Research on Key Technologies of Image Restoration

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Abstract

Computer vision and image processing experiment software for image processing technology has appeared in the market. For beginners and algorithm researchers of image processing, the system used to develop learning experiments has become a major research topic for professionals. This article focuses on the image restoration technology, and discusses the inverse filtering, Wiener filtering, and constrained least squares filtering algorithms in detail. This article mainly deals with the commonalities faced by beginners and algorithm researchers.

Keywords

Image Restoration ; Image Recovery; Inverse Filtering; Wiener Filter.

1. Introduction

The classical image restoration algorithm occupies an important position in image processing. The prerequisite for its restoration is the need to know the point spread function and the noise distribution. Some of these algorithms appear early and have achieved very good results. They have been widely used.

2. Classical algorithm introduction

There are two types of image restoration algorithms, linear and nonlinear. The linear algorithm achieves deconvolution through inverse filtering of the image. Such a method is convenient and quick, without loop or iteration, and the deconvolution result can be obtained directly. However, it has some limitations, such as the non-negativity of the image cannot be guaranteed. The non-linear algorithm continuously improves the quality of restoration through a continuous iterative process until the preset termination conditions are satisfied, and the results are often satisfactory. However, the iterative process leads to a large amount of calculations, and it takes a long time to recover the image, sometimes it may take several hours. Therefore, in practical applications, it is also necessary to consider the two processing methods comprehensively and select them.

The Wiener filtering method was proposed by Wiener and achieved good results in processing one-dimensional signals. After that, the Wiener filter method was used for two-dimensional signal processing, and it also achieved good results. Especially in the image restoration field, Wiener filtering has a small amount of calculation and a good recovery effect, and thus has been widely used and developed.

The EM algorithm is an iterative algorithm proposed by Dempster et al. for estimating the maximum likelihood of a parameter. It is an algorithm for estimating parameters of a probabilistic model from incomplete observation data. This algorithm is widely used in incomplete data processing and analysis.

For inverse filtering, inverse filtering is performed on the image to achieve deconvolution. This type of method is fast and convenient. It does not require looping or iteration. It can directly obtain deconvolution results.

The constrained least squares method is easily implemented by a simple program of the computer but cannot obtain such a certain solution of irrational roots.

Through comparison and analysis, the image restoration algorithm of this paper chooses inverse filtering algorithm, Wiener filtering algorithm and constrained least squares restoration algorithm. The above three algorithms are relatively simple and easy to understand, and user learning can be easily used.

3. Image degradation/restoration model

Image restoration technology is an attempt to use the prior knowledge of the degenerative process to restore the degraded image to its original condition, that is, to analyze the degraded environmental factors according to the cause of the degradation, establish a corresponding mathematical model, and restore it along the inverse process of degrading the image. The purpose is to eliminate or reduce the degradation of image quality caused by image acquisition and transmission, and to restore the original image. Therefore, the recovery technique is to model the degradation and use the opposite process to process the original image.

If an original image is used, an observation image is generated by a degenerate function and an additive noise term. In general, the degenerate function can be considered linear, position-invariant, and the noise is independent of the position and the current pixel value. The degradation process can be modeled as follows.

$$g(x,y) = f(x,y) * h(x,y) + n(x,y) \tag{1}$$

Where $h(x,y)$ is the spatial description of the degenerate function, also known as the PSF, the point spread function. * indicates space convolution and $n(x,y)$ is additive noise. The degenerate model of formula (1) can be expressed in the form of a vector matrix as the following equation.

$$g = Hf + n \tag{2}$$

In equation (2), g is the observation image, assuming that its size is N, f is a sample, and n is noise. g, f, and n are the same size, they are N^2*1 column vectors, and H is the $N^2 * N^2$ PSF parameter matrix. If it is a spatially invariant PSF, then H is a block circulant matrix. The problem of estimating f from the model is called the linear reversal problem, which is also the basis of classical image restoration research.

4. Principle of algorithm

4.1 Inverse filter algorithm

The inverse filter method is the earliest use of an unconstrained recovery method, which is commonly used to process degraded images transmitted from a spacecraft. The algorithm is as follows.

$$\| \mathrm{n} \|^2 = n^T n = \left\| g - H\hat{f} \right\|^2 = \left(g - H\hat{f} \right)^T \left(g - H\hat{f} \right) \tag{3}$$

For the image degeneration model of formula (3), when the statistical property of n is uncertain, one f-estimator needs to be searched so that H_f is the closest to g in the sense of the minimum mean square error, that is, the modulus or norm of *n* is to be minimized. .

According to the above formula, the recovery problem can be regarded as the minimum value for the f, as shown below.

$$L(f)=\left\|g-H\hat{f}\right\|^2 \tag{4}$$

Differentiating L and setting the result to zero, and then setting M=N and H-1 to exist, the unconstrained recovery formula can be obtained, as shown below.

$$\hat{f}=\left(H^TH\right)^{-1}H^Tg=H^{-1}\left(H^T\right)^{-1}H^Tg=H^{-1}g \tag{5}$$

According to the discussion of the circulant matrix diagonalization, formula (5) can be written as the following form of estimation.

$$\hat{F}(\mathrm{u,v})=\frac{G(u,v)}{H(u,v)} \tag{6}$$

Then use the inverse Fourier transform of $F(u,v)$ to obtain the corresponding estimate of the image. This method is called inverse filtering and the recovered image can be expressed by formula (7).

$$\hat{f}(x,y)=F^{-1}\left[\hat{F}(u,v)\right]=F^{-1}\left[\frac{G(u,v)}{H(u,v)}\right] \tag{7}$$

From equation (7) we can see that if $H(u,v)$ is zero or very small on the u plane, it will bring about computational difficulties. On the other hand, noise can also cause more serious problems. If noise is added, the following formula can be obtained.

$$\hat{F}(u,v)=F(u,v)+\frac{N(u,v)}{H(u,v)} \tag{8}$$

From equation (8), we can see that if $H(u,v)$ is zero or very small on the u and v planes, $N(u,v)$ and $H(u,v)$ will make the recovery result greatly different from the expected result. In practice, $H(u,v)$ decreases rapidly with the increase of the distance between the (u,v) and the origin, while the noise $N(u,v)$ changes slowly. In this case, recovery can only be done closer to the origin (closer to the center of the frequency domain), so in general the inverse filter does not happen to be exactly $H(u,v)$, and some function of u and v can be recorded. For $M(u,v)$, often referred to as the recovery transfer function, an improved method is shown below.

$$M(u,v)=\begin{cases} K, & H(u,v)\le d \\ H(u,v), & other \end{cases} \tag{9}$$

Where K and d are constants less than 1, and d is selected to be smaller.

4.2 Wiener filtering algorithm

Wiener filtering is one of the earliest and most well-known linear image restoration methods. Wiener filtering is based on the premise that the image signal can be approximated as a stationary random process. According to the statistical error between $F(x,y)$ and $\hat{f}(x,y)$, e^2 achieves the minimum criterion to achieve image recovery. The formula is as follows.

$$e^2=\min E\left[\left[f(x,y)\right]-\hat{f}(x,y)\right]^2 \tag{10}$$

In the formula, E represents the expectation operator, $f(x,y)$ represents the non-degraded image, and $\hat{f}(x,y)$ is the recovered image. If we consider the restoration as satisfying the condition of formula (10), we select the linear operator Q (transformation matrix) of known $\hat{f}$ so that Q and $\hat{f}$ are the smallest. You can usually solve this problem with the Lagrangian multiplier method. Let a be the Lagrangian multiplier and find $\hat{f}$ that minimizes the following criteria functions. The formula is as follows.

$$L\left(\hat{f}\right)=\left\|Q\hat{f}\right\|^2+a\left(\left\|g-H\hat{f}\right\|^2-\|n\|^2\right) \tag{11}$$

As with formula (11), there is a constraint recovery formula (let s=1/a), as shown below.

$$\hat{f}\left[H^TH+Q^TQ\right]^{-1}H^Tg \tag{12}$$

Define $R_f=E\left\{ff^T\right\}$, $R_n=E\left\{nn^T\right\}$, define $Q^TQ=R_f^{-1}R_n$, and substitute it into equation (12) to get the frequency domain expression, as shown below.

$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{\left|H(u,v)^2\right|}{\left|H(u,v)^2+S_n(u,v)S_f(u,v)\right|}\right]G(u,v) \tag{13}$$

Among them, $H(u,v)$ represents the degeneracy function, $S_n(u,v)$, $S_f(u,v)$ noise power ratio.

$S_n(u,v)=\left|N(u,v)\right|^2$ indicates the power spectrum of the noise.

$S_f(u,v)=\left|F(u,v)\right|^2$ represents the power spectrum of the non-degraded image.

As long as the inverse Fourier transform is obtained for $\hat{F}(u,v)$, the recovered image $\hat{F}(u,v)$ can be obtained. It can be seen that there is no pole in the Wiener filter. Even when $H(x,y)$ is equal to 0, the denominator of the Wiener filter is at least equal to the noise power ratio, so the noise is suppressed. The power of the signal and the noise is usually not known. Instead of $S_n(u,v)$, $S_j(u,v)$ is replaced by a constant array *K*. Then formula (13) is approximated by the following equation.

$$\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2+K}\right]G(u,v) \tag{14}$$

It can be seen that when K is 0, the Wiener filter is converted into a standard inverse filter, and the inverse filter is strictly derived from the degeneracy model. So when *K* is not equal to 0, although the noise can be suppressed from expanding, the recovered model does not have the deconvolution filter accurate, resulting in distortion of the restoration. The larger K is, the better the effect of suppressing noise is, but the recovery is inaccurate and the image is blurred. The smaller *K* is, the more accurate the restoration is, but the noise suppression effect is not good.

4.3 Constrained Least Square Filtering

The constrained least-squares filter formula starts from the following equation to determine the transformation matrix Q. In order to reduce the oscillation, an optimal criterion based on a smooth measure can be established. For example, some second-order differential functions may be minimized, and the second-order differential of $f(x,y)$ at (x,y) may be approximated as follows.

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial v^2}\approx 4f(x,y)-\left[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)\right] \tag{15}$$

The second-order differential can be obtained by convolving $f(x,y)$ with the following operators.

$$p(x,y)=\begin{pmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{pmatrix} \tag{16}$$

The optimal criterion based on this second-order differential is as follows.

$$\min\left[\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\right]^2 \tag{17}$$

The constraints of this function are as follows.

$$\left\| g-H\hat{f} \right\|^2=\|n\|^2 \tag{18}$$

The frequency domain solution to this optimization problem is given by equation (19).

$$\hat{F}(u,v)=\left[\frac{H*(u,v)}{|H(u,v)|^2+s|P(u,v)|^2}\right]G(u,v) \tag{19}$$

Where s is a parameter that must be adjusted so that the constraints are satisfied and $P(u,v)$ is the Fourier transform of the function $P(x,y)$.

5. Image restoration evaluation method

Artificial evaluation of the image restoration effect: The restoration degree of the restored image is evaluated by human being, and the results obtained by the evaluation are obtained by the average score of a certain number of observers.

$$C=\frac{\sum_{i=1}^{N} n_i c_i}{\sum_{i=1}^{N} n_i} \tag{20}$$

c_i is the score belonging to category *i*; n_i is the number of observers who judge the image to belong to category *i*. The objective evaluation method is as follows.

(a). Mean Squared Measure (MSE)

$$MSE=\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[f(i,j)-\overline{f}(i,j)\right]^2 \tag{21}$$

In the formula(21): $f(i,j)$ and $\overline{f}(i,j)$ are the original image and the restored image, respectively, and the definition of the relative mean square error MSE can also be obtained from the mean square error measure.

(b). Signal to Noise Ratio Measurement (SNR)

$$SNR=10\lg\left\{\frac{\sum_{i=1}^{M}\sum_{j=1}^{N} f(i,j)^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}\left[f(i,j)-\overline{f}(i,j)\right]^2}\right\} \tag{22}$$

(c). PSNR measurement: (PSNR)

$$PSNR = 10\lg\left\{\frac{2SS^2}{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\left[f(i,j)-\bar{f}(i,j)\right]^2}\right\} \tag{23}$$

(d). Signal to Noise Ratio Improvement Measure: (ISNR)

$$ISNR = 10\lg\left\{\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\left[f(i,j)-\bar{g}(i,j)\right]^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}\left[f(i,j)-\bar{f}(i,j)\right]^2}\right\} \tag{24}$$

6. Conclusion

This article mainly studies and expatiates on the following three aspects: the introduction of classical algorithms, image degradation/restoration model, and image restoration evaluation methods, and has achieved good results. In addition, the problem of further optimization of the algorithm, image processing algorithms developed rapidly in recent years, how to further study more optimized and efficient algorithms based on actual work, these are worthy of further exploration.

Acknowledgments

This work was financially supported by the National Spark Program (No.2015GA740054), the Binzhou University Research Foundation(No. BZXYG1713) and Binzhou University "Dual Service" Project (No. BZXYSFW201708).

References

[1] Shen Huan, Li Shunming, Mao Jianguo, Xin Jianghui. Digital Image Restoration Techniques: a review[J]. Chinese Journal of Image and Graphics, 2009,14(9): 1764-1775.

[2] Hao Jiankun, Huang Wei, Liujun, He Yang. Review of Non-blind deconvolution image restoration based on spatially-varying PSF[J]. Chinese Optics, 2016, 9(1):41-50.

[3] Jin Fei, Zhang Bin, Si Xuan, Yuan Congxin. Image Restoration Based on Wiener Filtering [J]. Journal of Communication University of China(Science and Technology),2011, 18(4):19-23,58.

[4] Zhu Ming, Yang Hang, He Bogen et al. Image Motion Blurring restoration of Joint Gradient Prediction and Guided Filter [J]. Chinese Optics, 2013, 6(6):850-855.

[5] Rudin S. Osher, E. Fatemi. Nonlinear total variation based noise removal algorithms [J]. Physical D: Nonlinear Phenomena, 2012, 60(4): 259-268.

[6] Chan, J. Shen. Mathematical models for local non-texture inpaintings [J]. SIAM J App, Math, 2013, 62(3): 1019-1043.

[7] Bao Songjian. Image Restoration Algorithm Research Based on Improved Non-local Average Wiener Filtering[J]. Journal of Chongqing Normal University (Natural Science), 2016, 33(5): 108-112.

[8] Yang Junfeng, Yin Wotao, Zhang Yin, Wang Yilun. A Fast Algorithm for Edge-Preserving Variational Multichannel Image Restoration[J]. Siam Journal on Imaging Sciences, 2009, 2(2):569-592.