in accordance with the rules of traffic we set in the cellular automation. Our rules are simple and it can extremely increase the traffic flow.

The cellular automata model has powerful complex computing functions, inherent parallel computing capability, high dynamic characteristics and spatial concepts, which make it possible to study the dynamic evolution of space-time complex systems.

## 7.2 Weaknesses

Cells have regular shapes and are regularly arranged in the cell space. However there are few such spatial systems that are so consistent and identical, in the real world. Toll plaza is clearly not such an ideal space.

In the process of driving, drivers are very vulnerable to the impact of the external environment, thus affecting the speed and the probability of lane changing.

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