Simulation and Comparison of Single Inverted Pendulum Based on MATLAB

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Abstract

The system was analyzed and mathematical models were used control the balance of the inverted pendulum. Try to design the PID controller and LQR controller based on the mathematical model. MATLAB was used for the simulation study of two models of the inverted pendulum. The results show that PID algorithm can effectively control the angle of swing rod, but the position of the car is out of control. LQR controller had the best effect, which has a shorter adjustment time, a smaller overshoot and better dynamic performance.

Keywords

Inverted pendulum; PID control; LQR control; simulation based on MATLAB.

1. Introduction

Inverted pendulum system is a typical nonlinear multivariable unstable system. It has been widely applied in aerospace, mechatronics and other fields, such as attitude stabilization control of rocket body and multi degree of freedom motion stability design of robot. Therefore, it is of great significance to carry out engineering application research and deeper theoretical research. With the rapid development of intelligent control technology in recent years, the inverted pendulum is used as the research object, and various intelligent control techniques have been used to solve the problem of stability control of nonlinear systems, which have been used by many scholars to study and verify.

In order to reflect the advantages and disadvantages of different inverted pendulum controllers, this paper will carry out modeling and evaluation of two kinds of controller ratio integral differential controller (PID controller) and linear two times regulator (LQR controller) through MATLAB. Furthermore, the simulation results of the controllers are compared and analyzed, so as to provide theoretical basis for the balance control of the inverted pendulum in the future.

2. Mathematical Modeling of Single Inverted Pendulum System

After careful assumption, neglecting some minor factors, such as ignoring air resistance, friction in the system, and not considering the deformation of the component, the linear single inverted pendulum system can be abstracted into a system composed of small cars and homogeneous rods, as shown in Figure 1. This is a typical motion rigid body system, which can be applied in the inertial coordinate system to establish the dynamic equations of the system.

The internal parameters of the system are defined as follows:

The quality of the car: M=0.5kg ;

The quality of the swing rod : m=0.2kg;

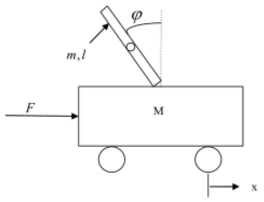
The friction coefficient of the car is b=0.1N/m/sec;

The inertia of the pendulum rod: I=0.006kg*m*m;

The length of the center of rotation of the swing rod to the center of mass is l=0.3m;

Sampling time: T=0.005s.

Figure 2 shows the force analysis of the trolley and swing rod. Among them, N and P are the horizontal and vertical components of the interaction force between the car and the swing rod.



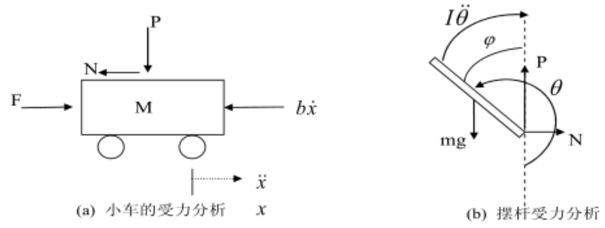


Fig 1. Inverted pendulum system   Fig 2. Force analysis of car and pendulum rod

The Newton method is used to establish the dynamic equations of the system as follows:

By analyzing the resultant force in the horizontal direction of the trolley, the following equations can be obtained.

$$\mathrm{M}\ddot{x} = F - b\dot{x} - N \tag{1}$$

The following equation can be obtained from the analysis of the horizontal force of the swing rod.

$$\mathrm{N} = \mathrm{m}\frac{d^2}{dt^2}(x + lsin\theta) \tag{2}$$

$$\mathrm{N} = \mathrm{m}\ddot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta \tag{3}$$

By substituting this equation in the upper form, the first motion equation of the system is obtained.

$$(\mathrm{M} + \mathrm{m})\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - \ ml\dot{\theta}^2 sin\theta = F \tag{4}$$

In order to derive the second equations of motion of the system, we can analyze the resultant forces in the vertical direction of the pendulum and obtain the following equations.

$$\mathrm{P} - \mathrm{mg} = -\mathrm{m}\frac{d^2}{dt^2}(lcos\theta) \tag{5}$$

$$\mathrm{P} - \mathrm{mg} = ml\ddot{\theta}sin\theta + \ ml\dot{\theta}^2 cos\theta \tag{6}$$

The moment equilibrium equation is as follows:

$$-\mathrm{P}lsin\theta - Nlcos\theta = I\ddot{\theta} \tag{7}$$

Combining two equations, P and N, we get second equations of motion.

$$(\mathrm{I} + \mathrm{m}l^2)\ddot{\theta} + mglsin\theta = -\mathrm{m}l\ddot{x}cos\theta \tag{8}$$

Let's suppose: $\theta = \pi + \varphi$. When the angle φ between the pendulum bar and the vertical upward direction is smaller than that of 1 (the unit is arc), the approximate treatment is done: $cos\theta = -1, sin\theta = -\varphi, (\mathrm{d}\theta/dt)^2 = 0$. The u is used to represent control volume, so the differential equation of the mathematical model of the system is:

$$\begin{cases} (\mathrm{I} + \mathrm{m}l^2)\ddot{\theta} - mgl\varphi = \mathrm{m}l\ddot{x} \\ (\mathrm{M} + \mathrm{m})\ddot{x} + b\dot{x} - \ ml\ddot{\varphi} = u \end{cases} \tag{9}$$

2.1 Transfer function model

The transfer function is:

$$\begin{cases} (\mathrm{I} + \mathrm{m}l^2)\Phi(\mathrm{s})\mathrm{s}^2 - mgl\Phi(\mathrm{s}) = \mathrm{m}l\mathrm{X}(\mathrm{s})\mathrm{s}^2 \\ (\mathrm{M} + \mathrm{m})\mathrm{X}(\mathrm{s})\mathrm{s}^2 + b\mathrm{X}(\mathrm{s})\mathrm{s} - ml\Phi(\mathrm{s})\mathrm{s}^2 = U(s) \end{cases} \tag{10}$$

After solving the collation, the transfer function with pendulum swing angle φ as output is obtained.

$$G_1(\mathrm{s}) = \frac{\Phi(\mathrm{s})}{U(s)} = \frac{\frac{\mathrm{m}l}{q}\mathrm{s}^2}{s^4 + \frac{b(\mathrm{I}+\mathrm{m}l^2)}{q}s^3 - \frac{(\mathrm{M}+\mathrm{m})mgl}{q}s^2 - \frac{\mathrm{bmg}l}{q}s} \tag{11}$$

In the formula above: $\mathrm{q} = [(\mathrm{M} + \mathrm{m})(\mathrm{I} + \mathrm{m}l^2) - (ml)^2]$.

If the car displacement is taken as output, the transfer function can be obtained as follows:

$$G_2(s)=\frac{X(s)}{U(s)}=\frac{\frac{(I+ml^2)}{q}s^2-\frac{mgl}{q}}{s^4+\frac{b(I+ml^2)}{q}s^3-\frac{(M+m)mgl}{q}s^2-\frac{bmgl}{q}s} \tag{12}$$

2.2 Mathematical model of state space

According to the modern control theory, the state space equation of control system can be written as follows:

$$\dot{X}=AX+Bu$$
$$Y=CX+Du$$

Solving algebraic equation (9), we get the following solution:

$$\begin{cases} \dot{x}=\dot{x} \\ \ddot{x}=\frac{-(I+ml^2)b}{I(m+M)+mMl^2}\dot{x}+\frac{m^2gl^2}{I(m+M)+mMl^2}\varphi+\frac{(I+ml^2)}{I(m+M)+mMl^2}u \\ \dot{\varphi}=\dot{\varphi} \\ \ddot{\varphi}=\frac{-mlb}{I(m+M)+mMl^2}\dot{x}+\frac{mgl(M+m)}{I(m+M)+mMl^2}\varphi+\frac{ml}{I(m+M)+mMl^2}u \end{cases} \tag{13}$$

After finishing, the system state space equation is obtained.

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\varphi}\\ \ddot{\varphi}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(m+M)+mMl^2} & \frac{m^2gl^2}{I(m+M)+mMl^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(m+M)+mMl^2} & \frac{mgl(M+m)}{I(m+M)+mMl^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \varphi\\ \dot{\varphi}\end{bmatrix}+\begin{bmatrix}0\\ \frac{(I+ml^2)}{I(m+M)+mMl^2}\\ 0\\ \frac{ml}{I(m+M)+mMl^2}\end{bmatrix}u \tag{14}$$

$$Y=\begin{bmatrix}x\\ \varphi\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \varphi\\ \dot{\varphi}\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}u \tag{15}$$

3. Design and Simulation of PID control method

The proportional differential integral controller is called PID controller for short. This controller has a simple operation principle, so it is easy to use and has a strong adaptability. It is widely used in all aspects of the manufacturing process. PID control has the characteristic of insensitivity to the change of the controlled object, it can be applied to automatic dynamic deviation and can reduce the adjustment time, and the integral action can eliminate the steady-state error, but the dynamic deviation of the response curve and the adjustment time are increased, so PID control is needed. PID control does not require accurate analysis of the system, so the experimental method can be chosen in the setting of controller parameters.

3.1 Control and Simulation of swing rod angle

The output is the position of the swing rod, and its initial position is vertical upward. We apply a disturbance to the system to observe the response of the swing rod. The system block diagram is shown in Figure 3.

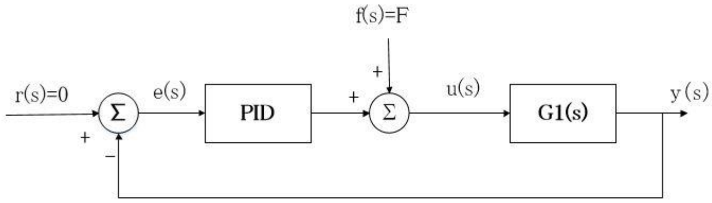


Fig 3. Angle control system block diagram

Consider the input: r(s)=0. The structure diagram can be transformed into Figure 4.

According to the pendulum swing angle output transfer function (11) of inverted pendulum, figure 5.is the structure diagram of PID control of single inverted pendulum rod angle in Matlab with Simulink. The specific structure of the PID encapsulation module diagram is shown in Figure 6.

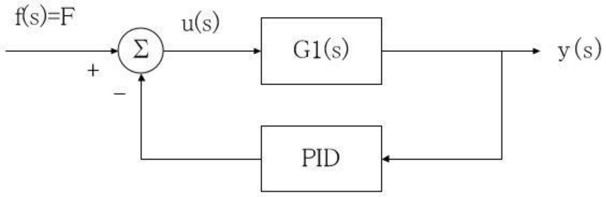


Fig 4. System block diagram after transformation

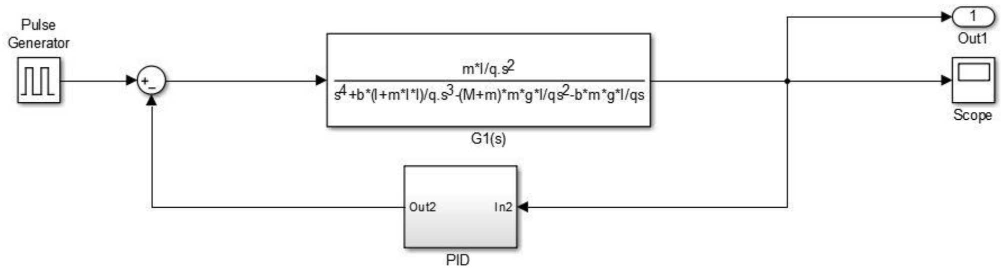


Fig 5. Simulink diagram of swing rod angle

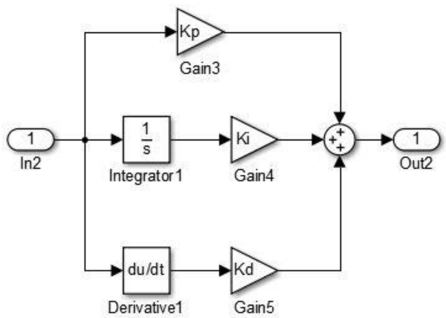


Fig 6. PID structure diagram

Among them, according to the parameters of the experimental PID, the values are as follows:

Kp=100; Ki=1; Kd=20.

Through Matlab simulation, the system impulse response curve is shown in Figure 7.

The simulation curves show that under the control of PID parameters, the response of the system meets the requirements of the index.

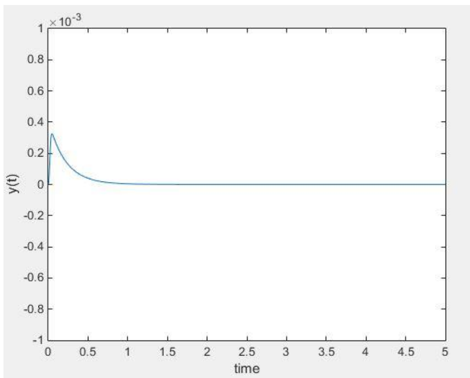


Fig 7. Impulse response curve of pendulum rod angle control

3.2 Position control and Simulation of a car

When the car position is output, the system block diagram is shown in Figure 8.

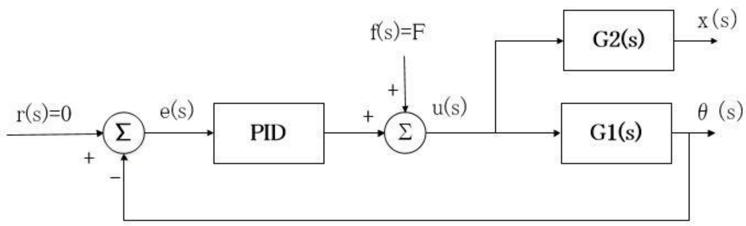


Fig 8. Block diagram of position control system

In the graph, G1(s) is the pendulum transfer function, and G2(s) is the trolley transfer function.

Because the input signal is r(s)=0, the structure diagram can be transformed into Figure 9.

The feedback represents the controller of the pendulum bar we designed earlier. From this block diagram, we can see that only the pendulum angle is controlled here, and the position of the trolley is not controlled. According to the transfer function of the car's displacement output of inverted pendulum (12), the displacement structure diagram of a single inverted pendulum car is built in Simulink in Matlab, as shown in Figure 10. Through Matlab simulation, the impulse response curve of car position control is shown in Figure 11.

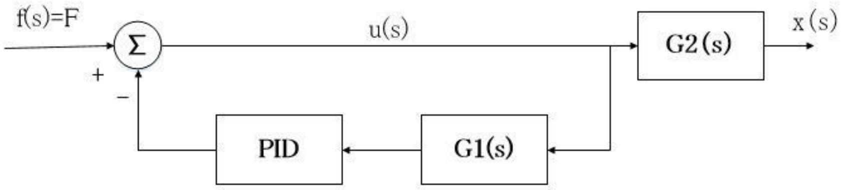


Fig 9. Block diagram of position control system

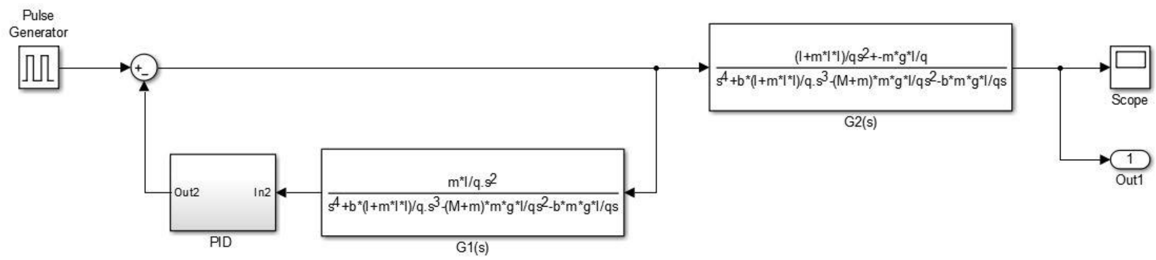


Fig 10. Simulink structure diagram of car displacement

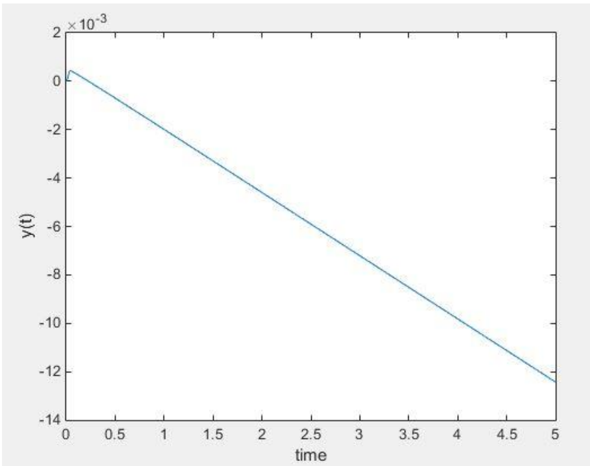


Fig 11. System impulse response curve of car displacement

It can be seen from the simulation that when the pendulum angle is in good closed loop control, the position curve of the cart is divergent. The car is out of control, and it will move in one direction and install the inverted pendulum limit switch, so the ordinary PID controller can not keep the car in a balanced position for a long time, which is determined by the characteristics of the single input and single output of the PID controller. For multiple input systems such as inverted pendulum, an improved PID control algorithm with multiple inputs and multiple outputs must be adopted.

4. Design and Simulation of LQR Control Method

The optimal control theory is an important part of modern control theory, and the research and application in recent decades make the optimal control theory become a major branch of modern cybernetics. The development of computer has made it easy to calculate the calculation that can not be realized in the past, so the idea and method of optimal control have been used more and more widely in the engineering practice.

In order to control the position of the pendulum swing rod and the displacement of the car, the state space method is used to analyze the system and to design the LQR controller. The block diagram of the LQR control system is shown in Figure 12.

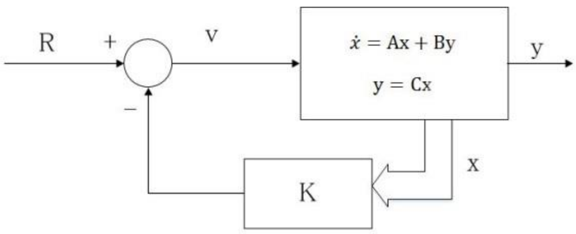


Fig 12. LQR control system block diagram

In the figure, R is a step input applied to a car. The four state variables , $x, \dot{x}, \varphi$ and $\dot{\varphi}$, represent car displacement, trolley speed, swing rod position and pendulum angle velocity. The output , $y = [x \quad , \varphi]'$, includes car position and swing angle. A controller is designed so that when a step input is applied to the system, the pendulum swing will swing, and then return to the vertical position, while the car reaches the new command position.

The coefficient matrix of state equation is obtained through Matlab simulation program as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix}; \mathrm{B} = \begin{bmatrix} 0 \\ 1.8182 \\ 0 \\ 4.5455 \end{bmatrix}; \mathrm{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \mathrm{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The precondition of optimal control is that the system is controllable. The controllability and observability of the system can be judged by the following formula.

The rank of the controllability matrix of the system: $\mathrm{rank}[\boldsymbol{B} \quad \boldsymbol{AB} \quad \boldsymbol{A}^2\boldsymbol{B} \quad \boldsymbol{A}^3\boldsymbol{B}] = 4.$

The rank of the observable matrix of the system: $\mathrm{rank}[\boldsymbol{C} \quad \boldsymbol{CA} \quad \boldsymbol{CA}^2 \quad \boldsymbol{CA}^3] = 4.$

Therefore, the system is controllable and observable. It can make the closed-loop system stable and optimal controller to the system, and meet the transient performance. In the use of two linear quadratic optimal control algorithm for the controller design, the main purpose is to obtain the feedback vector value of **K**. In order to simplify the problem and making the weighted matrix have a clearer physical meaning, **Q** is taken as diagonal matrix. We take the following values:

$$\boldsymbol{Q} = \begin{bmatrix} 5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \mathbf{R} = [1]$$

Write the m file of LQR. Use the LQR function in the Matlab control system toolbox to calculate, and get the feedback control vector as follows:

$$\mathrm{K} = [-70.7107 \quad -37.8345 \quad 105.5298 \quad 20.9238]$$

The simulation curve is obtained. LQR control step response curve is shown in Figure 13.

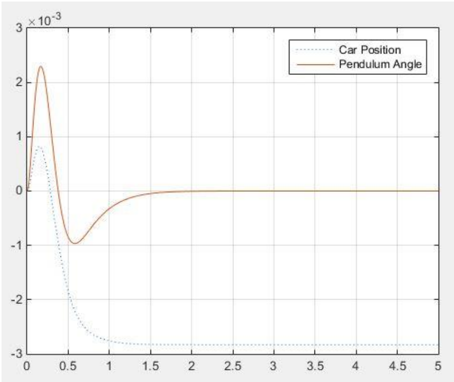


Fig 13. LQR control step response curve

Among them, the solid line represents the angle of the swing rod, and the dashed line represents the position of the cart. It can be seen from the diagram that the position of the trolley tracks the input signal, and at the same time, the overshoot of the swing rod is the most small, and the steady-state error meets the requirements. The stability time and the rise time are shorter, which also conform to the design target.

5. Summary

Inverted pendulum system is a complex unstable mechanical system with nonlinear and strong coupling. The study of the real-time stability of the inverted pendulum system is an important subject of modern control theory, and is also one of the most suitable experimental devices for the study and study of control theory. In this paper, the PID controller and the LQR optimal controller are designed by establishing the mathematical model of the single stage linear inverted pendulum system. By comparing the design process of the two controllers and choosing the reasonable control parameters and Matlab simulation, it is proved that the LQR controller has better control performance and can meet the design requirements for the complex system with multi input.

References

[1] Song Zhi'an, Zhang Xin, Song Yufeng. MATLAB/Simulink Modeling and Simulation of Mechanical and Electrical System [M]. Beijing: National Defense Industry Press, 2015.

[2] Song Shuang, Li Zexing. Practical Motion Control Technology [M]. Beijing: Electronic Industry Press,2006.

[3] Liu Jinkun. Advanced PID Control and Matlab Simulation [M]. Beijing: Electronic Industry Press, 2004.

[4] Wang Zhenglin, Wang Shengkai, Chen GuoShun, Matlab/Simulink and control system simulation[M]. Beijing: Electronic Industry Press, 2005.

[5] Li Junfang, Niu Wenxing, Zhang Zhendong. Stability Control of Inverted Pendulum System Based on State Space Method [J]. Journal of Tianjin Polytechnic College, 2003(2):34 −36.

[6] Lee G H, Jung S, Lee G H. Reference compensation technique of a fuzzy controlled inverted pendulum system for control education [J]. International Conference on Control,Automation & Systems, 2008:1704-1708.

[7] Bloch A M,leonard N E,Marsden J E. Controlled lagrangians and the stabilization of mechanical systems[J]. IEE Trans on Automatic Control,2000,45(12):2253-2260.

[8] Yavin Y. Control of a rotary inverted pendulum[J]. Applied Mathematics Letters, 1999, 12: 131-134.

[9] Sooyong J, Wen J T. Nonlinear model predictive control for the swing-up of a rotary inverted pendulum[J]. Journal of Dynamic Systems, Measurement & Control, 2004, 126(3): 666-674.

[10] Yang Shiyong, Xu Liping, Wang Peijin. Study on PID Control of Single Inverted Pendulum [J], Control Engineering, 2007,14(S1):23-25.