Research on Contract Coordination of Logistics Service Supply Chain Competing by Two Providers under Stochastic Demand

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Abstract
To enhance the competitiveness of the logistics service providers, and improve overall operating efficiency of the logistics, there is competition between the two logistics service provider and a logistics service integrator for the study of the composition, this paper studies the contract coordination problem of the secondary logistics service supply chain under stochastic demand. This paper expresses the demand of the two logistics service providers after the competition with effective demand, and establishes the stackelberg master-slave coordination model under the centralized decision model and the revenue sharing contract respectively, and obtains the conditions that the optimal solution needs to be satisfied under different coordination methods. The research results show that when the revenue sharing contract can coordinate the logistics service supply chain system composed of two competitive logistics service providers and one logistics service integrator, the supply chain system reaches the most in the coordination of revenue sharing contracts. By adjusting the revenue sharing factor, positive incentives can be achieved for each member of the system. The research results provide a theoretical reference for decision-making in logistics service supply chain related enterprises.

Keywords
Stochastic demand, logistics service supply chain, revenue sharing contract.

1. Introduction
In recent years, the One Belt and One Road Initiative has brought significant opportunities to the development of the domestic logistics industry. As the characteristics of China's service industry leading economic growth become more and more obvious, the coordinated operation of the logistics service supply chain has become particularly important. In September 2016, the National Development and Reform Commission promulgated the “Special Action Plan for Cost Reduction and Efficiency Improvement in Logistics Industry”, focusing on solving many existing problems in the logistics service supply chain, establishing a modern logistics service system, and improving the overall development level of the industry to better serve it. In the social and economic development. At present, how to effectively improve the quality and efficiency of logistics service, reasonably distribute the benefits among members of the logistics service supply chain, and meet the logistics demands of e-commerce enterprises for diversified, decentralized and timely sales has become an urgent problem for China's logistics industry.
Like product supply chain, logistics service supply chain is also composed of multiple members: functional logistics service provider (FLSP), logistics service integrator (LSI), etc. Due to the inconsistency of interests among members, an effective contract mechanism must be designed to coordinate the operation of logistics supply chain. As an important form of supply chain contract, revenue sharing contract can better solve the coordination problem of supply chain[1]. It has been
widely used in the management of product supply chain, but the characteristics of service make it impossible to copy the research results of product supply chain in the coordination research of logistics supply chain \[2\]. Therefore, it is particularly necessary to study the contract coordination of logistics service supply chain under the competition between two providers of random demand.

2. Logistics Service Supply Chain Coordination Research

Logistics service supply chain coordination means that after the establishment of logistics service supply chain, during the operation process of logistics projects, integrated management and decision-making are carried out on the interdependence among members of the operation activities, so as to achieve the goal of improving the performance of each member of the supply chain and the overall performance of the supply chain \[3\].

Scholars have studied the coordination of revenue-sharing contract on logistics service supply chain, and most of the research objects are concentrated in the system composed of a single logistics service integrator and a single logistics service provider: Tian Yu and Wu Peixun (2006), taking into account the two parameters of wholesale price and the quantitative ratio of sales revenue, respectively constructed the revenue sharing contract model of two-stage logistics service supply chain and three-stage logistics service supply chain, and provided the effective conditions under different revenue sharing models \[4]. He Meiling et al. (2010) took wholesale price as a constant, studied the input-sharing contract problem of determining the supply chain of secondary logistics service with uncertain demand, and provided the optimal conditions for the supply chain to reach \[1\]. Liu Weihua (2010) studied the determination method of the optimal revenue sharing coefficient of the three-level logistics service supply chain and obtained the optimal value of the revenue sharing coefficient \[2\]. Duan Huawei et al. (2016) focused on the impact of assuming social responsibility on the income of logistics service supply chain, and concluded that the coordination of income sharing contract on decentralized decision-making logistics service supply chain could achieve better results \[5\]. Meng Lijun et al. (2014) proved that revenue sharing contract can make logistics service supply chain reach the overall optimal state through comparative analysis of different coordination models. Compared with wholesale price contract, Pareto improvement was realized \[6\].

For the related research on the logistics service supply chain, there are relatively few articles considering two or more logistics service providers: Meng Lijun (2012) studied how to make contract selection in the secondary logistics service supply chain composed of "1LSI-2FLSPs" under the linear demand and from the perspective of provider of followers \[7\]. Jing Youguo (2015) studied the optimal decision-making of service effort level between a single logistics service integrator and two competing functional logistics service providers under linear demand \[8\]. Shi Kuiran et al. (2017) studied four modes of cooperation for the logistics service supply chain consisting of two functional logistics service providers and a single logistics service integrator involved in the supply chain service platform under the linear demand, and obtained the optimal conditions by building a game model \[9\].

These scholars have studied the logistics service supply chain composed of two suppliers from different aspects, but the demand assumptions of the supply chain are all linear, and few literatures consider the case of multiple logistics service providers under random demand. In fact, market demand will be affected by many factors, and overall there will be some randomness and uncertainty. With the increasingly fierce competition among enterprises, the supply chain structure of logistics services provided by a single provider will be gradually replaced by the structure of two or more competing suppliers. Therefore, it is of more practical significance to study the logistics supply chain in which multiple logistics service providers compete under stochastic demand. On the basis of Meng Lijun et al. \[6\], this study considers two logistics service providers with competitive relations and discusses how to coordinate the supply chain of secondary logistics service by revenue sharing contract.
3. Problem description and hypothesis

In this paper, a two-level LSSC system consisting of one logistics service integrator (LSI) and two functional logistics service providers (FLSP) is considered. The structure is shown in figure 1. Assuming that LSI and two FLSPs are both limited rationality and risk neutral, logistics customers outsource logistics business to LSI according to market demand, and LSI purchases logistics service capacity from FLSPi (i =1,2) according to customer demand.

\[
D = D_1 + D_2
\]

The logistics customer's total demand for logistics services of LSSC is \( D \geq 0 \), which is a random variable, and \( D_i \) is the initial logistics demand faced by FLSP\(_{(i=1,2)}\), and \( D = D_1 + D_2 \). \( F_i(x) \) and \( f_i(x)(x \geq 0) \) respectively represent the distribution function and density function of the stochastic demand faced by FLSP\(_{(i=1,2)}\). \( F_i(x) \) is a continuously differentiable strictly increasing function, \( F_i(0) = 0 \). Assume that a unit of logistics service demand requires one unit of logistics service capacity to meet. For LSI, the cost of purchasing each unit of logistics service from FLSP\(_{(i=1,2)}\) is different. FLSP\(_{(i=1,2)}\) provides alternative logistics services of the same type, so the two compete in customer share. The competitive behavior between FLSP\(_{(i=1,2)}\) will cause the LSI's demand for the two to change, and the changed demand is the effective demand. Assuming that after the competition between FLSP\(_1\) and FLSP\(_2\), FLSP\(_1\)'s logistics service is better than that of FLSP\(_2\), LSI will give more logistics service orders to FLSP\(_1\), and part of FLSP\(_2\)'s demand will be transferred to FLSP\(_1\), namely

\[
D_i^e = D_i + a_i (D_i - Q_i^0)^{+}
\]

\( a_i \) (\( 1 \geq a_i \geq 0 \)) represents the proportion of logistics service orders transferred from FLSP\(_2\) to FLSP\(_1\).

According to Lippman, McCadle (1997) \([10]\) and Anupindi, Bassok (1999) \([11]\), the probability distribution function of the market effective demand \( D^e \) is continuous, their distribution function is denoted by \( F^e_i(x) \), and the probability density is denoted by \( f^e_i(x) \). Use \( S(Q_i) \) to represent the volume of license plate logistics capacity of FLSP\(_i\), namely

\[
S(Q_i) = \int_0^Q x f^e_i(x)dx + \int_Q^\infty Q f^e_i(x)dx = Q_i - \int_0^Q F_i(x)dx
\]

\( p \) is the unit price of LSI to provide logistics services to logistics customers, \( c_i \) is the marginal cost of LSI, \( c_i \) is the marginal cost of FLSP\(_{(i=1,2)}\) to provide logistics services to LSI, and will be punished when LSI cannot meet customer logistics needs, unit penalty cost \( g \) (\( g > 0 \)). Under the centralized decision, LSI orders the logistics service capability of \( Q_{Ci} \) unit from FLSP\(_{(i=1,2)}\), and the total logistics service order quantity of LSI is \( Q_C \). Under the revenue sharing contract, LSI proposes a revenue sharing contract \( (w_i, k_i) \) to FLSP\(_{(i=1,2)}\), where \( w_i \) is the purchase price of LSI to FLSP\(_1\) order logistics service under the revenue sharing contract, and \( k_i \) (\( k_i \geq 0 \)) is the revenue sharing coefficient provided by LSI to FLSP\(_1\) under the revenue sharing contract. When LSI is punished, FLSP\(_{(i=1,2)}\) needs to share the penalty cost with LSI. The unit penalty cost of both FLSP\(_{(i=1,2)}\) is the same as \( g \) (\( g > 0 \)). FLSP\(_{(i=1,2)}\) considers the LSI to order \( Q_{RFi} \) logistics service from FLSP\(_{(i=1,2)}\) from the perspective of maximizing its own interests to benefit the most. Under the established conditions, LSI can directly benefit from the subscription of \( Q_{RLi} \) unit logistics services to FLSP\(_{(i=1,2)}\). \( Q_{RLi} \) is the total
logistics service order quantity of LSI in LSSC under the revenue sharing contract. LSI undertakes the integration of various service elements and links and carries out whole-process management, which is the core of LSSC and an absolute leading position.

4. Model establishment and analysis

4.1 Basic decision model

Centralized decision-making model

Centralized decision-making model is a common analysis method in the research of supply chain coordination. The integrator and the two providers are regarded as a whole and the overall profit maximization of LSSC is taken as the action goal to make decisions. The revenue obtained here is the optimal revenue that can be obtained by the LSSC system. The overall revenue function $\pi_C$ of the system is:

$$\pi_C = p \sum_{i=1}^{2} S(Q_{Ci}) - g \sum_{i=1}^{2} c_i Q_{Ci} - g \sum_{i=1}^{2} (D_i - Q_{Ci})^+$$

Theorem 1 Under the centralized decision, the expected return function $E(\pi_C)$ of the LSSC is a convex function of the logistics service supply quantity $Q_{Ci}$ of the two providers, that is, there is an optimal logistics service supply quantity $Q_{Ci}^*$ and the satisfaction of $F_i^e(Q_{Ci}^*) = 1 - \frac{c_i}{p + g}$ makes the overall expected supply revenue of the supply chain the largest.

Proof the expected return of LSSC

$$E(\pi_C) = p \sum_{i=1}^{2} (\int_{Q_{Ci}}^{Q_{Ci}^*} x f_i^e(x) dx + \int_{Q_{Ci}^*}^{\infty} Q_{Ci} f_i^e(x) dx) - g \sum_{i=1}^{2} c_i Q_{Ci} - g \sum_{i=1}^{2} \int_{Q_{Ci}}^{\infty} (x - Q_{Ci}) f_i^e(x) dx$$

Find a first-order partial derivative from $Q_{Ci}$ by equation (1):

$$\frac{\partial E(\pi_C)}{\partial Q_{Ci}} = (p + g)[1 - F_i^e(Q_{Ci})] - c_i$$

Further find the second-order partial derivative of equation (1):

$$\frac{\partial^2 E(\pi_C)}{\partial Q_{Ci}^2} = -(p + g) f_i^e(Q_{Ci})$$

Obviously, equation (3) is a negative value, so the expected return function $E(\pi_C)$ is a convex function about $Q_{Ci}$. According to $\frac{\partial E(\pi_C)}{\partial Q_{Ci}} = 0$, $F_i^e(Q_{Ci}^*) = 1 - \frac{c_i}{p + g}$, When both providers deliver the integrators according to their respective optimal logistics service offerings $Q_{Ci}^* = (F_i^e)^{-1}(1 - \frac{c_i}{p + g})$, the LSSC overall receives the maximum expected return. Under the centralized decision-making model, the total logistics service capacity of LSSC is $Q_C^* = Q_{C1}^* + Q_{C2}^* = \sum_{i=1}^{2} (F_i^e)^{-1}(1 - \frac{c_i}{p + g})$. The optimal expected return for LSSC is $E(\pi_C^*) = p \sum_{i=1}^{2} \int_{Q_{Ci}}^{\infty} x f_i^e(x) dx - (p + g) \sum_{i=1}^{2} \int_{Q_{Ci}}^{\infty} x f_i^e(x) dx$.

Logistics service integrators and two providers are rarely owned by one company in reality. All three parties pursue their respective revenue maximization, and the optimal logistics service quantity obtained by centralized decision-making and the optimal expected return of the supply chain can be realized. There are certain difficulties.

Decentralized decision-making model
Since LSI takes the leading position in LSSC and cannot meet the punishment of logistics service demand completely borne by LSI, without the decentralized decision of contract coordination, LSI first determines the optimal amount of logistics service to license $Q_{DL^1}$ from its own benefits, and then $FLSP_{i=(i=1,2)}$ chooses to follow the arrangement. The LSI's income function is

$$\pi_{DL} = \sum_{i=1}^{2} \left[ p \min(D_i^*, Q_{RL^i}) - (c + w_i)Q_{DL^i} - g(\mathbf{D}_i^* - Q_{RL^i})^+ \right]$$

(9)

Theorem 2 Under decentralized decision making, the integrator's expected return $E(\pi_{DL})$ is a convex function of the planned logistics service supply quantity $B$ of the two providers, that is, the existence of the optimal logistics service supply quantity $C$ and satisfying $D$ enables the logistics service integrator to obtain the optimal expected return.

The proof process of Theorem 2 is the same as Theorem 1.

The income function of $FLSP_{i=(i=1,2)}$ is: $\pi_{DF} = (w_i - c_i)Q_{DL^i}^*$. The optimal logistics service delivery under centralized decision-making model is $Q_{Ci}^* = (F_i^*)^{-1}(1 - \frac{c_i}{p + g})$. The optimal logistics service delivery under decentralized decision-making model is $Q_{DL^i}^* = (F_i^*)^{-1}(1 - \frac{c + w_i}{p + g})$. Due to $w_i > c_i$ and $F_i(x)$ is a continuously differentiable strictly increasing function, by comparison, $Q_{Ci}^* > Q_{DL^i}^*$. That is, at this time, the integrators obtained the optimal expected return, but the providers did not realize their optimal expected returns.

Theorem 3 Under the decentralized decision-making, the optimal logistics service supplies a determined from the interests of the logistics service integrator alone fails to optimize the overall LSSC.

Proof The income function of LSSC is: $\pi_D = \sum_{i=1}^{2} \left[ p \min(D_i^*, Q_{DL^i}) - (c + c_i)Q_{DL^i} - g(D_i^* - Q_{DL^i})^+ \right]$

Substituting the optimal logistics service provision $Q_{DL^i}^* = (F_i^*)^{-1}(1 - \frac{c + w_i}{p + g})$ determined by the integrator into the first derivative of the expected benefit of the LSSC, get result

$$\frac{\partial E(\pi_D)}{\partial Q_{DL^i}} \bigg|_{Q_{DL^i}^*, Q_{DL^i}} = w_i - c_i > 0 \text{ and } \frac{\partial^2 E(\pi_D)}{\partial Q_{DL^i}^2} < 0.$$ 

The expected benefit function of LSSC is a convex function, that is, for LSSC as a whole, there is a better decision than $Q_{DL^i}^*$. Therefore, decentralized decision-making without contract coordination has failed to optimize the overall benefits of LSSC.

Under decentralized decision-making, the penalties for failing to meet the demand for logistics services are entirely borne by the integrators. For the pursuit of maximizing their own profits, the decisions made by the dominant integrators will result in loose cooperation with the two providers, thus reducing the overall performance of LSSC. A certain contract coordination mechanism should be adopted to alleviate this situation, improve the overall performance of LSSC under decentralized decision-making and even achieve the performance results under centralized decision-making.

4.2 Revenue sharing contract model

This paper adopts the revenue sharing contract to coordinate the decentralized LSSC. Under the revenue sharing contract, LSI is still the absolute leader of LSSC. In order to cope with the random demand of the market, LSI builds a revenue sharing contract to share the risk with $FLSP_{i=(i=1,2)}$ and share the revenue. First, $FLSP_{i=(i=1,2)}$ analyzes the purchase price $w_i$ of the logistics service capability based on the revenue sharing contract given by LSI, and determines the amount of logistics service
capability $Q_{RFi}$ that maximizes its own revenue. Then, based on the reaction function of FLSP$_{i\in\{1,2\}}$, LSI determines the revenue sharing contract $(w_i, k_i)$ and the optimal logistics service capability $Q_{RL}$ that maximize its own revenue. Since FLSP$_{i\in\{1,2\}}$ provides logistics services for LSI at a relatively low price $w_i$ under the revenue sharing contract, LSI will compensate $(1-k_i)$ for sales revenue to FLSP$_{i\in\{1,2\}}$.

Under the given $(w_i, k_i)$, the income function of FLSP$_{i\in\{1,2\}}$ is:

$$\pi_{RFi} = (1-k_i) \min(D_{Ri}' \cdot Q_{RFi}) + (w_i - c_i)Q_{RFi} - g' (D_{Ri}' - Q_{RFi})^+$$

Theorem 4 Under the revenue sharing contract, the expected benefit function $E(\pi_{RFi})$ of the logistics service provider is a convex function of the logistics service supply quantity $Q_{RFi}$ of the two provider plans, that is, there is an optimal logistics service supply quantity $Q_{RFi}^*$ and meets $Q_{RFi}^* = (F_{i}'e_i)^{-1}(1 - \frac{c_i - w_i}{(1-k_i)p + g})$ to make the two logistics service providers expect the most benefits.

The proof process of Theorem 4 is the same as Theorem 1.

Under the revenue sharing contract, FLSP$_{i\in\{1,2\}}$'s optimal expected revenue is:

$$E(\pi_{RFi}^*) = (1-k_i) p \int_0^\infty x f_r(x) dx - \int_0^\infty [(1-k_i)p + g] \int_0^\infty x f_r(x) dx$$

Under the given revenue sharing contract $(w_i, k_i)$, the revenue function of LSI can be expressed as

$$\pi_{RL} = \sum_{i=1}^2 [k_i p \min(D_{Ri}' \cdot Q_{RLi}) - (c + w_i)Q_{RLi} - (g - g') (D_{Ri}' - Q_{RLi})^+]$$

And we can calculate that $Q_{RLi}^* = (F_{i}'e_i)^{-1}(1 - \frac{c + w_i}{k_i p + (g - g')})$, LSI gets the maximum expected revenue under the revenue sharing contract, and its expected revenue is:

$$E(\pi_{RLi}^*) = \sum_{i=1}^2 \{k_i p \int_0^\infty x f_r(x) dx - [k_i p + (g - g')] \int_0^\infty x f_r(x) dx\}$$

Then, the optimal order quantity of LSSC system is $Q_{RL}^* = Q_{RL1}^* + Q_{RL2}^* = \sum_{i=1}^2 (F_{i}'e_i)^{-1}(1 - \frac{c + w_i}{k_i p + (g - g')})$

Theorem 5 When $\frac{c_i - w_i}{(1-k_i)p + g} + \frac{c_i - w_i}{(1-k_2)p + g} = \frac{c + w_i}{k_i p + (g - g')} + \frac{c + w_i}{k_2 p + (g - g')}$, under the revenue sharing contract, LSI and B get the optimal expected revenue.

Theorem 6 When the revenue sharing coefficient $k_i = \frac{(c + w_i)(p + g') - (c_i - w_i)(g - g')}{p(c + c_i)}$, the revenue sharing contract can coordinate the logistics service supply chain composed of two competitive logistics service providers and one logistics service integrator.

Proof When the supply chain system reaches a coordinated state, the amount of logistics service ordered by the integrator from the two providers is equal to the amount of logistics service provided by the two providers to the integrator. And there is a unique solution for the optimal logistics service order quantity from the perspective of logistics service integrators. That is, Equation $Q_{RLi}^* = Q_{RFi}^* + Q_{RFi}^*$ is established, and thus Theorem 6 is proved.

Theorem 7 By changing the size of the revenue sharing coefficient $k_i$, the benefits of each member of the system can be adjusted.

Proof From $\frac{\partial Q_{RFi}^*}{\partial k_i} = (F_{i}'e_i)^{-1} \frac{(w_i - c_i)p}{[(1-k_i)p + g]}$, it can be seen that the growth rate of $Q_{RFi}^*$ on the optimal logistics service of FLSP$_{i\in\{1,2\}}$ increases with the increase of $k_i$, that is, the proportion of logistics service integrators to share and transfer their profits to the logistics service providers in the
The system is larger, and the two logistics service providers can provide more logistics services. Similarly, it can be concluded from the expected revenue formula of $FLSP_{i=1,2}$ that the revenue of logistics service providers increases with the increase of $Q_{RFi}^*$. Therefore, the revenue of each member in the system can be adjusted by changing the value of revenue sharing coefficient.

Theorem 8 when revenue sharing coefficient $k_i = \frac{(p+c+w_i)}{p} - \frac{g-g_i}{p}$, the logistics service supply chain system achieves the optimal state under the coordination of revenue sharing contract.

Proof Under the centralized decision model, the obtained benefit is the maximum expected benefit of the entire logistics service supply chain system. When the system under the revenue sharing contract is optimal, there is $Q_{RLC} = Q_{C}^*$, that is, the optimal under the revenue sharing contract. The volume of logistics service orders is consistent with the optimal logistics service order quantity under centralized decision.

5. Summary

In this paper, the research on the secondary logistics service supply chain under stochastic demand is carried out. The two logistics service providers with competition and an integrator system are taken as the research objects and the coordination effect of the revenue sharing contract on the logistics service supply chain is analyzed.

Firstly, the model under centralized decision-making is constructed, and the optimal logistics service order quantity and optimal expected return of the logistics service supply chain are considered from the perspective of stochastic demand. Secondly, under the stochastic demand, the revenue sharing contract is used to coordinate the decentralized logistics service supply chain, and the optimal logistics service order quantity that takes into account the interests of all parties in the system is obtained, and at this time, two logistics service providers and logistics service integrators optimal expected return. Finally, through the comparative analysis between the models, it is proved that the revenue sharing contract can effectively coordinate the two-level logistics service supply chain system composed of two competitive logistics service providers and one logistics service integrator. The results show that when the revenue sharing contract makes the LSSC of "1LSI-2FLSPs" realize effective coordination, the supply chain system is in the optimal state of revenue sharing contract coordination. In addition, by changing the size of revenue sharing coefficient, the revenue of each member in the system can be adjusted to play a positive role in motivating.

This research explores the solution from the theoretical research level, in order to give a certain theoretical reference in the actual operation, and has a certain positive effect on enriching the contract coordination theory and guiding practice of the two logistics service providers with competitive relationship in the logistics service supply chain. However, there are still many expandable research points: in this study, the model assumes that the demand function obeys a random distribution, but the actual demand of the market may be closely related to factors such as time and price. Further research can assume that demand is a correlation function of a variable to discuss the problems of two providers in the logistics service supply chain.

References


