
Efficient Phase-field modeling of interfacial dynamics in multiphase flows

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Abstract

We propose an efficient formulation of the scale-separation approach which has been developed by Han et al. [1] for multi-scale sharp interface modeling of multi-phase flows based on the level-set technique. Instead of shifting the entire level-set field twice as in the original method, the improved method identifies the non-resolved interface structures from two auxiliary level-sets close to the interface. Non-resolved structures are separated from the interface by a localized re-distancing method, which increases the computational efficiency considerably compared to the original global reinitialization procedure. Several tests for two-phase flow problems, involving simple and complex interface structures, are carried out to show that the present method maintains sharper interface structures than the original method, and achieves effective scale-separation.

Keywords

Multi-scale; Multi-phase; Interfaces; Scale separation; Level-set method.

1. Introduction

Simulations of two-phase flow, such as bubble interaction[2], drop impact[3] and spray atomization[4-5], need to resolve length scales that can span several orders of magnitude, which poses a great computational challenge. Adaptive mesh refinement (AMR)[6] and multi-resolution (MR) methods[11]; [9], even with local time stepping, do not sufficiently reduce computational cost to enable accurate routine simulations of complex interfacial flows. A promising approach to improve efficiency is offered by multi-scale sharp-interface methods[9-10]; [1]. One essential procedure in multi-scale modeling is scale separation. For a given spatial resolution with grid size h , the interface segments with characteristic size can be categorized as resolved if $B > h$, or non-resolved if $B < h$. In previous work, two approaches have been proposed for scale separation: one is based on the refined level-set grid method (RLSG)[9-11], the other is the constrained stimulus response procedure (CSRP)[1]. While RLSG requires a two-grid system where a higher-resolution grid is used for representing the interface, CSRP uses a single grid for representing both the interface and the individual fluids. CSRP identifies the resolved and non-resolved interface segments based on the different responses they exhibit when subjected to a small shift of the level-set field, and separates these scales with a two-step level-set re-initialization procedure. Although CSRP is effective, in particular the additional re-initialization operation significantly increases computational cost.

In this paper, an improved scale-separation method is proposed to increase computational efficiency. Using two auxiliary level-set fields instead of level-set shifting, the new method identifies non-resolved interface structures by examining the topological consistency between the auxiliary and zero level-sets. Non-resolved structures are subsequently separated by a localized re-distancing approach to avoid additional re-initialization operations. The performance of the new method is evaluated

through several tests, including single-vortex flow, underwater explosion, shock–bubble interaction, and liquid-filament breakup.

2. Simulation

Non-resolved interface segments often manifest themselves as drops and thin filaments with a characteristic length scale of about the grid size h . In the level set field they share a common property, that is the lack of topological consistency among the level-sets near the interface. Such a property corresponds to the large discrepancies induced by the non-resolved interface segments when the reinitialization operations are subjected to a small constant shift of the level set field [1].

Based on this criterion we now develop a new identification method, first in one dimension and then in two dimensions. Assuming a distribution of the level set field on cells near the interface, as sketched in Fig. 1(a), cut cells are those that contain segments of the zero level-set, i.e. A,B,C and D. Other than shifting the entire level-set field by ε as in Ref. [1], we use two auxiliary level-sets, $\varphi=+\varepsilon$ and $\varphi=-\varepsilon$, as shown in Fig. 1(b). As pointed out in Ref.[1], all small interface segments with length scale less than h should be removed, so we also choose $\varepsilon=0.6h$. We can define two types of cut cells associated with the auxiliary level-sets, the positive auxiliary cut cells $A+\varepsilon, B+\varepsilon, C+\varepsilon, D+\varepsilon$ containing segments of $\varphi=+\varepsilon$ and the negative auxiliary cut cells $A-\varepsilon, D-\varepsilon$ containing segments of $\varphi=-\varepsilon$, as shown in Fig. 1(b).

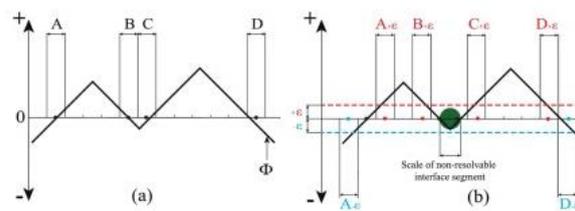


Fig. 1. Schematic of identifying the non-resolved interface segments in one dimension: (a) initial interface level set field and cut cells (block solid dots); (b) corresponding auxiliary cut cells crossed by $\Gamma+\varepsilon$ (red solid dots) and by $\Gamma-\varepsilon$ (blue solid dots). The tick marks indicate the cell faces, and the green mark the non-resolved interface segment.

The extension of such an identification algorithm to two dimensions is illustrated in Fig. 2. First, let S be defined as cut cells crossed by the zero level-set Γ_0 .

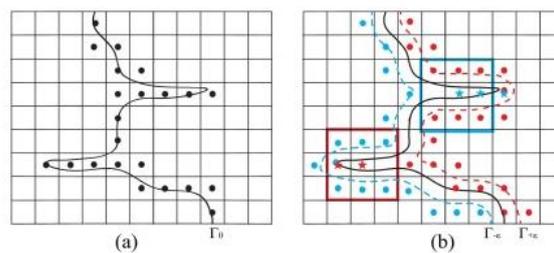


Fig. 2. Identification of the non-resolved interface segments in two dimensions: (a) the initial interface Γ_0 (black solid line) and corresponding cut cells (black solid dots); (b) the auxiliary level-sets $\Gamma+\varepsilon$ (red dash line) and $\Gamma-\varepsilon$ (blue dash line), and corresponding auxiliary cut cells crossed by $\Gamma+\varepsilon$ (red solid dots) and by $\Gamma-\varepsilon$ (blue solid dots). Cut cells whose neighbors contain no red solid dots are indicated by a red star, and Cut cells whose neighbors contain no blue solid dots are indicated by a blue star.

The level set values for cut cells in $S_{1\neq P}$ and $S_{1\neq N}$ are assigned from an estimate of their distance to $\Gamma+\varepsilon$ and $\Gamma-\varepsilon$, respectively. In one dimension, as shown in Fig. 3(a), Cells B and C are in set $S_{1\neq N}$ and their the level-set values can be estimated from the minimum distance to Cell $A-\varepsilon$ and to $D-\varepsilon$. For the setup in Fig. 3(a), we can obtain corrected level set values $\varphi_B=i_B-i_{A-\varepsilon}+\varphi_{A-\varepsilon}$ and $\varphi_C=i_C-i_{A-\varepsilon}+\varphi_{A-\varepsilon}$, where i_B, i_C and $i_{A-\varepsilon}$ are the indexes of Cells B, C and $A-\varepsilon$, respectively. By

correcting the level set values of all cells in S1, and a well-resolved interface, indicated by a black solid line in Fig. 3(b), is obtained.

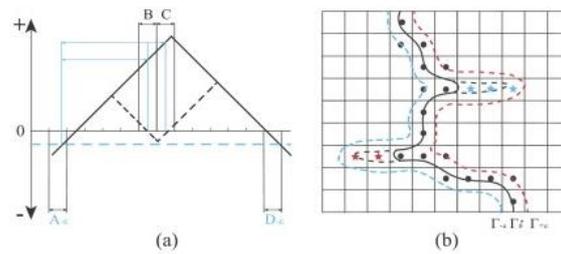


Fig. 3. Interface reconstruction in one and two dimensions: (a) level-set values for the cut cells B and C are calculated from auxiliary cut cell $A-\epsilon$, (b) level-set values of the cells marked with red stars are calculated from $\Gamma+\epsilon$ and level-set values of the cells marked with blue stars are calculated from $\Gamma-\epsilon$.

We consider a pure interface-transport case, where the interface deforms by a single-vortex flow [1]. A circle of radius $R_0=0.15$ is located at $(0.5,0.75)$ in a unit square computational domain, as shown in Fig. 4 at the initial time. To compare the results with those in Ref.

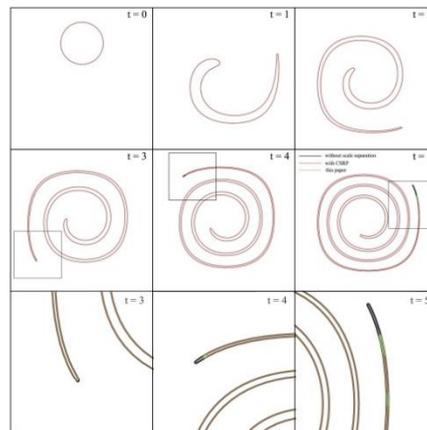


Fig. 4. Single-vortex flow: the evolution of interface deformation (first and second row) without scale separation (black line) and with CSRP (red line), and the method in this paper (green line). The third row shows zooms of the interfaces as indicated at times $t = 3, 4$ and 5 .

3. Results And Discussion

The results given in Fig. 5 show that the present method achieves less area loss than the CSRP. Furthermore, our experiment shows that the present method achieves a speed up factor of 2.3 compared to the CSRP method for the scale-separation operations.

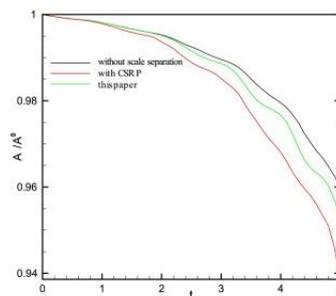


Fig. 5. Single-vortex flow: variation of the total area.

Fig. 6 shows the interface evolution as well as Schlieren-type images of density at several different time instances. The results are identical to those in Ref. [8]. As shown in Fig. 6, one can observe that, since the interface is smooth and well-resolved, no interface segments are removed by the present

scale separation method. Compared to the CSR method, our results show that the present method achieves a speed up factor of 2.5.

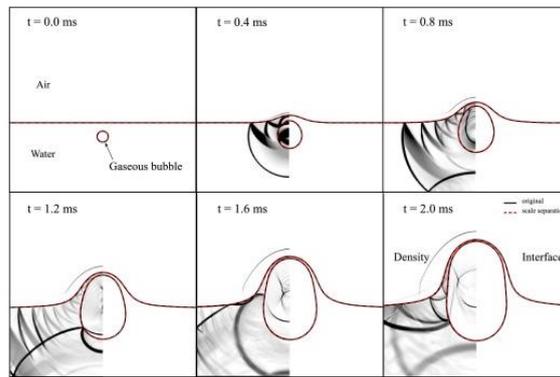


Fig. 6. Underwater explosion: Schlieren-type image for density (the left half) and interface position (the right half) at different time instances.

Fig. 7 shows the Schlieren-type images of density gradient $|\Delta\rho|$ at the several time instants. It can be observed that the results are in good agreement with previous numerical work of Refs.

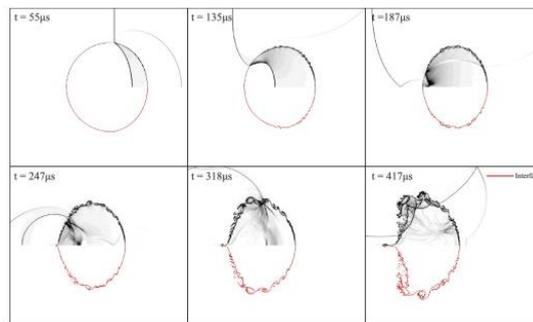


Fig. 7. Shock–bubble interaction: Schlieren-type images of density (the top half) and interface position (the bottom half) at different time instances.

Fig. 8 shows the interface at several time instants of the simulation with the resolution 1024×6144 . As reported in Ref.

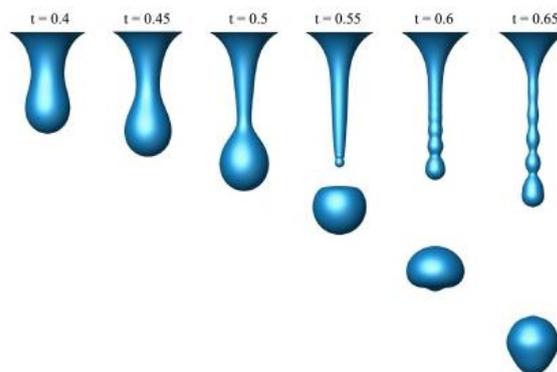


Fig. 8. Liquid-filament breakup: evolution of the interface.

The evolution of the neck diameter is shown in Fig. 9. Fig. 9(a) illustrates the breakup times with different resolution, which implies about second order convergence with mesh refinement. Furthermore, the results recover the expected potential-flow scaling law, as shown in Fig. 9.

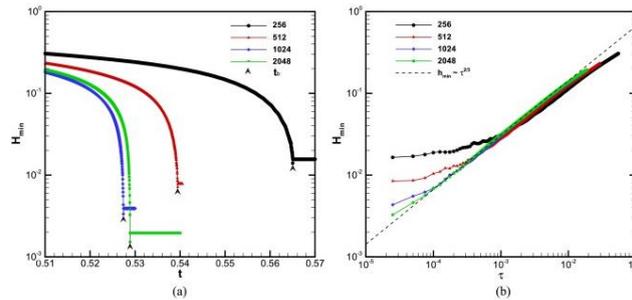


Fig. 9. Liquid-filament breakup at different resolutions: (a) evolution of the neck diameter h_{min} with time t ; (b) evolution of h_{min} with the dimensionless time τ and the potential flow scaling law $h_{min} \sim \tau^{2/3}$.

4. Conclusion

We have developed a new formulation for scale separation to improve the computational efficiency for multi-scale sharp interface modeling of multi-phase flows. Based on the observation that there is lack of topological consistency between the auxiliary and zero level-sets for the non-resolved interface segments, the cut cell associated with non-resolved interface segments are identified explicitly by checking the auxiliary cut cells in its neighborhood. Non-resolved structures are removed and the corresponding level-set field is reconstructed by a localized re-distancing method. Several typical numerical examples are simulated to validate the performance of the present scale separation method. The results show that the current approach is able to handle complex interface evolution. Compared to Han et al. [1], the present method decreases the over-smoothing effect of the original method and achieves a considerable computational speed up. Furthermore, the simulations of the filament-breakup case on with increasing grid resolutions verifies the physical consistency of the present scale-separation method.

Acknowledgements

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