Research on Risk Management of CPPI Strategy Based on Johnson Distribution

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Abstract

Constant proportion portfolio insurance (CPPI) is one of the most commonly used strategies in various portfolio insurance strategies.The biggest characteristic of this strategy is to dynamically adjust the proportion of each asset in the portfolio so that it can not only protect the downside risk but also maximize the potential income.However, due to the discrete nature of the transaction, there is a "gap risk" in CPPI strategy, that is, the maturity date can not realize the possibility of capital preservation.In order to solve this problem, this paper introduces Johnson distribution, deduces dynamic risk multiplier by using local quantile criterion and value at risk (VaR), and controls "gap risk" by allowing constant change of risk multiplier.By the empirical analysis of four typical markets in the history data of the CSI 300 , the risk multiplier values under different market conditions and different market conditions are obtained , and how to choose the portfolio insurance strategy under different market conditions is analyzed.

Keywords

CPPI strategy; Johnson distribution; gap risk; local quantile; GARCH model.

1. Introduction

The primary goal of portfolio insurance is to control the downside risk of a bear market, followed by gains when the market is rising.There are a variety of portfolio insurance strategies in the market: European protective put option (OBPI), Constant proportion portfolio insurance strategy (CPPI strategy), stop loss strategy and so on.Among the different portfolio insurance strategies, the constant proportion portfolio insurance (CPPI) strategy has become the most commonly used strategy because of its simple and flexible operation and no complicated calculation formula.The CPPI strategy involves only two types of assets, a risk-free asset, B, which rises at a fixed interested rate r, usually a one-year Treasury rate, and a risky asset, S.

The main idea of CPPI strategy is that an initial investment V0 with a fixed ratio p is set as the guaranteed (floor), so that the portfolio value Vt is always above the insured (floor) / Ft=p*V0*e-r（T-t） at any time during the investment period [0, T].Then we decide on the position of investing in risky assets (risk exposure et), makes risk exposure equal to the product of fixed constant m (also known as risk multiplier) and the difference between portfolio value and insured amount (i.e. Ct=Vt-Ft).Therefore, the value of Ct must be positive at any time during the investment period.Finally, the remaining assets will be invested in the risk free assets Bt.Compared with other portfolio strategies, CPPI strategy has the advantage of simplicity and maneuverability.The initial insured amount and risk multiplier can be determined by the degree of risk preference of investors.However, as the market suddenly falls sharply, investors may have to assume the risk of failure of portfolio insurance, that is, "gap risk".

So we have a crucial question: the position of a risky asset, or the value of an acceptable risk multiplier m?On the one hand, the expected return of the portfolio will increase with the increase of the risk multiplier m, and investors are eager to obtain a higher return from the largest possible risk multiplier.On the other hand, due to the imperfect market mechanism, investors must set a reasonable upper bound of risk multiplier.There are a lot of studies on risk multipliers in the existing literature, such as setting $m \leq 1 / d$ (d is the largest daily decline in forecast or history), as well as dynamic study of risk multipliers. Considering the possibility of extreme risk (i.e., a sharp fall or jump in the price of risky assets) over the entire investment period,Prigent (2001) adopts extreme theory and Engel (2004) to study the maximum value of risk multiplier using conditional self - regression VaR model .Benjamin Hamidian, Emmanuel Jurczenko and Bertrand Maillet (2009) use the risk multiplier to determine a fixed risk exposure in the CPPI strategy, and construct a conditional risk multiplier estimation model based on the dynamic partial bit autoregressive method. And compare the performance of portfolio value under conditional risk multiplier and non-conditional risk multiplier.Yao Yuan, Zhai Jia and Fan Mingqi (2015) introduced the MACD index as a new risk multiplier, in addition, the amount to be insured was also turned into a dynamic one. A DM-CPPI strategy was constructed to realize the dynamic adjustment of the risk multiplier and the insured amount. The empirical analysis shows that the DM-CPPI strategy performs better than the traditional CPPI strategy.

In this paper, Johnson distribution system is introduced, Johnson distribution is used to depict the return rate of risk assets, and then GARCH model is used to estimate the volatility of return on risky assets. In this paper, we introduce the local quantile criterion and the value of risk (VaR), and discuss the conditions based on the local quantile criterion and the value of risk. The choice of conditional risk multiplier depends on the management requirement of "gap risk".The empirical results show that the upper bound of the conditional risk multiplier is the function of the return rate and volatility of the risk asset, the given probability level and the threshold value.

The structure of the paper is as follows: in the second part, Johnson distribution system is introduced to simulate the return rate of risky assets;The third part introduces the traditional constant proportion portfolio insurance strategy and constructs CPPI strategy under Johnson framework.The fourth part introduces local quantile criterion and risk value conditions to construct CPPI strategy based on conditional risk multiplier in Johnson framework.The fifth part of the empirical analysis, solve the Johnson distribution system of four parameters, analyze the investment returns of CPPI strategy under the conditions of different quantile , and compare the investment returns under the conditions of quantile of bureau and traditional CPPI.The sixth part is the conclusion.

2. Introduction of Johnson Distribution Family

Many studies on the return of risky assets show that the mean-variance method is not sufficient to evaluate the risk and return of specific assets such as the CSI 300 index. In fact, these asset returns are distributed in a negative skewness and excessive kurtosis. As documented by Fung and Hsieh（1997）, Ackerman et al （1999）, Brown et al （1999）, Brooks and Kat（2001）, Caglayan and Edwards（2001）, Bacmann and Scholz（2003）, Agarwal and Naik（2004）.... It is thus concluded that, for such assets , the challenge lies not only in the first two moments but also at a higher time . In this framework, it is necessary to select a flexible distribution family to consider asymmetry and kurtosis, which is a necessary condition to study the income behavior of CSI 300 index. Johnson distribution system (1949) can meet this requirement.

2.1 Johnson Distribution System

The Johnson system covers three main families of probability distributions that can model a wide variety of empirical distributions.

The three standard types of Johnson distribution correspond to a function g (.) defined by:

- A lognormal distribution:

$$g(y)=\ln(y)$$

-An unbounded distribution:

$$g(y)=\ln\left(y+\sqrt{1+y^2}\right)=\sinh^{-1}(y)$$

-A bounded distribution:

$$g(y)=\ln\left(\frac{y}{1-y}\right)$$

2.2 Selection of Johnson Distribution System

Some scholars have studied the use of sample quantiles to select the Johnson distribution family and estimate unknown parameters. Slifker and Shapiro choose four symmetrical, isometric standard normal deviations: -3Z, -Z, Z and 3Z, Z is any positive number. The quantiles corresponding to the X distribution are X-3Z, X-Z, XZ and X3Z.Devoted:

l=X3Z-XZ, m=X-Z-X-3Z, p=XZ-X-Z (1)

Defining a quantile ratio, lm / p2, Slifker and Shapiro, indicates that this ratio can be used to distinguish the three distribution families in the Johnson distribution system with the following criteria:

If X has SB distribution , then lm / p2 = 1 ;

If X has SL distribution, then lm/p2=1;

If X has SU distribution, then lm/p2>1 (2)

Here, l, m and P are estimated by using the quantile value of samples, and the resulting ratio $\hat{l}\hat{m}/\hat{p}^2$ is the consistent estimate of lm/ p2.

Because the estimator xi is related to z, it is a function of z, sample size n and sample data. In order to ensure that the Johnson distribution can fit the non-normal data well, a suitable z value must be selected to obtain a better transformation. The ideal range of z values suggested by Chou and others is s={z: z= 0.25, 0.26, … , 1.25}.

Set up a set of data X1, X2… , Xn. The unknown parameters of the Johnson distribution family can be estimated according to the following program.

(1) For medium size data groups, select z less than 1. It is difficult to estimate the quantile value corresponding to ±3z by choosing Z to be 1 or larger. For example, when z = 1, the area under the standard normal curve below -3Z, -Z, Z and 3Z is 0.00130.1587, 0.8413 and 0.9987, respectively, corresponding to the quantile values in the data set $\hat{x}_{(0.0013)}, \hat{x}_{(0.1587)}, \hat{x}_{(0.8413)}, \hat{x}_{(0.9987)}$. When n = 100, $\hat{l}\hat{m}/\hat{p}^2$ depends on the minimum and maximum values in the data set, ignoring all the values between the $\hat{x}_{(0.1587)}$ and $\hat{x}_{(0.8413)}$. In contrast, if a small z value, such as z= 0.3, is obtained, the value other than $\hat{x}_{(0.1481)}$ and $\hat{x}_{(0.8159)}$ is ignored. Therefore, it is more representative to select z value near 0.5. For example, take z= 0.5483, it meets $\Phi(3z)$ =90%. When n is larger, a larger z value should be selected. The parameters in the Johnson distribution family are calculated by using the z value and the estimated value of p, l, m, so the selected z value should make the data and empirical distribution fit well.

(2) According to the normal distribution table, the percentages corresponding to ζ=-3Z, -Z, Z and 3Z $\Phi(\zeta)$ are determined. For example, when z = 0.5, $\Phi(0.5)$=0. 6915.

(3) For each ζ, the corresponding quantile value is obtained from the data by using the relational formula (i-0.5)/n= $\Phi(\zeta)$. So, $\hat{x}_\zeta$ is the I order of observations, where i=n$\Phi(\zeta)$+0.5. In general, I is not an integer and must be interpolated by the frequency distribution table.

(4) The estimated values of l, m and p are calculated from the values $\hat{x}_\zeta$ obtained from the previous step, and then the appropriate Johnson distribution family is selected by applying criterion 2.

It must be pointed out that because xζ is a random variable, the probability of lm/p2=1 is zero. If you want to use a SL distribution, you should allow lm/p2 to be located in an interval near 1.

2.3 Johnson Distribution Parameter Estimation

After selecting the distribution form, the parameters in the distribution are estimated . The parameters η, γ, λ and ε in the distribution can be calculated according to the selected z value and the estimated value of l, m, p. For convenience, the following parameter expressions are functions of l, m and p, as well as functions of the total values of -3Z, -Z, Z and 3Z. In fact, the corresponding parameter estimation is calculated based on the sample x value. Given the values of z and l, m, p, the estimators of the parameters in the distribution are described below. These estimators can be used to complete the calculation.

(1) SB distribution: The solution of the parameters depends on p/l and p/m:

$$\delta=\frac{z}{\cosh^{-1}\left\{\frac{1}{2}\left[\left(1+{p}/{l}\right)\left(1+{p}/{m}\right)\right]^{\frac{1}{2}}\right\}}$$

$$\gamma=\delta\sinh^{-1}\left\{\frac{\left(\frac{p}{m}-\frac{p}{l}\right)\left[\left(1+{p}/{l}\right)\left(1+{p}/{m}\right)-4\right]^{\frac{1}{2}}}{2\left[{p^2}/{lm}-1\right]}\right\} \tag{3}$$

$$\lambda=\frac{p\left\{\left[\left(1+{p}/{l}\right)\left(1+{p}/{m}\right)-2\right]^{2}-4\right\}^{\frac{1}{2}}}{{p^2}/{lm}-1}$$

$$\varepsilon=\frac{x_z+x_{-z}}{2}-\frac{\lambda}{2}+\frac{p\left(\frac{p}{m}-\frac{p}{l}\right)}{2\left[{p^2}/{lm}-1\right]}$$

(2) SL distribution (logarithmic normal distribution):

$$\delta=\frac{2z}{\ln\left({l}/{p}\right)}$$

$$\gamma=\delta\ln\left[\frac{{l}/{p}-1}{p\left({l}/{p}\right)^{\frac{1}{2}}}\right] \tag{4}$$

$$\varepsilon=\frac{x_z+x_{-z}}{2}-\frac{p\left({l}/{p}+1\right)}{2\left({l}/{p}-1\right)}$$

(3) Su distribution: the solution of the parameters depends on l / p and m / p:

$$\delta = \frac{2z}{\cosh^{-1}\left[\frac{1}{2}\left(l/p + m/p \right)\right]}$$

$$\gamma = \delta \sinh^{-1}\left[\frac{m/p - l/p}{2\left(lm/p^2 - 1 \right)^{\frac{1}{2}}} \right]$$

$$\lambda = \frac{2p\left(lm/p^2 - 1 \right)^{\frac{1}{2}}}{\left(l/p + m/p - 2 \right)\left(l/p + m/p + 2 \right)^{\frac{1}{2}}} \tag{5}$$

$$\varepsilon = \frac{x_z + x_{-z}}{2} + \frac{p\left(m/p - l/p \right)}{2\left(l/p + m/p - 2 \right)}$$

3. CPPI Strategy Based on Johnson Distribution

In CPPI strategy, it is generally assumed that the distribution of return on risk assets follows logarithmic normal distribution, but empirical evidence has shown that the distribution of return rate of risk assets has negative bias and asymmetry, that is, "peak and thick tail". This shows that the distribution of return rate of risk assets can not be regarded as logarithmic normal distribution. In order to solve this problem, we introduce Johnson distribution. In this chapter, we study how to determine the risk multiplier based on Johnson distribution, and then construct the CPPI model of fixed risk multiplier based on Johnson distribution.

3.1 Risk Multiplier Analysis of Traditional CPPI Strategy

CPPI is the most widely used strategy in portfolio insurance.Two types of assets are typically involved: risk-free assets (growing at risk-free interest rates) and risky assets (usually stocks or price indices).

During the whole investment period, the initial investment of the investor is V_0, at the end of the investment period, the investor expects to obtain a fixed proportion θ of the initial investment.The value of the portfolio at the end of the period is V_T ,In order to achieve the final value of the portfolio V_T than the amount to be insured θV_0 ,At any time t throughout the investment period $[0,T]$, to maintain the value of the portfolio $V_t \geq F_t = \theta \cdot V_0 \cdot e^{-r(T-t)}$ .That is, at any time t_k , the value of the portfolio V_{t_k} must be above the fixed amount to be insured F_{t_k} .

The number of investment in risk assets is E_{t_k} , $E_{t_k} = mC_{t_k}$ .Among $C_{t_k} = V_{t_k} - F_{t_k}$, the difference between the value of the portfolio and the amount to be insured, Constants, often called risk multipliers, are nonnegative constants. E_{t_k} is Risk exposure, C_{t_k} is safety pad.The share of investments in riskless assets is $D_{t_k} = V_{t_k} - E_{t_k}$,The risk-free interest rate is r_{t_k} .

It is assumed that during the entire investment period $[0,T]$,Changes in asset prices occur at discrete times, and riskless assets grow at risk-free interest rates. The price changes of risk assets between intervals $[t_k, t_{k+1}]$ are recorded as $\Delta S_{t_{k+1}} = S_{t_{k+1}} - S_{t_k}$ .In addition, the opposite number N_{t_k} representing the relative jump of risky assets at the moment t_k is introduced, $N_{t_k} = -\Delta S_{t_k} / S_{t_{k-1}} = (S_{t_{k-1}} - S_{t_k}) / S_{t_{k-1}}$. To represent the maximum value W_T of a finite sequence $\left(N_{t_k}\right)_{1 \le k \le n}$, $W_T = Max(N_{t_1},...,N_{t_n})$.

Based on the above, the value of the portfolio can be derived:

$$V_{t_k} = V_{t_{k-1}} - E_{t_{k-1}} N_{t_k} + \left(V_{t_{k-1}} - E_{t_{k-1}}\right) r_{t_k} \quad (6)$$

Thus:

$$C_{t_k} = C_{t_{k-1}} \left[1 - mN_{t_k} + (1-m) r_{t_k}\right] \quad (7)$$

And because the safety cushion must be positive at all times of the investment period in the CPPI policy, the end result is: for all $k \le n$,

$$-mN_{t_k} + (1-m) r_{t_k} \ge -1 \quad (8)$$

In this case, since the risk-free interest rate is relatively small, assuming that the effect of the risk-free interest rate is not taken into account, the above formula（3-3）could read as follows:

$$\forall k \le n, N_{t_k} \le \frac{1}{m} \quad (9)$$

Or equivalent to

$$W_T = Max(N_{t_k})_{k \le n} \le \frac{1}{m} \quad (10)$$

An upper bound of the risk multiplier is determined by formulas（3-4）or（3-5）.At the same time, Let the right bound point of the ordinary distribution of variables N_{t_k} be a L（$L > 0$）.In this case, as long as the risk multiplier m is smaller than $1/L$,In the whole investment period $[0,T]$, the safety cushion is guaranteed to be positive, that is, the portfolio value is higher than the insured amount.

3.2 Analysis of CPPI Policy Risk Multiplier Based on Johnson Distribution

We introduce a gross rate of return R_{t_k} that represents the price of risky assets, $R_{t_k} = \left(\frac{S_{t_k}}{S_{t_{k-1}}}\right)$ Z_{t_k} represents the logarithmic rate of return on the price of risky assets, $Z_{t_k} = \ln\left(R_{t_k}\right)$.

According to the CPPI strategy, we can know that portfolio value at the moment t_{k+1}:

$$V_{t_{k+1}} = V_{t_k} + e_{t_k}\left(\frac{\Delta S_{t_{k+1}}}{S_{t_k}}\right) + \left(V_{t_k} - e_{t_k}\right) r_{t_{k+1}} \quad (11)$$

Thus:

$$C_{t_k} = C_{t_{k-1}}\left[1 + m\left(\frac{\Delta S_{t_{k+1}}}{S_{t_k}}\right) + (1-m) r_{t_{k+1}}\right] \quad (12)$$

Also because , in CPPI policy , for all times during the investment period , the security mat requires a positive number and is thus finally obtained : for all , $k \le n$:

$$m\left(\frac{\Delta S_{t_{k+1}}}{S_{t_k}}\right) + (1-m) r_{t_k} \ge -1 \quad (13)$$

In this case, since the risk-free interest rate is relatively small, assuming that the effect of the risk-free interest rate is not taken into account, the above formula（3-8）could read as follows:

$$\forall k \le n, -\frac{\Delta S_{t_{k+1}}}{S_{t_k}} \le \frac{1}{m} \tag{14}$$

Or equivalent to:

$$M_{\mathrm{n}} = Max(-\frac{\Delta S_{t_{k+1}}}{S_{t_k}})_{k \le n} \le \frac{1}{m} \tag{15}$$

It is also equivalent to:

$$Max(1-R_{t_k})_{k\le n} \le \frac{1}{m}, Min(R_{t_k})_{k\le n} \ge \left(1-\frac{1}{m}\right) \tag{16}$$

Now letundefineds assume that the return on risky assets is based on a generalized Johnson distribution, Introducing innovations $\in_{t_k}$ that represent the logarithmic rate of return on risk assets at the time t_{k-1}:

$$\in_{t_k} = \frac{\ln R_{t_k} - \mu_{t_k}}{\sigma_{t_k}},$$

Thus, at the moment t_{k+1}, the value of a risky asset:

$$S_{t_{k+1}} = S_{t_k} \exp\left[\mu_{t_k} + \sigma_{t_k} \in_{t_k}\right],$$

Assuming that the probability distribution $\in_{t_k}$ is the generalized Johnson distribution, we can obtain:

$$\varepsilon_{t_k} = h_{t_k}\left(\in_{t_k}\right), \text{ where } h_{t_k}\left(x\right) = \gamma_{t_k} + \delta_{t_k} g\left(\frac{x-\xi_{t_k}}{\lambda_{t_k}}\right),$$

Since the four parameters of the Johnson distribution are known, we can get:

$$\in_{t_k} = h_{t_k}^{-1}\left(\varepsilon_{t_k}\right), \text{ where } h_{t_k}^{-1}(z) = \lambda_{t_k} g^{-1}(\frac{z-\gamma_{t_k}}{\delta_{t_k}}) + \xi_{t_k},$$

From (3-11) We can get:

$$Min\left(\varepsilon_{t_k}\right) \ge \delta_{t_k} g\left(\frac{\left[\frac{\ln\left(1-\frac{1}{m}\right)-\mu_{t_k}}{\sigma_{t_k}}\right]-\xi_{t_k}}{\lambda_{t_k}}\right)+\gamma_{t_k} \tag{17}$$

Formula (3-12) is stringent,Whether the upper bound of the risk multiplier can be improved by introducing a new method to improve the selection of the risk multiplier. Just as the risk value represents the maximum possible loss of a financial asset or portfolio at a certain level of confidence, we consider whether we can guarantee a given level of probability $1-a$,Portfolio value is always above the amount to be insured. Therefore, the quantile method is introduced to improve the selection of risk multipliers. That is, during the investment period $\left[0,T\right]$, the probability of ensuring that the safety cushion is greater than zero is as shown in formula (3-13):

$$P\left[C_{t_k} \ge 0, \forall t_k \in \{1,...,T\}\right] \ge 1-a \tag{18}$$

Or equivalent to:

$$P\left[\forall t_k \in \{1,...,T\}, -\frac{\Delta S_{t_{k+1}}}{S_{t_k}} \le \frac{1}{m}\right] \ge 1-a \quad (19)$$

Under the generalized Johnson distribution , (3 - 14) is equivalent to :

$$P\left[\forall t_k \in \{1,...,T\}, \varepsilon_{t_k} \ge \delta_{t_k} g\left(\frac{\left[\frac{\ln\left(1-\frac{1}{m}\right)-\mu_{t_k}}{\sigma_{t_k}}\right]-\xi_{t_k}}{\lambda_{t_k}}\right)+\gamma_{t_k}\right] \ge 1-a \quad (20)$$

N is introduced to represent the cumulative distribution function of standard Gao Si distribution. (3-15) is equivalent to :

$$1-N\left[\delta_{t_k} g\left(\frac{\left[\frac{\ln\left(1-\frac{1}{m}\right)-\mu_{t_k}}{\sigma_{t_k}}\right]-\xi_{t_k}}{\lambda_{t_k}}\right)+\gamma_{t_k}\right] \ge (1-a)^{\frac{1}{T}} \quad (21)$$

From which we can find:

$$\delta_{t_k} g\left(\frac{\left[\frac{\ln\left(1-\frac{1}{m}\right)-\mu_{t_k}}{\sigma_{t_k}}\right]-\xi_{t_k}}{\lambda_{t_k}}\right)+\gamma_{t_k} \le q_a,T \text{ , } q_a,T = N^{-1}\left[1-(1-a)^{\frac{1}{T}}\right] \quad (22)$$

That is, we finally get the upper bound of the risk multiplier:

$$m \le Min_k\left[\frac{1}{1-\exp\left[h_{t_k}^{-1}(q_a,T)\sigma_{t_k}+\mu_{t_k}\right]}\right] \quad (23)$$

Through comparative analysis, it is found that the upper bound of the risk multiplier based on the Johnson distribution is obviously larger than that of the traditional CPPI strategy risk multiplier. When the probability level is constant and the Johnson distribution parameter is known, the upper bound of the risk multiplier depends on the expected rate of return μ_{t_k} and volatility of the risk asset price σ_{t_k} .Generally, the higher the expected rate of return μ_{t_k} , the lower the probability level, the higher the upper bound of the risk multiplier; The higher the volatility σ_{t_k} , the higher the probability level and the lower the upper bound of the risk multiplier.

4. Conditional Risk Multiplier CPPI Strategy Based on Johnson Distribution

In chapter 3, the risk multiplier is considered to be fixed. At the beginning of the investment, it is selected and determined according to the preference of the investor. In the following analysis, we allow risk multipliers to change dynamically according to market volatility, determine conditional risk multipliers based on local quantile and risk value conditions, and then avoid "gap risk"and improve the overall earnings of CPPI strategy.

4.1 Conditional Risk Multiplie

As time invariant portfolio insurance strategy (TIPP) will change with the change of market, the risk multiplier of constant proportion portfolio insurance strategy will change with the market fluctuation. Agent (2001) proposed a risk exposure function $e_t = e(t, C_t)$,in which the defined domain of the function is positive and continuous $[0,T]\times\mathbb{R}^+$ and the range requires that the cushion must always be positive. If the cushion is zero, the risk exposure must be equal to zero (i. $e(t, C_t) = 0$);If the relative jump value of a risky asset has a left bound d (negative value), then, at any point, there is:

$$e(t,C) \le -\frac{1}{d} C$$

In the following analysis we still assume that the cushion is a linear function but now the risk multiplier is a random variable whose value depends on a given state variable such as the volatility of a risky asset. Then this paper will analyze how the conditional risk multiplier is determined as a function of state variables in the generalized Johnson distribution system.

4.2 The Risk Multiplier under the Condition of Local Quantile

Considering the flow of information generated by the return on risky assets and a sequence of thresholds (which can be fixed values or random variables), the local quantile is defined as follows:

$$P^{\mathcal{F}_{t_{k-1}}}\left[C_{t_k} > L_{t_{k-1}}\right] \ge (1-a) \tag{24}$$

Where the conditional probability $P^{\mathcal{F}_{t_{k-1}}}$ of information flow $\mathcal{F}_{t_{k-1}}$ is represented.

In addition, during the investment period, the expression for the cushion as mentioned earlier, because the time interval is small, the risk-free interest rate for the investment period is zero, that is,

$$C_{t_k} = C_{t_{k-1}}.\left(1 + m_{t_{k-1}} * \frac{\Delta S_{t_k}}{S_{t_{k-1}}}\right) \tag{25}$$

And at any moment, $C_{t_k} > L_{t_{k-1}}$ is always existed.

When $C_{t_{k-1}} > 0$,

$$C_{t_k} > L_{t_{k-1}} \Leftrightarrow 1 + m_{t_{k-1}}.\frac{\Delta S_{t_k}}{S_{t_{k-1}}} > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}$$

$$C_{t_k} > L_{t_{k-1}} \Leftrightarrow 1 + m_{t_{k-1}}.\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}$$

$$C_{t_k} > L_{t_{k-1}} \Leftrightarrow 1 + m_{t_{k-1}}.\left[\exp\left[h_{t_k}^{-1}\left(\varepsilon_{t_k}\right)\sigma_{t_k} + \mu_{t_k}\right] - 1\right] > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}$$

At that time $C_{t_{k-1}} < 0$, there was no real condition to be guaranteed. Therefore, one possible strategy is to keep the previous strategy unchanged when the cushion is positive; Once the cushion becomes negative, investors need to shift the entire portfolio to riskless assets. So, at any moment t_{k-1}, the risk multiplier m_{k-1} has to depend on, at the moment t_k, the cushion is satisfied: $C_{t_k} > L_{t_{k-1}}$, Accordingly, the following functional relations are satisfied:

$$m_{t_{k-1}} = m_{t_{k-1}}^{(1)}.\prod C_{t_{k-1}} > L_{t_{k-1}} + m_{t_{k-1}}^{(2)}.\prod C_{t_{k-1}} \le L_{t_{k-1}} \tag{26}$$

Where $m_{t_{k-1}}^{(1)}$ and $m_{t_{k-1}}^{(2)}$ are arandom variable of the measurable value $\mathcal{F}_{t_{k-1}}$.

4.3 Determination of Conditional Risk Multipliers

As we all know, the ARCH model and the GARCH model are two special nonlinear time series models. This kind of dynamic type is very suitable for describing the fluctuation of assets in the discrete time environment. They are widely used in macroeconomics and statistical theory. A large number of domestic studies show that most of the asset returns are derived from the GARCH family model, so this part introduces the GARCH model. In CPPI strategy, it is generally assumed that the distribution of return on risk assets follows logarithmic normal distribution, but empirical evidence has shown that the distribution of return rate of risk assets has negative bias and asymmetry, that is, "peak and thick tail". This shows that the distribution of return rate of risk assets can not be regarded as logarithmic normal distribution, but when we use GARCH model to estimate the volatility of return rate of risk assets, we must choose a kind of distribution to describe the changing process of return rate of risk assets. In recent years, the t distribution or GED distribution used in the literature research can only describe the "thick tail" phenomenon of the risk asset price, but a simple distribution has not been found to take into account the asymmetry and spike of the risk asset return distribution. To solve this problem, we use the Johnson distribution. The logarithmic return rate of risk assets transformed by Johnson distribution is based on GARCH model, and the concrete expression form of conditional risk multiplier is determined.

The GARCH model takes the following form:

$$R_{t_k} = \left(\frac{S_{t_k}}{S_{t_{k-1}}}\right) = \exp\left[Z_{t_k}\right], Z_{t_k} = \mu_{t_k} + \sigma_{t_k} h_{t_k}^{-1}\left(\varepsilon_{t_k}\right)$$

Where Z_{t_k} is the solution of the following equation of self - regression :

$$\begin{cases} Z_{t_k} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot Z_{t_{k-i}} + \sigma_{t_k} \cdot \in_{t_k} \\ \Lambda(\sigma_{t_k}) = \beta + C_0\left(\in_{t_{k-1}}\right) + C_1\left(\in_{t_{k-1}}\right) \cdot \Lambda(\sigma_{t_{k-1}}) \end{cases} \tag{27}$$

Where, σ_{t_k} Represent volatility,

the series $\left(\in_{t_k}\right)_k$ is the independent same distribution sequence under the ordinary probability distribution, Λ, $C_0(.)$ and $C_1(.)$ are certain function, and the sum is a certain function. Assume that the function Λ increases monotonously in the set of positive real numbers. Set

$\mu_{t_k} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot Z_{t_{k-i}}$

Sequence $\left(\in_{t_1}, \ldots\ldots, \in_{t_{k-1}}\right)_k$ generates a stream of information that conveys the observed value of return on risky assets, recorded as:

$$\mathcal{F}_{t_{k-1}} = \sigma - a\lg ebra\left(\in_{t_1}, \ldots\ldots, \in_{t_{k-1}}\right) \tag{28}$$

Then $m_{t_{k-1}}^{(1)}$ and $m_{t_{k-1}}^{(2)}$ can be derived with respect to the definite function of the $\left(\in_{t_1}, \ldots\ldots, \in_{t_{k-1}}\right)$. Namely:

$$m_{t_{k-1}} = m^{(1)}.\left(t_{k-1}, Z_{t_1}, ..., Z_{t_{k-1}}, \sigma_{t_1}, ..., \sigma_{t_k}\right).\prod C_{t_{k-1}} > L_{t_{k-1}}$$
$$+m^{(2)}.\left(t_{k-1}, Z_{t_1}, ..., Z_{t_{k-1}}, \sigma_{t_1}, ..., \sigma_{t_k}\right).\prod C_{t_{k-1}} \leq L_{t_{k-1}} \quad (29)$$

Where there is a given threshold $L_{t_{k-1}} > 0$

In summary, in order to determine the specific form of expression of the risk multiplier, there are two scenarios to be discussed:

The first case: Cushion is non-negative at the moment $t_{k-1} : C_{t_{k-1}} > L_{t_{k-1}}$

The second situation: the Cushion is satisfied at all times $t_{k-1} : 0 < C_{t_{k-1}} < L_{t_{k-1}}$

When $C_{t_{k-1}} > L_{t_{k-1}}$,the condition for local quantile is as follows: $0 < C_{t_{k-1}} < L_{t_{k-1}}$

$$P^{\mathcal{F}_{t_{k-1}}}\left[1 + m_{t_{k-1}}.\frac{\Delta S_{t_k}}{S_{t_{k-1}}} > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}\right] \geq (1-a) \quad (30)$$

Equivalent to:

$$P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}.\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1\right] \geq (1-a)$$

At the moment, risk assets are both rising and falling, so there are also two situations.

Assume that the risk multiplier $m_{t_{k-1}}$ is non-negative, and then get:

$$P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1\right]$$
$$= P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1 \cap \left(R_{t_k} - 1\right) > 0\right] + \quad (31)$$
$$P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1 \cap \left(R_{t_k} - 1\right) < 0\right]$$

Because

$$m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1 \Rightarrow R_{t_k} > \frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1}{m_{t_{k-1}}} + 1 \quad (32)$$

-i)When $\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1}{m_{t_{k-1}}} + 1 < 0$, $R_{t_k} > \frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1}{m_{t_{k-1}}} + 1$ is satisfied, local quantile is as follows:

$$P^{\mathcal{F}_{t_{k-1}}}\left[1 + m_{t_{k-1}}.\frac{\Delta S_{t_k}}{S_{t_{k-1}}} > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}\right] = 1 > (1-a)$$

-ii)When $C_{t_{k-1}} > 0$, $C_{t_{k-1}} > L_{t_{k-1}}$ and $\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1}{m_{t_{k-1}}} + 1 > 0$,

$$P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}.\left(R_{t_k}-1\right)>\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1\right]$$

$$=P^{\mathcal{F}_{t_{k-1}}}\left[R_{t_k}-1>0\right]+P^{\mathcal{F}_{t_{k-1}}}\left[1>R_{t_k}>\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right]$$

$$=P^{\mathcal{F}_{t_{k-1}}}\left[R_{t_k}>\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right]$$

Equivalent to:

$$P^{\mathcal{F}_{t_{k-1}}}\left[\exp\left[h_{t_k}^{-1}\left(\varepsilon_{t_k}\right)\sigma_{t_k}+\mu_{t_k}\right]>\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right]\geq(1-a)$$

$$=P^{\mathcal{F}_{t_{k-1}}}\left[\varepsilon_{t_k}<h_{t_k}\left(\frac{1}{\sigma_{t_k}}\ln\left(\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right)-\frac{\mu_{t_k}}{\sigma_{t_k}}\right)\right]\leq 1-(1-a) \tag{33}$$

N Represents the cumulative distribution function of a standard normal random variable (ε_{t_k}). As a result:

$$N\left[h_{t_k}\left(\frac{1}{\sigma_{t_k}}\ln\left(\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right)-\frac{\mu_{t_k}}{\sigma_{t_k}}\right)\right]\leq 1-(1-a) \tag{34}$$

Also equivalent to:

$$\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}\leq\exp\left[\sigma_{t_k}h_{t_k}^{-1}\left(\left(N\right)^{-1}\left(1-(1-a)\right)\right)+\mu_{t_k}\right]-1 \tag{35}$$

Set: $q_{(\in,T)}=\left(N\right)^{-1}\left[\left(1-(1-a)\right)\right]^{\frac{1}{T}}$

As a result:

-a)When $\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}{}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]<1$, if $\frac{L_{t_{k-1}}}{C_{t_{k-1}}}<\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}{}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]$, $m_{t_{k-1}}$ must satisfy:

$$m_{t_{k-1}}\leq\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}{}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]-1} \tag{36}$$

If $\frac{L_{t_{k-1}}}{C_{t_{k-1}}} > \exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k} + \mu_{t_k}\right]$, $m_{t_{k-1}}$ must satisfy $m_{t_{k-1}} < 1$

-b)When $\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k} + \mu_{t_k}\right] > 1$, $m_{t_{k-1}}$ has no constraint.

It can be deduced from the previous formula that if the risk multiplication value m is small, the local quantile condition will automatically be satisfied (see argument i).)If the risk multiplication value is larger (see argument ii), and the risk asset price falls (the ii-a case), the risk multiplier is bounded, and if the buffer value at the last moment is not high, the risk multiplier is likely to be less than 1.

(2) When $0 < C_{t_{k-1}} < L_{t_{k-1}}$, the condition for local quantile is as follows:

$$P^{\mathcal{F}_{t_{k-1}}}\left[1 + m_{t_{k-1}} . \frac{\Delta S_{t_k}}{S_{t_{k-1}}} > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}\right] \geq (1-a) \tag{37}$$

Equivalent to:

$$P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}} .\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1\right] \geq (1-a)$$

At the moment, risk assets are both rising and falling, so there are also two situations.

Assume that the risk multiplier $m_{t_{k-1}}$ is non-negative, and then get:

$$\begin{aligned} & P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1\right] \\ & = P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1 \cap \left(R_{t_k} - 1\right) > 0\right] + \\ & P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1 \cap \left(R_{t_k} - 1\right) < 0\right] \end{aligned} \tag{38}$$

Then get:

$$\begin{aligned} & P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1\right] \\ & = P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1 \cap \left(R_{t_k} - 1\right) > 0\right] + \\ & P^{\mathcal{F}_{t_{k-1}}}\left[m_{t_{k-1}}\left(R_{t_k} - 1\right) > \frac{L_{t_{k-1}}}{C_{t_{k-1}}} - 1\right] \end{aligned} \tag{39}$$

Thus, two equivalent conditions are obtained:

$$P^{\mathcal{F}_{t_{k-1}}}\left[1 + m_{t_{k-1}} . \frac{\Delta S_{t_k}}{S_{t_{k-1}}} > \frac{L_{t_{k-1}}}{C_{t_{k-1}}}\right] \geq (1-a),$$

$$P^{\mathcal{F}_{t_{k-1}}}\left[R_{t_k} > \frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right] \geq (1-a) \tag{40}$$

Therefore, the local quantile is equivalent to:

$$P^{\mathcal{F}_{t_{k-1}}}\left[\varepsilon_{t_k} > h_{t_k}\left(\frac{1}{\sigma_{t_k}}\ln\left(\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right)-\frac{\mu_{t_k}}{\sigma_{t_k}}\right)\right] \geq (1-a) \tag{41}$$

It is also equivalent to:

$$h_{t_k}\left(\frac{1}{\sigma_{t_k}}\ln\left(\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}}+1\right)-\frac{\mu_{t_k}}{\sigma_{t_k}}\right) \leq \left(N\right)^{-1}\left(1-(1-a)\right) \tag{42}$$

This means that:

$$\frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{m_{t_{k-1}}} \leq \exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]-1 \tag{43}$$

Eventually:

-i) When $\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]<1$, $m_{t_{k-1}}$ has no positive solution.

-ii) When $\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]>1$,

If $\frac{L_{t_{k-1}}}{C_{t_{k-1}}}<\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]$, $m_{t_{k-1}}$ has no constraint;

If $\frac{L_{t_{k-1}}}{C_{t_{k-1}}}>\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]$, $m_{t_{k-1}}$ must satisfy:

$$m_{t_{k-1}} \geq \frac{\frac{L_{t_{k-1}}}{C_{t_{k-1}}}-1}{\exp\left[h_{t_k}^{-1}\left(q^{\varepsilon}_{(a,T)}\right)\sigma_{t_k}+\mu_{t_k}\right]-1} \tag{44}$$

The second case analysis corresponds to finding the lowest portfolio performance. In fact, as long as the threshold remains positive, even if investors are likely to take more risk of volatility in risky assets, the cushion will remain above the minimum positive. Therefore, if the initial value of the cushion is less than the threshold and the return rate of the risk asset is very low, the risk multiplier has no positive solution. Conversely, if the return on risk assets is high, the risk multiplier has no constraints. But when the return rate of risk assets is in the middle value, in order to pursue better investment income, more risk exposure is needed, so the risk multiplication value must be larger than the minimum value.

5. Empirical Analysis

5.1 Estimation of Yield of CSI 300 Index Under Johnson Distribution

This article selects the daily closing data of The CSI 300 Index for research and analysis. Data from NetEase financial history trading data.Time selected January 2011 to December 2014, a total of 969 daily yield data. During this period, four typical risk asset volatility characteristics are included in the stock market. In 2011 , the market was down and the maximum single - day drop was 3.87 % ;2012 was a shock, the biggest one-day drop was 2.88%; 2013 was a shock and fell, the biggest one-day drop of 6.52%; 2014 was a rise, the biggest one-day drop of 4.59%.

Figure 1. The CSI 300 index trend chart of 2011-2014

(a)Statistical characteristics analysis of sample data

From the histogram of logarithmic yield series of The CSI 300 index, we can see that there is a typical "sharp peak thick tai l" phenomenon in the return rate series. The deviation value is 0.0974, larger than 0, and the kurtosis is 4.772, which is 3 larger than the standard peak value of normal distribution. In addition, the statistical value of JB is 128.3373, p = 0, so the logarithmic income sequence is refused to accept the hypothesis of normal distribution.

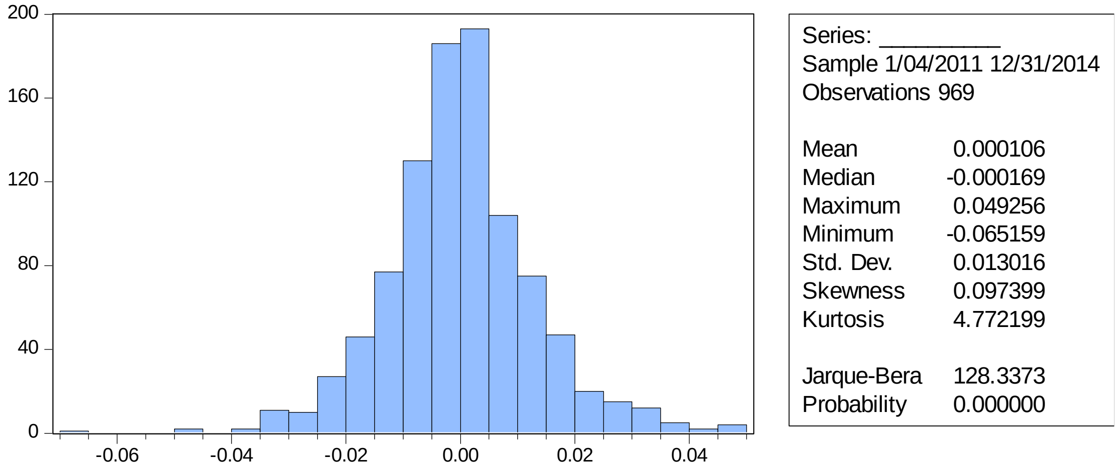

Fig 2. Histogram of logarithmic yield sequence of CSI 300 index

(b)Estimation of Johnson Distribution types and corresponding parameters

From Figure 4-1, we can see that the yield of The CSI 300 index does not obey normal distribution, so it is not consistent with the assumption that the price distribution of risky assets is a normal distribution in the EGARCH model. According to Section 2.2, the Johnson distribution is used to fit the CSI 300 index. Set z=0.74,thus $\Phi(-3Z)$ =0.0132, $\Phi(-Z)$ =0.2296, $\Phi(Z)$ =0.7704, $\Phi(3Z)$ =0.9868. According to the result of the calculation on the basis of (i-0.5)/n, the data x1, x2, x3, x4are located at the position of 13.2908,222.9824and 747.0176,956.7092, respectively, and the corresponding data are -181.382212,-44.100548,51.023498,202.466265, respectively. Thus, l=x4-x3=151.442767, m=x2-x1=143.281663, p=x3-x2=95.124047. Obtained by discriminant 2-2:lm/p2=2.398052,So the transformation equation is chosen as Su distribution. According to the parameter estimation formula provided by 2-3, the results are obtained: δ =1.472410889, γ =-0.053407, λ =90.707016, ε=-0.253803, So the normal transformation is:

$$h_{t_k}(x) = -0.053407 + 1.472411 \sinh^{-1}\left[\frac{x+0.253803}{90.707016}\right]$$

To ensure the validity of the Johnson transformation, we examined the histogram of the transformed sample data and observed whether the JB values fit the normal distribution. Figure 5-3 shows that the CSI 300 index after Johnson transformation is in line with normal distribution.

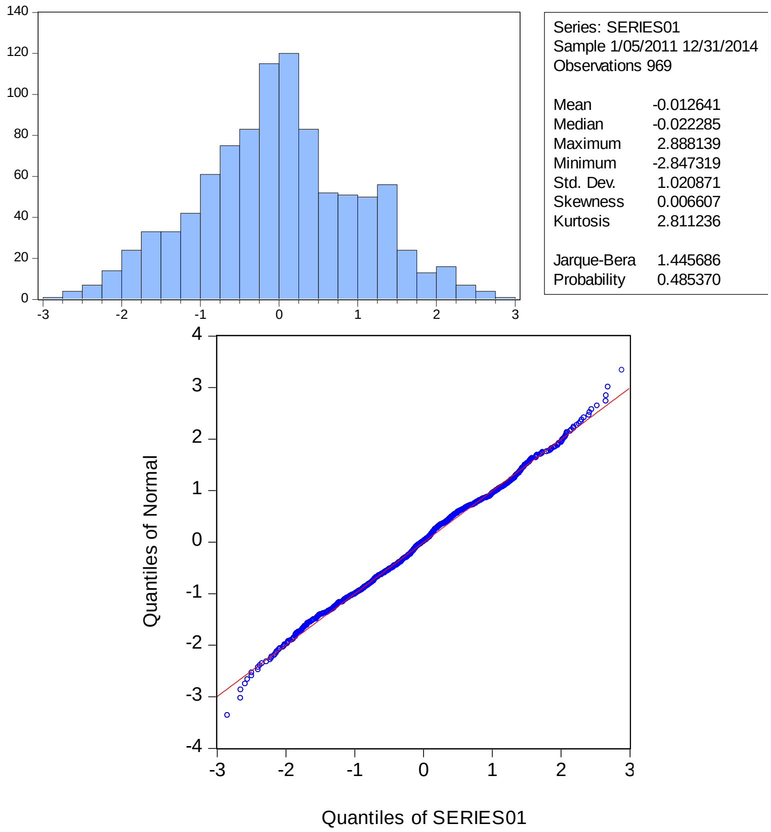

Figure 3. Histogram and QQ plot of The CSI300 Exponential Series after Johnson Transformation

5.2 Portfolio Value of Conditional Risk Multiplier CPPI Strategy Based on Johnson Distribution

The performance of the stock market in the past four years has exactly four typical risk asset volatility characteristics. Therefore, the four years are taken as the data basis of empirical analysis, and one year as the reference time for investment, and then the empirical analysis is carried out year by year. And the yield of one-year national debt as a risk-free rate of return.

Table 1. The CSI 300 Index related indicators of 2011-2014

	Annualized return (%)	Daily of return (%)	Daily volatility (%)	Number of annual transaction days (day)	Daily yield of one-year national debt (%)
2011	-26.46	-0.118	1.3043	244	0.0124278
2012	9.75	0.03	1.281	243	0.0106907
2013	-7.7	-0.0334	1.4024	238	0.0130489
2014	52.18	0.17	1.2084	245	0.0137166

Given different risk tolerance (probability level) and different threshold values, GARCH GARCH（1, 1）model is used to estimate the logarithmic return and volatility of risk assets on Johnsonundefineds converted sample data. The upper bound of the risk multiplier of the market portfolio insurance can be calculated, and the minimum value of the upper bound sequence of the risk multiplier can be selected to analyze the sample data.

Let the value of the portfolio at the beginning of the period be 100, the limit of the end of the term shall be 100 of the value of the portfolio at the beginning of the period, set a threshold $L_{t_{k-1}} = 0.75C_{t_{k-1}}$ for the trading of risky assets, and consider that the commission in the Shanghai and Shenzhen stock exchanges is 3/1000 of the transaction amount. Stamp duty is 1/1000 of the transaction amount, so the total transaction cost is set at 4/1000 of the risk asset adjustment quota.

Comparison of conditional risk multiplier CPPI strategy under Johnson distribution and traditional CPPI strategy in 2011.

Table 2. Comparative analysis of CPPI strategy based on local quintile condition and traditional CPPI strategy

	Proportion to be insured	quintile	risk multiplier	rate of return	fluctuation ratio	Final portfolio value
CPPI Strategy for quintile conditions	1	95%	1.73	2.0002%	1.86%	102.0002
		90%	1.93	1.0001%	1.87%	101.0001
Traditional CPPI strategy	1	-	2	0.2772%	1.52%	100.2772
		-	3	0.0500%	1.72%	100.0500
		-	4	0.0068%	1.81%	100.0068
		-	5	0.0007%	1.86%	100.0007

Select the ratio to be insured is 1,the CPPI strategy of conditional risk multiplier selection based on Johnson distribution is compared with the traditional CPPI strategy. It can be seen from Table 5-2 that under the condition of market decline, the selection of the risk multiplier is reduced under the same condition of the proportion to be preserved by introducing the Johnson distribution and the local quantile conditions, and the higher the quantile, the smaller the conditional risk multiplier. The 95% of the portfolio performed better than the 90% of the portfolio.This is because under the condition that the price of risky assets in the short market continues to fall, the selection of risk multipliers is reduced by introducing the Johnson distribution and local quantile conditions. The conditional risk multiplication of 95 percent of the portfolio is small, which makes more assets transfer to riskless assets, thus enabling investors to meet the requirements of capital preservation and obtain positive returns. At the end of the period, the value of portfolio is greater than the amount to be insured, and there is no gap risk.

Comparison of conditional risk multiplier CPPI strategy under Johnson distribution and traditional CPPI strategy in 2012

Table 3. Comparative analysis of CPPI strategy based on local quintile condition and traditional CPPI strategy

	Proportion to be insured	quintile	risk multiplier	rate of return	fluctuation ratio	Final portfolio value
CPPI Strategy for quintile conditions	1	95%	3.50	18.96%	14.04%	118.9645
		90%	3.83	27.93%	17.84%	127.9320
Traditional CPPI strategy	1	-	2	7.89%	1.17%	103.4534
		-	3	10.12%	2.31%	107.9870
		-	4	15.02%	3.38%	115.0215
		-	5	36.81%	8.37%	136.8089

Select the ratio to be insured is 1,the CPPI strategy of conditional risk multiplier selection based on Johnson distribution is compared with the traditional CPPI strategy. It can be seen from Table 5-3 that under the condition of oscillating closing up market, the selection of conditional risk multiplier can be improved relatively under the same condition of proportion to be preserved by introducing the Johnson distribution and local quantile conditions, and the smaller the quantile is, the greater the conditional risk multiplier, The 90% portfolio performed better than the 95% portfolio because the Johnson distribution and local quartile conditions were introduced to improve the selection of risk multipliers under conditions of consolidation of risk asset prices in the volatile closing market. A 90 percent quartile portfolio has a larger conditional risk multiplier, allowing more assets to be transferred to risky assets, which in turn allows investors to reap gains from the market rally. At the end of the period, the value of portfolio is greater than the amount to be insured, and there is no gap risk.

Comparison of conditional risk multiplier CPPI strategy under Johnson distribution and traditional CPPI strategy in 2013

Table 4. Comparative analysis of CPPI strategy based on local quintile condition and traditional CPPI strategy

	Proportion to be insured	quintile	risk multiplier	rate of return	fluctuation ratio	Final portfolio value
CPPI Strategy for quintile conditions	1	95%	3.46	6.04%	4.45%	106.0443
		90%	3.89	2.04%	4.61%	102.0361
Traditional CPPI strategy	1	-	2	1.33%	1.43%	101.3306
		-	3	0.83%	2.09%	100.8331
		-	4	0.46%	2.69%	100.4615
		-	5	0.22%	3.26%	100.2243

Select the ratio to be insured is 1,the CPPI strategy of conditional risk multiplier selection based on Johnson distribution is compared with the traditional CPPI strategy. It can be seen from Table 5-4 that under the condition of concussion closing and falling market, after introducing Johnson distribution and local quantile conditions, the selection of risk multiplier is relatively reduced under the same condition of proportion to be preserved, and the smaller the quantile is, the bigger the conditional risk multiplier is. The 95% quartile portfolio performed better than the 90% portfolio because the Johnson distribution and local quartile conditions reduced the choice of risk multipliers under conditions of consolidation of risk asset prices in volatile closing markets. The conditional risk multiplication of 95 percent of the portfolio is small, which makes more assets transfer to riskless assets, thus enabling investors to meet the requirements of capital preservation and obtain positive returns. At the end of the period, the value of portfolio is greater than the amount to be insured, and there is no gap risk.

Comparison of conditional risk multiplier CPPI strategy under Johnson distribution and traditional CPPI strategy in 2013

Table 5. Comparative analysis of CPPI strategy based on local quintile condition and traditional CPPI strategy

	Proportion to be insured	quintile	risk multiplier	rate of return	fluctuation ratio	Final portfolio value
CPPI Strategy for quintile conditions	1	95%	5.23	112.48%	22.82%	212.4774
		90%	5.89	114.05%	22.89%	214.0502
Traditional CPPI strategy	1	-	13.02%	3.38%	113.0215	13.02%
		-	26.81%	8.37%	126.8089	26.81%
		-	51.76%	15.64%	151.7570	51.76%
		-	69.73%	19.06%	169.7348	69.73%

Select the ratio to be insured is 1,the CPPI strategy of conditional risk multiplier selection based on Johnson distribution is compared with the traditional CPPI strategy. It can be seen from Table 5-5 that under the condition of long market, after introducing Johnson distribution, the selection of risk multiplier can be improved under the same condition of proportion to be guaranteed, thus increasing the return rate and the value of portfolio at the end of the term. And the smaller the quantile, the bigger the conditional risk multiplier, the better the performance of 90% portfolio is than the 95% portfolio, which is because of the rising risk asset price in the long market. The selection of risk multipliers is improved by introducing Johnson distribution and local quantile conditions. The conditional risk multiplication of 90 percent of the portfolio is larger, which makes more assets in the portfolio transfer to the risk assets, which will enable investors to gain the return when the market goes up. At the end of the period, the value of portfolio is greater than the amount to be insured, and there is no gap risk.

6. Conclusion

Because of the non-normality of the return on risk assets, we introduce the Johnson distribution, and transform the non-normal distribution of the return on risk assets into normal distribution. Furthermore, when using GARCH model to estimate the volatility of return on risky assets, the standard normal distribution can be used directly, and the accurate volatility of return on risky assets can be obtained. Under Johnson distribution system, the upper bound of conditional risk multiplier can be obtained by introducing local quantile condition, that is, conditional risk multiplier is a function of given probability level, threshold value and volatility rate of return on risky assets. The empirical results show that the threshold $L_{t_{k-1}} = 0.75C_{t_{k-1}}$,the lower the probability level (quantile), the higher the conditional risk multiplier and the higher the return rate of portfolio when the market is in long and oscillating. When the market is in short and oscillatory closing, the threshold $L_{t_{k-1}} = 0.75C_{t_{k-1}}$, the higher the probability level (quantile), the smaller the conditional risk multiplier and the higher the portfolio return. No matter which market the market is in, the portfolio end value of conditional risk multiplier CPPI strategy based on Johnson distribution is above the guaranteed value, and there is no gap risk.

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