Analytical and variational numerical methods for multiphase flow conservation Instantaneous equation

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Abstract

The aim of this paper is to propose a framework to obtain a new formulation for multiphase flow conservation equations without length-scale restrictions, based on the non-local form of the averaged volume conservation equations. The simplification of the local averaging volume of the conservation equations to obtain practical equations is subject to the following length-scale restrictions: d ≪ ℓ ≪ L, where d is the characteristic length of the dispersed phases, ℓ is the characteristic length of the averaging volume, and L is the characteristic length of the physical system. If the foregoing inequality does not hold, or if the scale of the problem of interest is of the order of ℓ, the averaging technique and therefore, the macroscopic theories of multiphase flow should be modified in order to include appropriate considerations and terms in the corresponding equations. In these cases the local form of the averaged volume conservation equations are not appropriate to describe the multiphase system. As an example of the conservation equations without length-scale restrictions, the natural circulation boiling water reactor was consider to study the non-local effects on the thermal-hydraulic core performance during steady-state and transient behaviors, and the results were compared with the classic local averaging volume conservation equations.

1. Introduction

In a recent paper, Sha and Chao[1] presented a detailed derivation of the porous media formulation for multiphase flow conservation equations. Unfortunately, the derivation is limited to the following length-scale restrictions:d ≪ ℓ ≪ L

where d is the characteristic length of the dispersed phases, ℓ is the characteristic length of the averaging volume, and L is the characteristic length of the physical system. According to these authors the length-scale restrictions are inherent in the *local* volume averaging theorem given by Whitaker [2]. In general for a sparsely dispersed flow of a discrete phase, the length-scale restrictions are true.

In earlier studies of the closure problems, Carbonell and Whitaker[3] showed that the average of a spatial deviation.

the position vector relative to the centroid of the averaging volume as illustrated in Fig. 1 (Section 2) under normal circumstances, i.e., in the homogenous regions of the multiphase flow the average deviation is zero.

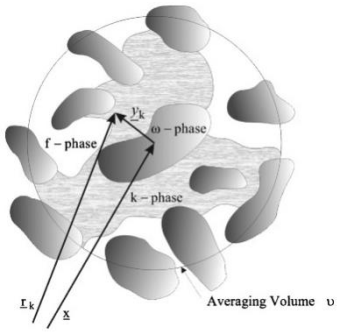

Fig. 1. Position vectors associated with the averaging volume.

In this work we developed applicable averaged transport equations for multiphase flow without length-scale restrictions. This formulation is based on the *non-local* form of the volume-averaged conservation equations.

2. Preliminaries

The method of volume averaging is a technique that can be used to rigorously derive continuum equations for multiphase systems. This means that equations which are valid for a particular phase can be spatially smoothed to produce equations that are valid everywhere[4-20], except in the boundaries which contain the multiphase systems.

As mentioned above υ is a constant, which is invariant in both space and time, and its orientation relative to inertial frame of reference fixed, as illustrated in Fig. 1.

The natural circulation model of the NRBWR numerical code was considered in this analysis in order to evaluate the effects of the non-local parameters. Balancing the gravity head available and total loop pressure drop obtained by integration of the momentum balance leads to the model for the natural circulation. The natural circulation model includes the pressure drops and flows from the downcomer, lower and upper plenum (known as chimney), reactor core and steam separators, in order to obtain the following momentum balance. The flow path of the NRL is shown in Fig. C.1.

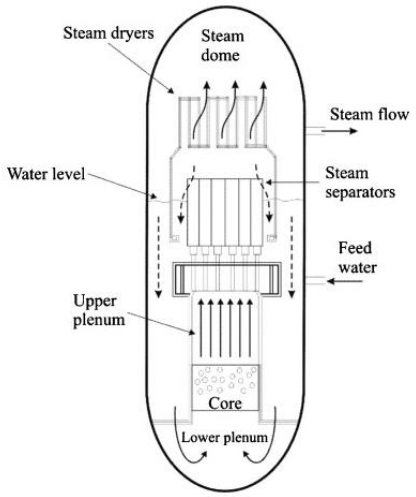

Fig. C.1. NRBWR configuration and flow paths [5]

Fig. C.2 is a schematic diagram of the boiling water reactor where the arrangement of the computational cells of the NRBWR model is shown. The reactor vessel was divided into five zones. Two of these zones, the vessel dome and the downcomer, have a variable volume according to the vessel water level. The three fixed volume zones are the lower plenum, the upper plenum and steam separators, and the reactor core. Due to its importance on the model performance, the latter was subdivided into twelve one-dimensional nodes.

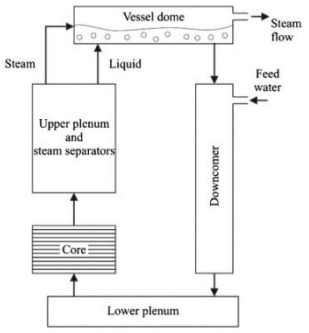

Fig. C.2. Arrangement of the computational nodes in the NRBWR model [5].

3. Simulation And Conclution

Fig. C.3, Fig. C.4, Fig. C.5, Fig. C.6, Fig. C.7 ; Fig. C.8 shows the transient behaviors of the core performance of the SBWR during a reactor Scram for η′za=0, η′za=2 and η′za=4. It is important to note that the classic formulation does not have the term given by Eq.(C.45). The core flow is major for η′za=4 (Fig. C.3) due that the global average void fraction in the core is less (Fig. C.4) therefore it presents less hydraulic resistance, with respect to η′za=0 and η′za=2.

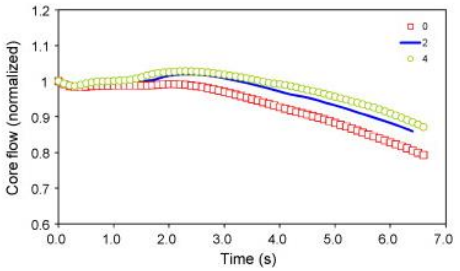

Fig. C.3. Core flow transient behavior for η′za=0, η′za=2 and η′za=4.

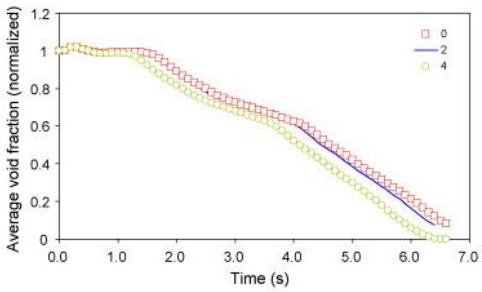

Fig. C.4. Global average void fraction transient behavior for η′za=0, η′za=2 and η′za=4.

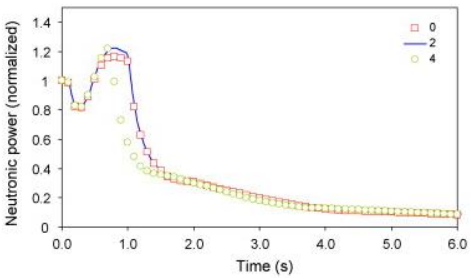

Fig. C.5. Neutronic power transient behavior for η′za=0, η′za=2 and η′za=4

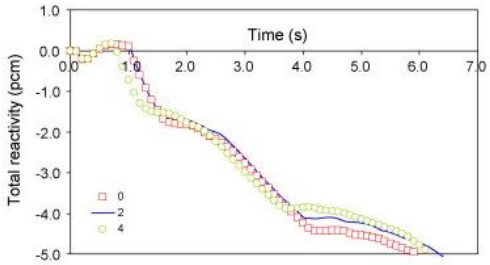

Fig. C.6. Total reactivity during reactor scram for η′za=0, η′za=2 and η′za=4.

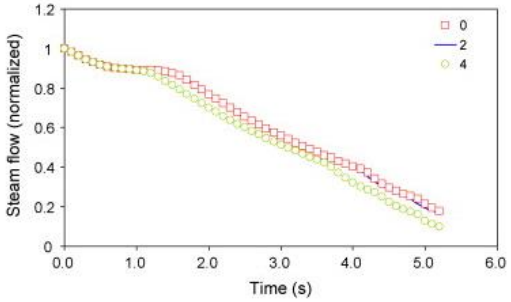

Fig. C.7. Steam flow transient behavior for η′za=0, η′za=2 and η′za=4.

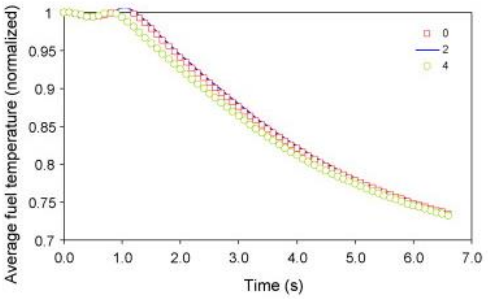

Fig. C.8. Global average fuel temperature transient behavior for η′za=0, η′za=2 and η′za=4.

Acknowledgements

This work is supported by the National Natural Science Foundation of China(No. 11602066) and the National Science Foundation of Heilongjiang Province of China (QC2015058 and 42400621-1-15047), the Fundamental Research Funds for the Central Universities.

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