# Biarticular robot sliding mode control

Guojing Li [a], Shengbin Hu [b]
Shanghai University of Engineering Science, Shanghai 201620, China.
[a]575992367@qq.com, [b]65740824@qq.com,

## Abstract

Sliding mode control is a special class of nonlinear control with strong anti-interference capability and strong robustness. For nonlinear dynamic model is set up two joint robot, the sliding mode controller is designed, the stability of system is proved by using Lyapunov theorem, the simulation results show that the design of sliding mode control can well realize the tracking control of robot two joint performance.

## Keywords

Sliding mode control; Biarticular robot; Sliding mode control rate.

## 1. Introduction

Robot system is a time-varying, strong coupling, multi-input multi-output nonlinear systems, nonlinear dynamic characteristics, and inevitably exists uncertainty in actual application, its control is rather complicated [1]. Sliding Mode Control (SMC) is essentially a special type of nonlinear Control, which has become an effective Control method due to its strong robustness [2]. In this paper, a sliding mode control method is proposed and a simulation is carried out for the two-joint robot.

## 2. Biarticular robot dynamics model

The biarticular robot is shown below see Fig1.

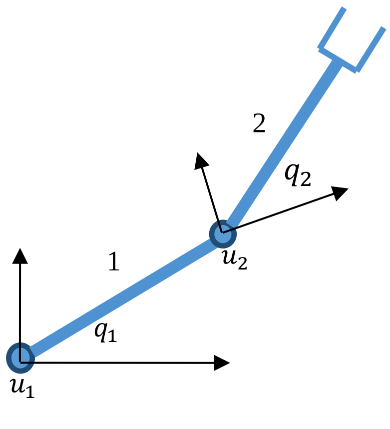


Fig. 1 biarticular robot

The d-h coordinate was established, and the Lagrange modeling method was used to deduce its nonlinear dynamic model as follows:

$$M\ddot{q}+C\dot{q}+G+\tau_d=\tau \tag{1}$$

$\tau_d$ is modeling error and external interference,Set it to zero, where,

$$M=\begin{bmatrix} a_1+2a_4\cos q_2 & a_2+a_4\cos q_2 \\ a_2+a_4\cos q_2 & a_3 \end{bmatrix}$$

$$C=a_4\sin q_2\begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1+\dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}$$

$$G=\begin{bmatrix} b_1\cos q_1+b_2\cos(\mathrm{q}_1+\mathrm{q}_2) \\ \mathrm{b}_2\cos(\mathrm{q}_1+\mathrm{q}_2) \end{bmatrix}$$

$$\ddot{q}=\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, \dot{q}=\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

then:

$$\begin{bmatrix} a_1+2a_4\cos q_2 & a_2+a_4\cos q_2 \\ a_2+a_4\cos q_2 & a_3 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}+a_4\sin q_2\begin{bmatrix} -\dot{q}_2 & -(\dot{q}_1+\dot{q}_2) \\ \dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}+ \begin{bmatrix} b_1\cos q_1+b_2\cos(\mathrm{q}_1+\mathrm{q}_2) \\ \mathrm{b}_2\cos(\mathrm{q}_1+\mathrm{q}_2) \end{bmatrix}=\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{2}$$

Where, $q_1, q_2$ are the rotation angles of link 1 and link 2 respectively, and $\tau_1, \tau_2$ are the motor rotation moments of link 1 and link 2 respectively. $a_1, a_2, a_3, a_4, b_1, b_2$ are normal numbers related to the mass of the two links the moment of inertia the length and the gravitational acceleration. Where, are the rotation angles of link 1 and link 2 respectively, and are the motor rotation moments of link 1 and link 2 respectively. Are normal Numbers related to the mass of the two links the moment of inertia the length and the gravitational acceleration.

## 3. Sliding mode controller design

### 3.1 Sliding mode control

The goal of robot tracking control is to ask the vector $q$ to track the specified amount of angular displacement $q_r$ as well as possible.

For the nonlinear dynamic model shown in equation (1), the feedback line technology can be adopted. It may be set $z_1=q, z_2=\dot{q}$,the feedback controller can be designed:

$$\tau=Mu+C\dot{q}+G \tag{3}$$

To convert equation (1) into the following linear form:

$$\begin{cases} \dot{z}_1=z_2 \\ \dot{z}_2=u \end{cases} \tag{4}$$

where $u$ is the control quantity that needs design. Define tracking error:

$$e=q_r-q \tag{5}$$

Where, $q=\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, q_r=\begin{bmatrix} q_{r1} \\ q_{r2} \end{bmatrix}$.

The design sliding surface is:

$$s=\dot{e}+\Lambda e \tag{6}$$

Where, $\Lambda=diag\left(\Lambda_1,\Lambda_2\right), e=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}, \dot{e}=\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix}$.

Using the above linear feedback method, the sliding mode control rate is designed as:

$$u = M_0(\ddot{q}_r + \Lambda \dot{e}) + C_0\dot{q} + G_0 + K\,\mathrm{sgn}(s) \tag{7}$$

Where, $\mathrm{sgn}(s) = \begin{bmatrix} \mathrm{sgn}(s_1) \\ \mathrm{sgn}(s_2) \end{bmatrix}$, $\mathrm{sgn}(s_i) = \begin{cases} 1 & s_i > 0 \\ 0 & s_i = 0 \\ -1 & s_i < 0 \end{cases}$ $(i = 1, 2)$ , $K = diag(k_1, k_2)$ , $M_0$ , $G_0$ , $C_0$ is the nominal value of $M$ , $G$ , $C$ .set $M_0 = 0.8M$ , $G_0 = 0.8G$ , $C_0 = 0.8C$ .

### 3.2 Stability analysis

Define the Lyapunov function:

$$V = \frac{1}{2}s^T s \tag{8}$$

$$\begin{aligned} \dot{V} &= s^T \dot{s} \\ &= s^T(\ddot{q}_r - u + \Lambda \dot{e}) \end{aligned} \tag{9}$$

So let's put equation (7) in,

$$\begin{aligned} \dot{V} &\le -K|s| \\ &\le 0 \end{aligned} \tag{10}$$

The Lyapunov theorem shows that the system is stable.

## 4. The simulation

Trajectory tracking control simulation was carried out for the two-joint robot to verify the effectiveness of control rate. MATLAB language was used to write the program and the simulation was carried out on the MATLAB R2014a platform.

Simulation parameters are as follows:

$a_1 = 200.1$, $a_2 = 23.5$, $a_3 = 122.5$, $a_4 = 25$, $b_1 = 784.8$, $b_2 = 245.25$,

$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$,The reference trajectory is: $q_{r1}(t) = q_{r2}(t) = \frac{\pi}{2}\left[1 - e^{(-5t)(1+5t)}\right]$.

The simulation results are shown below,see Fig.1,Fig.2.

## 5. Conclusion

For trajectory tracking of robot manipulators with two joints, a sliding mode controller design, can be seen from the simulation results, under the control of the position of the two joints, the robot tracking effect is very good, the method for this kind of multiple input multiple output nonlinear system effect is very good.

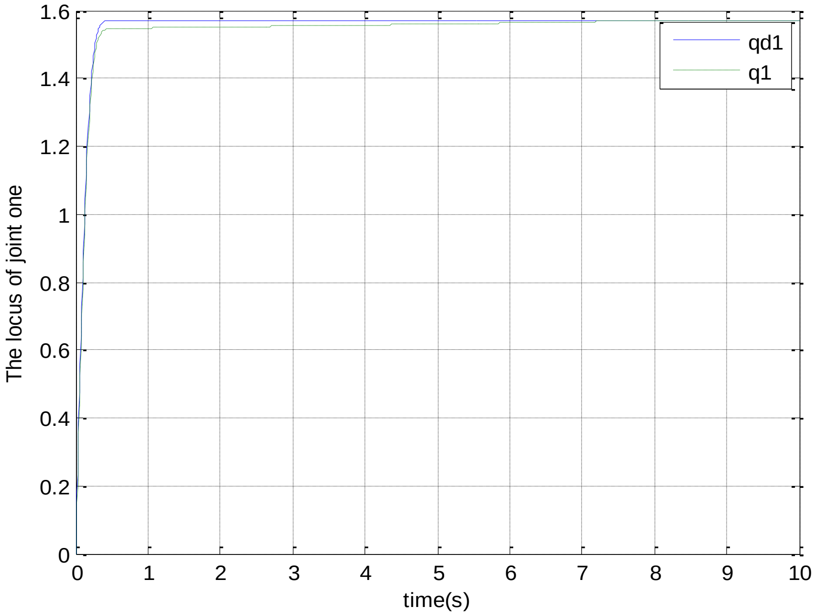


Fig. 2 joint 1 position tracking

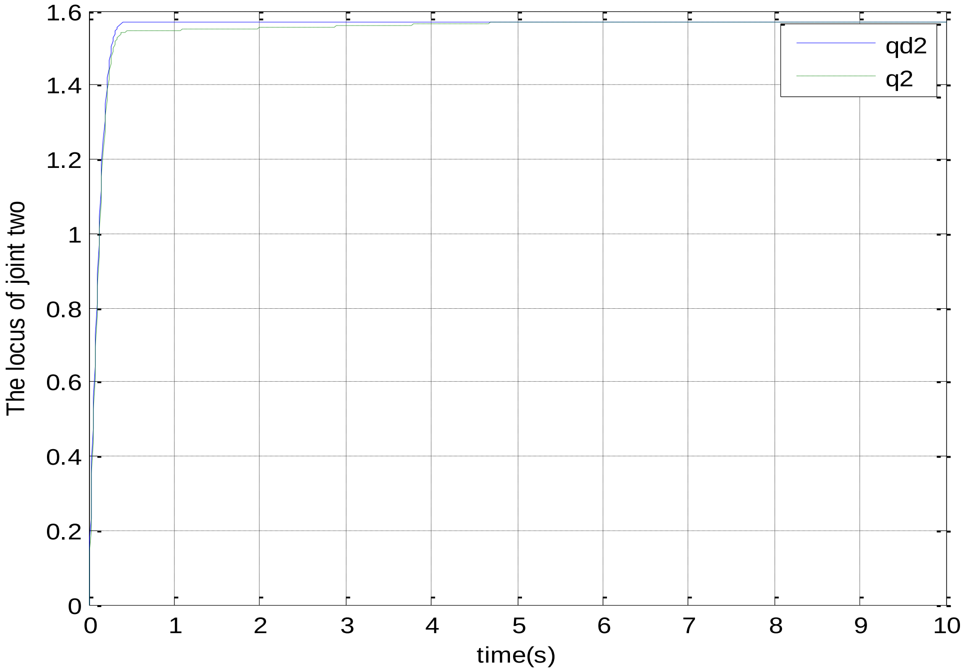


Fig. 3 joint 2 position tracking

## Acknowledgements

First and foremost, I appreciate my country who give me a comfortable learning atmosphere. Second, I would like to show my deepest gratitude to my supervisor, Prof. Hu Shengbin, who has walked me through all the stages of the writing of this paper. Without his illuminating instruction and patience, this paper could not have reached its present form.

## References

[1] YU Di. Nonsingular Terminal Neural Sliding Mode Control for Trajectory Tracking of Robot Manipulators [J]. Computer Simulation,2009,26(8):162-166

[2] LIU Jinkun, SUN Fuchun. Research and development on theory and algorithms of sliding mode control [J]. Control Theory and Applications,2007,24(3):407-418

[3] LIU Jinkun.Sliding Mode Control and MATLAB Simulation (Third Edition)The Basic Theory and Design Method[M].BeiJing:TSINGHUA UNIVERSITY PRESS,2015.

[4] HU Shengbin. Fuzzy Integral Sliding Mode Control for Three－Links Spatial Robot [J]. Computer Simulation,2012,29(2):162-166