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# Study on the Problem of VLF Underwater Emission

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## Abstract

Consider adopting the way of VLF emission source under water VLF communication, with very low frequency symmetric diploes antenna as an example to calculate the underwater radiation of the very low frequency signal source problem, given symmetric diploes antenna underwater launch related parameters, calculating the current distribution on the transmitting antenna, and according to the current torque calculation of different distance of electromagnetic field intensity on the surface.

## Keywords

VLF; communication; intensity.

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## 1. The Introduction

When a submarine is underwater, VLF communication is the main communication method. When the VLF electromagnetic waves enter the sea from the air, they will have a strong attenuation due to the action of the sea water, so the electromagnetic waves emitted by the transmitting source above the water surface are difficult to enter the water. In order to solve this problem, the communication distance and communication depth of VLF to submarine are improved by increasing transmitting power and antenna transmitting efficiency. A VLF communication station built on land has a large size of transmitting antenna, which is vulnerable to damage in wartime, and usually has a large attenuation distance from the station to the ocean. Another way is considered in this paper, the way of locating VLF communication emission source under water. The underwater symmetric diploes antenna launch related parameters is given, the current distribution on the transmitting antenna is calculated, then according to the current moment, the electric field intensity above the ocean surface of different distance is calculated. Then according to atmospheric noise level, communication distance and depth is estimated.

## 2. Calculation of Parameters of Very Low Frequency Underwater Transmitting Antenna

Taking the VLF symmetric diploes antenna placing under the sea surface as an example, the input impedance, the current distribution on the antenna, and the current moment of the antenna are calculated.

### 2.1 The Propagation Constant of Electromagnetic Waves Along the Antenna

Assume that transmitting antenna is symmetric diploes antenna, located below the ocean surface depth  $h$ , arm length  $l$ , wire radius for  $a$ , in sea water transmitting antenna to be coated a layer of insulation, insulation outer radius to  $b$ , for relative dielectric coefficient  $\epsilon_2$ , can export the electromagnetic wave propagation constant along the antenna[1]

$$\gamma = \frac{2\pi\sqrt{\epsilon_2}}{\lambda} \sqrt{\frac{-\ln(b/a) - \ln(\delta/b) + 0.23 - A/2}{\ln(b/a)}} + j \frac{\pi/4 - B/2}{\ln(b/a)} \tag{1}$$

Among that

$$\delta = 1/\sqrt{\pi f \mu_0 \sigma} \tag{2}$$

$\delta$  represents depth of the water,  $\sigma$  represents the conductivity of seawater,  $\lambda$  represents the wavelength of free space,

$$A = \frac{1}{16(h/\delta)^2 + 1.8(h/\delta) + 1} \tag{3}$$

$$B = \frac{-0.3(h/\delta)}{31.2(h/\delta - 0.3)^4 + h/\delta} \tag{4}$$

When  $h \geq \delta$ , the parameters A and B in equation (1) can be ignored.

The real part of the propagation constant is the attenuation coefficient, expressed as  $\beta$ , and the imaginary part is the phase coefficient, expressed as  $\alpha$ .

$$\gamma = \beta + j\alpha \tag{5}$$

### 2.2 Antenna Impedance

The calculation formula of characteristic impedance of symmetric diploes antenna under sea surface is

$$Z_c = -j \frac{\gamma}{2\pi\omega\epsilon_2\epsilon_0} \ln\left(\frac{b}{a}\right) = \frac{\alpha - j\beta}{2\pi\omega\epsilon_2\epsilon_0} \ln\left(\frac{b}{a}\right) \tag{6}$$

The input impedance of the symmetric diploes antenna can be calculated as follows[2]:

$$Z_{in} = Z_c \frac{1 + e^{-2\gamma l}}{1 - e^{-2\gamma l}} = R_{in} + jX_{in} \tag{7}$$

In the above equation,  $R_{in}$  is the input resistance of the antenna, and  $X_{in}$  is the input reactance of the antenna. For symmetric diploes antenna with short length, the input impedance is generally large capacitive reactance, needed to tune with coils.

### 2.3 Antenna Current Distribution

If the input power of the antenna is P, and the antenna is tuned to resonance with a tuning coil, the antenna inputs current can be expressed

$$I_{in} = \sqrt{P/(R_{in} + R_t)} \tag{8}$$

$R_t = X_{in}/Q$  is resistance of the tuning coil,  $Q$  is quality factor of the tuning coil.

The current distribution on the antenna

$$\begin{aligned} I_x &= I_{in} \frac{sh\gamma(l-x)}{sh\gamma l} = I_{in} \frac{sh\beta(l-x)\cos\alpha(l-x) + jch\beta(l-x)\sin\alpha(l-x)}{sh\beta l \cos\alpha l + jch\beta l \sin\alpha l} \\ &= I_{in} \left[ \frac{sh\beta(l-x)\cos\alpha(l-x)sh\beta l \cos\alpha l + ch\beta(l-x)\sin\alpha(l-x)ch\beta l \sin\alpha l}{(sh\beta l \cos\alpha l)^2 + (ch\beta l \sin\alpha l)^2} \right. \\ &\quad \left. - j \frac{sh\beta(l-x)\cos\alpha(l-x)ch\beta l \sin\alpha l - ch\beta(l-x)\sin\alpha(l-x)sh\beta l \cos\alpha l}{(sh\beta l \cos\alpha l)^2 + (ch\beta l \sin\alpha l)^2} \right] \end{aligned} \tag{9}$$

In the above equation, x is the distance from the point in the symmetric oscillator. The moment of current in the antenna is equal to

$$\mathfrak{I} = 2 \int_0^l I dx = \mathfrak{I}_r - j\mathfrak{I}_i \tag{10}$$

$$\mathfrak{I}_r = 2I_{in} \int_0^l \left[ \frac{sh\beta(l-x)\cos\alpha(l-x)sh\beta l \cos\alpha l + ch\beta(l-x)\sin\alpha(l-x)ch\beta l \sin\alpha l}{(sh\beta l \cos\alpha l)^2 + (ch\beta l \sin\alpha l)^2} \right] dx \tag{11}$$

$$\mathfrak{I}_i = 2I_{in} \int_0^l \left[ \frac{sh\beta(l-x)\cos\alpha(l-x)ch\beta l \sin\alpha l - ch\beta(l-x)\sin\alpha(l-x)sh\beta l \cos\alpha l}{(sh\beta l \cos\alpha l)^2 + (ch\beta l \sin\alpha l)^2} \right] dx \tag{12}$$

For example, the numerical calculation assumes  $l = 50m$ ,  $\epsilon_2 = 10$ ,  $a = 2.5mm$ ,  $b = 7.5mm$ ,  $h = 10m$ , the electrical conductivity of seawater  $\sigma = 4S/m$ , frequency  $f = 15kHz$ . Due to the large attenuation of VLF in seawater, the depth of antenna cannot be very deep.

According to equation (2), the skin depth of sea water to 15kHz electromagnetic waves was given. According to equation (1), the influence of interface is ignored[3]. According to the equation (6), the characteristic impedance can be calculated  $Z_c = 52.26 - j2.7 \Omega$ . According to the equation (7), calculate the input impedance  $\Omega$ . Assuming that the power of the transmitter is equal to 5kW and the tuning coil is tuned, the input current of the antenna is calculated according to equation (8). According to equation (9), we draw the current distribution curve of the real part on the vibrator arm (figure 1) and the virtual part current distribution curve (figure 2). Both figure 1 and figure 2 are normalized relative to the maximum current at the real part. It can be seen that the virtual current is much smaller than the real current and can be ignored.

In principle, the current moment of the antenna should be calculated according to equation (10) ~ (12), but since we have calculated the current distribution; It is easier to calculate the current moment from the current distribution [4,5]. The imaginary part of the current can be ignored, and the real part of the current is basically a straight line, so the moment of current is equal to the input current times the arm length, which is equal to 1686.5A.m.

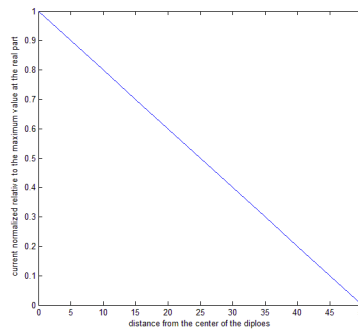


Fig 1. The actual current distribution on the vibrator arm (relative to the maximum normalization)

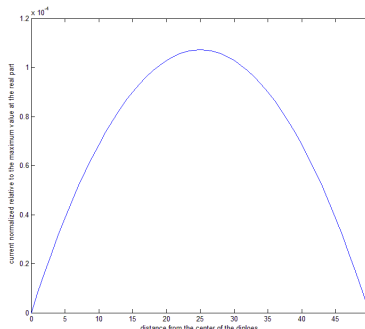


Fig 2. The current distribution on the imaginary part of the vibrator arm (relative to the maximum of the real part current)

### 3. Calculation of Electromagnetic Field Intensity of VLF Underwater Emission

According to the literature[2,3], the formula of the electromagnetic field component over the sea surface generated by the horizontal moment of current in the ocean is given. In general, the distance  $\rho$  is much greater than the depth  $h$  of the transmitting source, so these formulas are simplified to

$$E_z = -\frac{\omega\mu_0 P}{2\pi k_1} e^{-jk_1 h} e^{-jk_2 \rho} \left[ \frac{jk_2}{\rho} + \frac{1}{\rho^2} + \frac{k_2^3}{k_1} \sqrt{\frac{\pi}{k_2 \rho}} e^{jR} \mathfrak{Z}(R) \right] \cos \varphi \tag{13}$$

$$E_\rho = \frac{\omega\mu_0 k_2 P}{2\pi k_1^2} e^{-jk_1 h} e^{-jk_2 \rho} \left[ \frac{jk_2}{\rho} + \frac{1}{\rho^2} - \frac{j}{k_2 \rho^3} + \frac{k_2^3}{k_1} \sqrt{\frac{\pi}{k_2 \rho}} e^{jR} \mathfrak{Z}(R) \right] \cos \varphi \tag{14}$$

$$E_\varphi = \frac{\omega\mu_0 k_2 P}{2\pi k_1^2} e^{-jk_1 h} e^{-jk_2 \rho} \left[ \frac{2}{\rho^2} - \frac{2j}{k_2 \rho^3} - \frac{jk_2^2}{k_1 \rho} \sqrt{\frac{\pi}{k_2 \rho}} e^{jR} \mathfrak{Z}(R) \right] \sin \varphi \tag{15}$$

$$H_\rho = -\frac{k_2 P}{2\pi k_1} e^{-jk_1 h} e^{-jk_2 \rho} \left[ \frac{2}{\rho^2} - \frac{2j}{k_2 \rho^3} - \frac{jk_2^2}{k_1 \rho} \sqrt{\frac{\pi}{k_2 \rho}} e^{jR} \mathfrak{Z}(R) \right] \sin \varphi \tag{16}$$

$$H_\varphi = \frac{k_2 P}{2\pi k_1} e^{-jk_1 h} e^{-jk_2 \rho} \left[ \frac{jk_2}{\rho} + \frac{1}{\rho^2} - \frac{j}{k_2 \rho^3} + \frac{k_2^3}{k_1} \sqrt{\frac{\pi}{k_2 \rho}} e^{jR} \mathfrak{Z}(R) \right] \cos \varphi \tag{17}$$

$$H_z = \frac{k_2^2 P}{2\pi k_1^2} e^{-jk_1 h} e^{-jk_2 \rho} \left[ \frac{1}{\rho^2} - \frac{3j}{k_2 \rho^3} - \frac{3}{k_2^2 \rho^4} \right] \sin \varphi \tag{18}$$

where  $\mathfrak{Z}(R)$  is Fresnel integral, defined as[6]

$$\mathfrak{Z}(R) = \int_R^\infty \frac{e^{-jt}}{\sqrt{2\pi t}} dt = \frac{1-j}{2} - \int_0^R \frac{e^{-jt}}{\sqrt{2\pi t}} dt \tag{19}$$

$$R = \frac{k_2^3 \rho}{2k_1^2} \tag{20}$$

These formulas all contain multiplication factors  $e^{-jk_1 h}$ , wave number of sea water

$$k_1 = \sqrt{-j\omega\mu_0\sigma} \tag{21}$$

The conductivity of seawater is  $\sigma = 4S/m$ , for 15kHz electromagnetic waves, there is  $k_1 = 0.4867 - j0.4867$ . The electromagnetic field decays by about 130 times through 10m layer of seawater, and by 16882 times through 20m seawater. This indicates that with the increase of depth  $h$ , the attenuation of factors  $e^{-jk_1 h}$  increases rapidly, so the antenna depth cannot be very deep.

As can be seen from equation (20), for 15kHz frequency,  $k_2^3 / (2k_1^2) = j3.2725 \times 10^{-11}$ .

Within the communication range on the ground,  $R$  is very small, and the Fresnel integral is approximately as

$$\mathfrak{Z}(R) = \int_R^\infty \frac{e^{-jt}}{\sqrt{2\pi t}} dt \approx \frac{1-j}{2} \tag{22}$$

In addition, for vertical electric field, radial electric field and azimuth magnetic field, when the distance

$$\rho \ll \frac{2|k_1|^2}{\pi k_2^3} \tag{23}$$

For azimuth electric field and radial magnetic field, when the distance

$$\rho \ll \frac{8 |k_1|^2}{\pi k_2^3} \quad (24)$$

The values of terms including Fresnel integrals are far less than those of the first term, so the former can be ignored. For 15kHz frequencies. It can be seen that the Fresnel term in the formula above the VLF frequency band can be ignored. According to equation (13), the absolute value of vertical electric field with different distances in the maximum direction is calculated, as shown in table 1.

Tab 1. Different distance maximum absolute value direction vertical electric field

Distance (km)	5	10	25	50	100	200	300
$E_z (\mu V/m)$	2.6479 $\times 10^{-2}$	1.1721 $\times 10^{-2}$	4.5034 $\times 10^{-3}$	2.2382 $\times 10^{-3}$	1.1174 $\times 10^{-3}$	5.5848 $\times 10^{-4}$	3.7229 $\times 10^{-4}$
$E_z (dB)$	-151.54	-158.62	-166.93	-173	-179.04	-185.06	-188.6

#### 4. Conclusion

Putting VLF communication emission source underwater can improve the concealment of the launching site, the launch of the very low frequency underwater communication mode, can be used as backup of land VLF communication station, and used as emergency communication mode. But erection transmitting antenna underwater, such as antenna should not be too big, the marine environmental factors, this thesis discusses the symmetric diploes antenna underwater transmitting antenna propagation constant, the calculation method of antenna impedance and current distribution, and according to the current moment calculation of different distance of electromagnetic field intensity on the surface of the process, and in 50 meters symmetric diploes antenna as an example for the numerical calculation.

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