High Accuracy DOA Estimation Based on MUSIC Algorithm

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Abstract

The MUSIC algorithm is widely used in the DOA estimation, but this algorithm cannot distinguish the coherent signal source and small SNR signal source with and smaller incident angle spacing. Some scholars have adopted the conjugate rearranged method to improve the MUSIC algorithm, but still did not achieve very good results. This paper use the orthogonal property of noise subspace and signal direction subspace, and by weighting the noise subspace, and the centrosymmetric non-uniform linear array method is used to effectively solve the DOA estimation problem of coherent signal source and the adjacent small SNR signal source. In this paper, the applicability of new covariance matrix obtained after centrosymmetric non-uniform linear array to rearrange the conjugate is given proof, and the verification of simulation experiment is carried out. When SNR=0db, it can still distinguish two coherent signals with 1° interval, the experimental results show that the method proposed by paper can improve the direction-finding performance of the algorithm dramatically.

Keywords

MUSIC algorithm; signal DOA; noise subspace.

1. Introduction

Array signal processing is an important part of signal processing field. With the rapid development of microelectronic technology, signal processing technology and computer science technology, array signal processing is also constantly improving and developing, its application involves radar, positioning, communication, ranging, tracking, sensor network and other fields [1-5].

Spatial spectrum estimation is the main content of array signal processing. The main object of spatial spectrum estimation research is to handle DOA (Direction of Arrival) in signal within the bandwidth. In the traditional DOA estimation algorithm, MUSIC (Multiple Signal Classification) algorithm [6] is one of the most classical algorithms. The orthogonality of signal subspace and noise subspace is used to form the spatial spectrum function, and the spectral peak search is used to detect DOA of signal. However, there are large amounts of calculation, coherent, low SNR, smaller angle interval insufficient resolution problems in the MUSIC algorithm, the scholars propose a root-MUSIC algorithm that use root ways replace spectral peak search to reduce the amount of calculation for the above problems [7]; MUSIC algorithm based on spatial smoothing technique [8], which distinguishes signal with the coherent signal, low SNR, small interval [8], and an IMUSIC algorithm that optimizes the covariance matrix by received data with conjugate rearrangement method.

The above algorithm cannot tell the signal as the SNR decreases for the coherent signal with signal interval less than 5°,. For that reason, the paper uses the characteristic that MUSIC resolution ratio of algorithm rises as array spacing increases, the layout of the sensor linear array is changed, and a non-uniform linear array structure based on the symmetric array method is put forward, and MUSIC algorithm (W-NMUSIC) that weights noise subspace. This method further improves the direction-finding performance of the algorithm by improving the radius wavelength ratio.

2. Brief Introduction to MUSIC Algorithm

The array model is shown in Fig.1, and the array is set as an even linear array, the array number is M , the array spacing is d , and the wavelength of incident wave is λ .

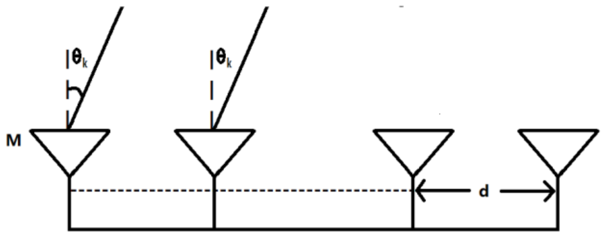

Figure.1 uniform linear array

Assuming there are $K(K<M)$ narrow band signal source into the line array, the incident angle is $\theta_1,\theta_2,\ldots,\theta_K$. The radiation signals from the far-field signal source can be regarded as a plane wave in the ideal case when the radiation signal reach the line array, the array element in the array are co-rotating and isotropic and there is no channel inconsistency or mutual coupling and other factors. Taking the first array on the left as a reference, if the wavefront signal that signal from the k radiation element reaches array element 1 is $S_k(t)$, then the signal received from the i array element is:

$$a_k S_k(t)\exp(-j\omega_0(i-1)d\sin\theta_k/c) \tag{1}$$

In the formula, a_k is response of the array element i to the first k signal source, ω_0 is the center frequency of the signal, c is the spread velocity of the wave, and θ_k is the included angle of the incident angle and antenna normal of the first k signal source. Then output signal of the first i array element:

$$x_i(t)=\sum_{k=1}^{D} a_k S_k(t)\exp(-j\omega_0(i-1)d\sin\theta_k/c)+n_i(t) \tag{2}$$

In the formula, $n_i(t)$ is noise, the label i shows that this variable belongs to the first i array element, and the label k indicates the first k signal source. Then the signals that the first M array element received are arranged in a column vector in a particular time, the vector form is:

$$X(t)=AS(t)+N(t) \tag{3}$$

In the formula, $X(t)=[x_1(t),x_1(t),\ldots x_M(t)]^T$ is vector tha M dimension receive data, $S(t)=[S_1(t),S_2(t),\ldots,S_K(t)]^T$ is vector of K dimension; $A=[a(\theta_1),a(\theta_2),\ldots,a(\theta_K)]$ is the $M\times K$ is the vector of dimension array, $a(\theta_k)=[1,e^{-j\omega_0\tau_k},\ldots e^{-j\omega_0(M-1)\tau_k}]$ is the direction vector of M dimension; the $N(t)=[n_1(t),n_2(t),\ldots n_M(t)]^T$ is dimension vector of M dimension.

Assuming that the noise of each element is a stationary white-noise process with mean zero, the variance is σ^2 , and the noise is uncorrelated and is not correlated with the signal. When the number of snapshots is N , the covariance matrix of the line array output signal is:

$$RX=E[X(t)X^H(t)]=APA^H+\sigma^2 I \tag{4}$$

In the formula, $P=[S(t)S(t)^T]$ is the covariance of the signal, I is the unit matrix of M . And R is made feature decomposition and eigenvalue and eigenvalue vector are obtained:

$$RX\cdot V=V\cdot\Lambda \tag{5}$$

$V=[v_1 v_2\ldots v_M]$, $diag(\Lambda)=\{\lambda_1,\lambda_2,\ldots\lambda_M\}$. The subspace that formed by corresponding eigenvector of the $M-K$ small eigenvalue in $diag(\Lambda)$ is the noise subspace $E=[v_{K+1},v_{K+2},\ldots v_M]$. In the ideal case, the direction vector of the signal subspace is orthogonal to the noise subspace:

$$EA=0 \tag{6}$$

In the actual case, due to the existence of noise, E and A cannot be completely orthogonal, namely $EA \neq 0$, The classical MUSIC algorithm achieve DOA estimation by the minimum optimization search based on the above characteristics. The formula of spatial spectrum estimation of MUSIC algorithm:

$$P_{MUSIC}(\theta) = \frac{1}{A^H(\theta)EE^H A(\theta)} \tag{7}$$

3. W-NMUSIC Algorithm

3.1 Weight the MUSIC algorithm

The weighted MUSIC algorithm make preprocesses for the coherent signal from the noise subspace. First, the received data is conjugated and rearranged to obtain a new covariance matrix, which can effectively remove the coherence of the signal [10], the covariance matrix are made eigenvalue decomposition to obtain a new noise subspace matrix, and the noise subspace matrix are done weights operation to further improve the orthogonality of the noise subspace and the signal direction vector [11]. First, the data collected by the line array are conjugated and rearranged:

$$Y(t) = JX^*(t) \tag{8}$$

In the formula, $X^*(t)$ is the conjugate transpose of $X(t)$, J is the inverse unit matrix of $M \times M$ order:

$$J = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 1 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix}_{M\times M} \tag{9}$$

RY form covariance matrix of $Y(t)$:

$$RY = E[Y(t)Y^H(t)] = JA^*P^*(A^*)^H J + \sigma^2 I = JRX^*J \tag{10}$$

A new covariance matrix is obtained:

$$R = RX + RY \tag{11}$$

Because the matrix A is composed of many signal direction vectors, assume $1 \le i \le K$, then $a_i = [1 \quad e^{-j\frac{2\pi d}{\lambda}\sin\theta_i} \quad \cdots \quad e^{-j\frac{2\pi d}{\lambda}(M-1)\sin\theta_i}]^T$ can be written in the form $a_i = [1 \quad e^{-j\tau_i} \quad \cdots \quad (e^{-j\tau_i})^{M-1}]^T$, τ_k represent $\tau_i = -\frac{2\pi d}{\lambda}\sin\theta_i$, matrix can be written as follows:

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\tau_1} & e^{j\tau_2} & \cdots & e^{j\tau_K} \\ \vdots & \vdots & & \vdots \\ (e^{j\tau_1})^{M-1} & (e^{j\tau_2})^{M-1} & \cdots & (e^{j\tau_K})^{M-1} \end{bmatrix} \tag{12}$$

For any noise subspace matrix v_j^T, which is orthogonal to any signal direction vector, $v_j^T \perp a_i$ can be written in the form of a polynomial:

$$F(z) = v_{j,0} + v_{j,1}z^1 + ... + v_{j,M-1}z^{M-1} = 0 \tag{13}$$

There is noise subspace, $z_1 = e^{j\tau_1}$ $z_2 = e^{j\tau_2}$, $z_{M-1} = e^{j\tau_{M-1}}$ can make polynomials work. It can be known from the characteristics of complex multiplication,

$$(v_j^T)^* \perp a_i^* \tag{14}$$

Because $J*J = I$, then $JA^*P^*(A^*)^H JJ(v_j^T)^* = 0$. The formula (14) can obtain polynomial $Z(z) = F^*(z) = 0$

$$Z(z) = F^*(z) = v_{j,0}^* + v_{j,1}^* z^* + ... + v_{j,M-1}^*(z^*)^{M-1} = 0 \tag{15}$$

And because the roots of $F(z)$ in the unit circle, then the polynomial $Z(z)$:

$$z^{M-1}Z(z) = v_{j,M-1}^* + v_{j,K-1}^* z + ... + v_{j,0}^* z^{M-1} = 0 \tag{16}$$

It can be inferred from (16):

$$J(v_j^T)^* \perp a_i \tag{17}$$

It can be deduced from formula (17) that we can obtain $APA^H J(v_j^T)^* = 0$, the orthogonality of signal subspace and noise subspace is enhanced after conjugate rearrangement. However, since the covariance matrix R is obtained in accordance with the finite observation data in the actual operation, when R is done eigendecomposition, the determination of the minimum eigenvalue (noise variance) and the multiplicity and the estimation of the minimum eigenvector all have errors, the $M \times (M-K)$ weight matrix is constructed for the new noise subspace, the noise subspace matrix adjusts the sensitivity of direction vector of target signal in signal subspace matrix by weight matrix, the orthogonality of the signal subspace and the noise subspace are increased. The steps as follows:

$$\begin{cases} [u,s] = eig(R) \\ S = diag(s) \\ U_n = u(:,1:M-K) \\ w = power(S(K+1:M,1)^T, p) \end{cases} \tag{18}$$

weight matrix W of M*(M-K) is constructed, it can see from the formula (18) that each row of the matrix W is composed of matrix w, and the matrix $w = power(S(K+1:M,1), p)$, p is adjustable parameter.

$$W = \begin{bmatrix} w \\ w \\ \vdots \\ w \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & \cdots & w_{M-K} \\ w_1 & w_2 & \cdots & w_{M-K} \\ \vdots & \vdots & \cdots & w_{M-K} \\ w_1 & w_2 & \cdots & w_{M-K} \end{bmatrix}_{M\times(M-K)} \tag{19}$$

The weight matrix W is multiplied by the corresponding bit of the noise subspace matrix U_n to obtain a new noise subspace:

$$U_{new} = W.*U_n \tag{20}$$

The spatial spectral estimation function based on the weighted MUSIC algorithm is:

$$P_{W-MUSIC}(\theta) = \frac{1}{A^H(\theta)U_{new}U_{new}^H A(\theta)} \tag{21}$$

3.2 Improved linear array structure of sensor

Under this premise of the same array number, the non-uniform linear array (NLA) can achieve greater aperture and resolution than the uniform linear array (ULA,). However, the weighted MUSIC algorithm by using the autocorrelation matrix does not apply to the irregular non-uniform linear array [12-13], the increase of the array spacing of the non-uniform linear array will cause appearance of the false spectral peaks [14-15]. Therefore, this paper designs a non-uniform linear array based on the symmetric distribution of the array element center, This linear array maintains a similar complementary correlation with the uniform linear array, and gives the applicability proof and the increase of the element spacing and cause the real spectral peak program is extract under the case of false spectral peak.

3.2.1 Non-uniform linear array based on central symmetry

First, the coordination of array element and the wavelength radius ratio are normalized, and the set $L = \{l_1, l_2, .. l_m \ldots, l_{M-1}\}$ is the distance of the first m array element to the first array element, and meet formula (22):

$$l_{m+1} - l_m = l_{M-m+1} - l_{M-m}, m = 1,2,\ldots,M-2 \tag{22}$$

The coordination of array element and the wavelength radius ratio are normalized, Table 1 gives some examples of the array.

Table 1 Non-uniform center-symmetric linear array based on symmetric distribution of array element centers

Array number M	coordination of normalized element
4	{0,1,3,4}
5	{0,1,3,5,6}
6	{0,1,3,5,7,8}
7	{0,1,3,6,9,11,12}
8	{0,1,3,6,9,12,14,15}
9	{0,1,3,6,10,14,17,19,20}
10	{0,1,3,6,10,14,18,21,23,24}

In this paper, we design a signal vector matrix $A=[a_0,a_1,..a_k...,a_K]$ of signal direction of non-uniform linear array based on central symmetry, in which the matrix a_m can be written as follows $a_k=[1 \quad (e^{-j\tau_k})^{l_1} \quad \cdots \quad (e^{-j\tau_k})^{l_{M-1}}]^T$: it can be seen from the $v_j^T \perp a_k$ and polynomial (13)

$$F(z)=v_{j,0}+v_{j,1}z^{l_1}+...+v_{j,M-1}z^{l_{M-1}}=0 \tag{23}$$

It can be obtained from formula (23) $Z(z)$ is:

$$Z(z)=F^*(z)=v_{j,0}^*+v_{j,1}^*(z^*)^{l_1}+...+v_{j,M-1}^*(z^*)^{l_{M-1}}=0 \tag{24}$$

It can be obtained from the characteristics of the array element spacing of formula (22) and formula (24)

$$z^{l_{M-1}}Z(z)=v_{j,M-1}^*+v_{j,D-1}^*z^{l_1}+...+v_{j,0}^*z^{l_{M-1}}=0 \tag{25}$$

It can be known from the formula (23), (24) and (25) that we can see that the non-uniform linear array based on the symmetric distribution of the array element center not only meets characteristics of $(v_j^T)^* \perp a_k^*$ and $J(v_j^T)^* \perp a_k$. But the irregular non-uniform linear array of vector can only meet $(v_j^T)^* \perp a_k^*$, and does not meet $J(v_j^T)^* \perp a_k$, so the weighted MUSIC algorithm can be applied to the non-uniform linear array based on the symmetric distribution of the element center designed by this experiment.

3.2.2 Extract the program of real spectral peak

For the specific conditions of the experiment, when $d=\lambda$, the number of false spectral peak is analyzed, and the program of spectral peak is extracted.

For a one-dimensional MUSIC algorithm, it is assumed that $\tilde{\theta}$ is estimation angle, and if there is a spectral false peak, then $\tilde{\theta}$ should include the true arrival direction of the information source θ) in the spectral peak search interval and direction and angle of all false sound sources θ_i $i=1,2,\ldots,n-1$. The direction vector phase of the array structure of the one-dimensional linear array is $\varphi=2\pi(d/\lambda)\sin(\tilde{\theta})$, and φ is located in the interval $[-2\pi d/\lambda, 2\pi d/\lambda]$:

$$2\pi(d/\lambda)\sin(\tilde{\theta})=2\pi(d/\lambda)\sin(\theta)+2k\pi \tag{26}$$

It can be seen from formula (20):

$$\tilde{\theta}=\arcsin(\sin\theta+(\frac{k}{d/\lambda})) \tag{27}$$

It can be seen from the experimental conditions $d=\lambda$ and formula (21):

$$(\sin(\theta)+k)\in[-1,1] \tag{28}$$

It can be known from formula (22) that if $\theta\in[0,\pi/2]$, k can only take 0 or -1. $\tilde{\theta}=\theta$ is the true signal direction angle; if $k=-1$, $\tilde{\theta}=\theta_1$ is direction angle of the false sound source. Thus, when $d=\lambda$, there is only a false sound source θ_1, and $\theta_1\in[-\pi/2,0]$, similarly, when $\theta\in[-\pi/2,0)$, $\theta_1\in[0,\pi/2]$. Therefore, when not increasing the calculation amount and other equipment, the search scope can be reduced to eliminate the effect of the false spectral peaks.

4. Simulation Experiment

Simulation 1: Assume there are three narrowband coherent signals, they are incident on the antenna array at 30 °, 45 °, 50 °, respectively, the sampling rate is 1024, MUSIC algorithm and IMUSIC algorithm are run in 9-element uniform linear array simulation model whose array spacing is equal to the half-wavelength. The W-NMUSIC algorithm is run on a 9-element central symmetric non-uniform linear array simulation model arranged in Table 1. The simulation results of Figure 2~Figure in 5 SNR = 9dB, 7dB, 5dB, 3dB.

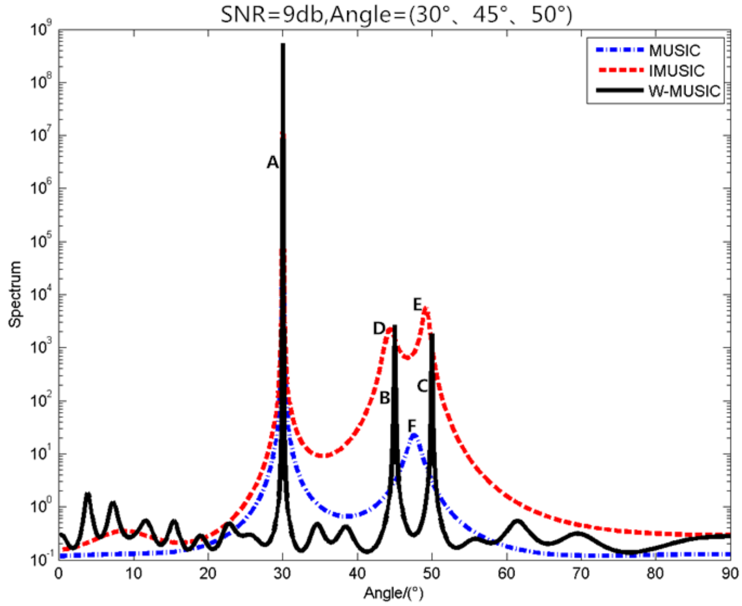

Figure.2 SNR = 9, Angle = (30 °, 45 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

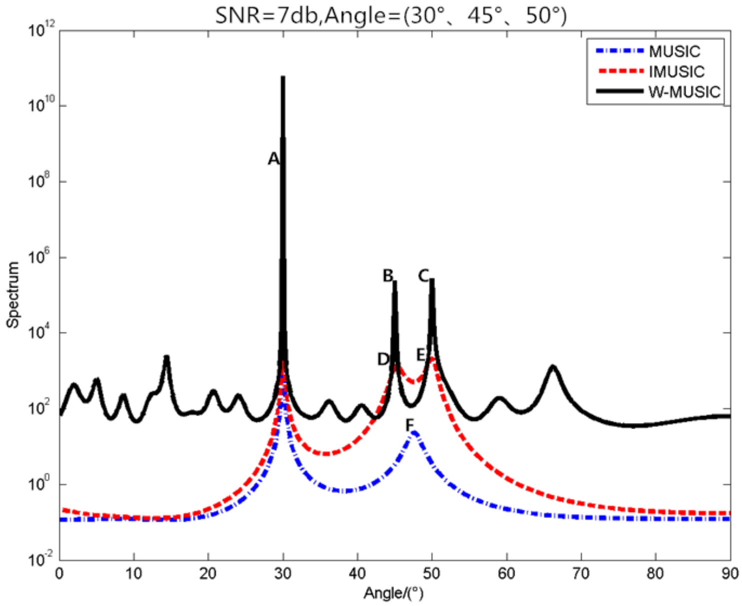

Figure.3 SNR = 7, Angle = (30 °, 45 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

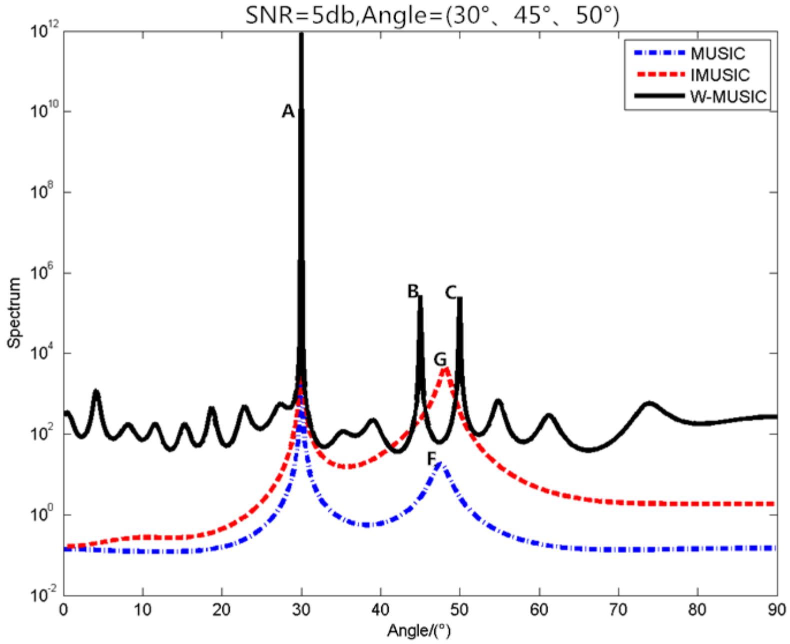

Figure.4 SNR = 5, Angle = (30 °, 45 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

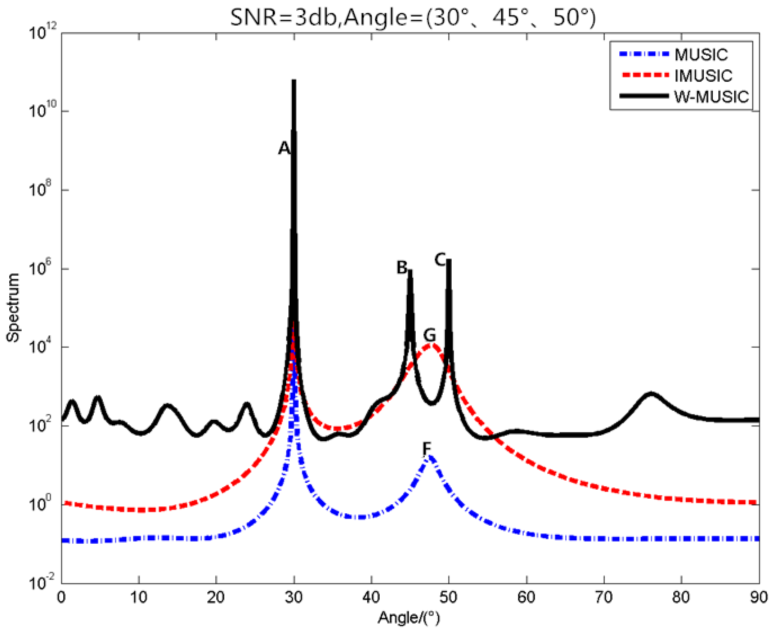

Figure 5.SNR = 3, Angle = (30 °, 45 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

The results analysis of simulation 1: Figure 2 to Figure 5 is the experimental results under the condition of same incident angle and different SNR, A, B, C, D, E, F, G are the peaks of the three algorithms after spectral peak search. When the incident signal is a coherent signal, with the reduction of SNR, the three algorithms can distinguish the signal whose incident angle is 30 ° and peak at A. The coherent signal whose signal interval is 5 °, incident angle is 45 °, 50 °, respectively, spectral peak search of MUSIC algorithm fail, the DOA of signal source cannot be distinguished; with the decreases of SNR, resolution of IMUSIC algorithm has declined as well, the peaks at D and E are gradually merged into a peak G, W-NMUSIC algorithm has a good ability to distinguish the signal, with the decrease of the SNR, it is still can distinguish the peak at the B and C, and contrast the peak at B and D, C and E, it can be seen that the W-NMUSIC algorithm is sharper than the peak of the spectral peak search of the IMUSIC algorithm.

Simulation 2: Assume that three narrowband coherent signals, which are incident on the antenna array with SNR=10db, the sampling rate is 1024, and the MUS-IC algorithm and the IMUSIC algorithm are run on a 9-element uniform array simulation model whose array element spacing is equal to half-wavelength. The W-NMUSIC algorithm is run on a 9-element central symmetric non-uniform array simulation model arranged in Table 1. The simulation results under the condition that incidence angles of Figs. 6 to 9 are (30 °, 45 °, 50 °), (30 °, 46 °, 50 °), (30 °, 47 °, 50 °), (30 °, 50 °).

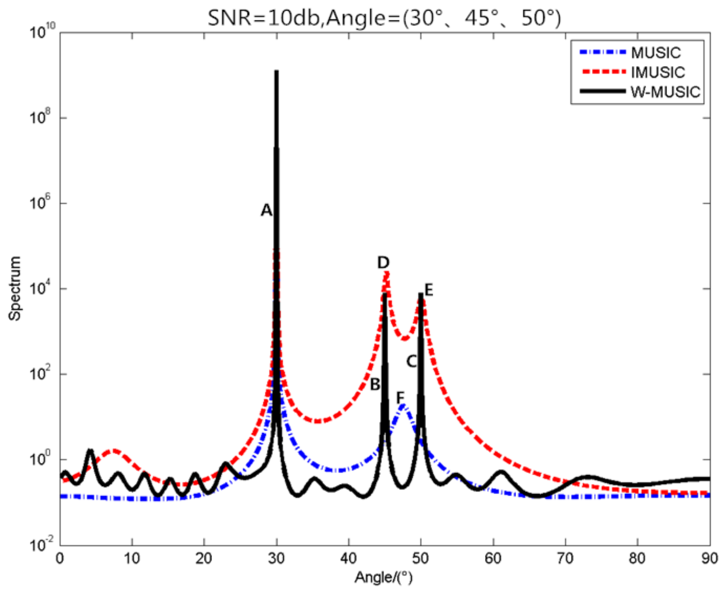

Figure.6 SNR = 10, Angle = (30 °, 45 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

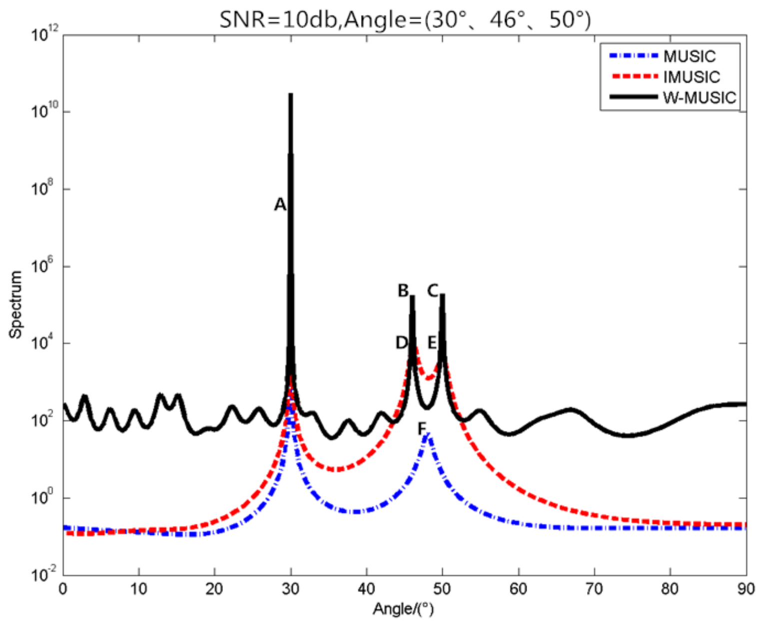

Figure 7 SNR = 10, Angle = (30 °, 46 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

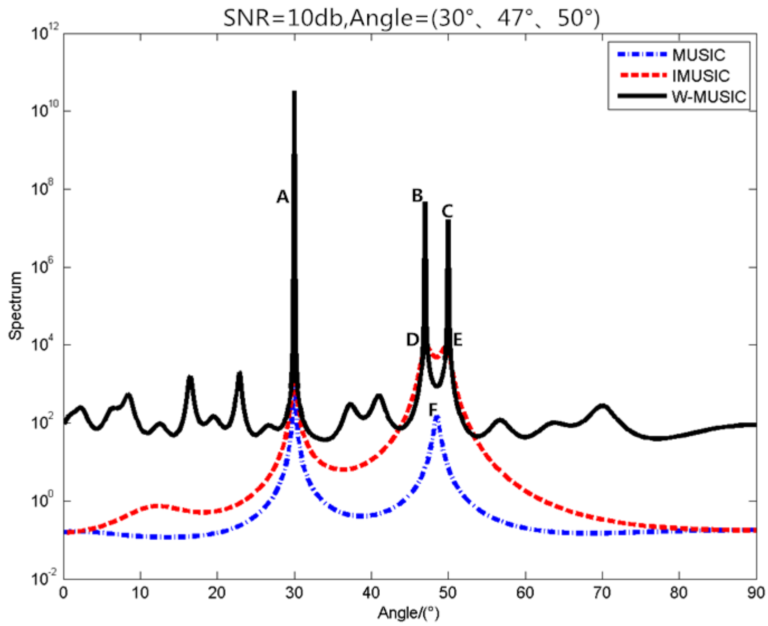

Figure 8 SNR = 10, Angle = (30 °, 47 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

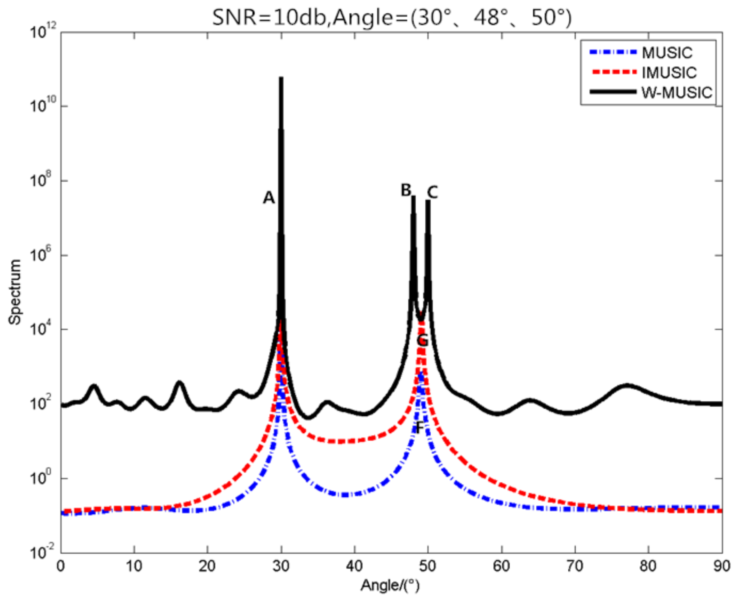

Figure 9 SNR = 10, Angle = (30 °, 48 °, 50 °), MUSIC algorithm, IMUSIC algorithm, W-NMUSIC algorithm of coherent signal

The results analysis of simulation 2: Figure 6 to Figure 9 is the experimental results under the condition of same SNR and different incidence angle, A, B, C, D, E, F, G are the peaks of three algorithms after spectral peak search, when the SNR ratio is the same, the three algorithms can distinguish the signal whose incident angle is 30° and peak at A, and for the other two signals with less than 5 ° interval, the spectral peak search of MUSIC algorithm fails, the DOA of the signal source cannot be distinguished, as the interval of incidence angle of the coherent signal becomes smaller and smaller, the resolution of the IMUSIC algorithm is also decreasing, and the peak at D and E is gradually merged into a peak G, the W-NMUSIC algorithm has good ability to distinguish signal, when the signal interval is 2 °, it still can distinguish the peak between the B and C, and compare the peak between B and D, C and E, it can be seen that W-NMUSIC algorithm is sharper than the peak of the spectral peak search of the IMUSIC algorithm.

5. Conclusion

This paper aims the problem that the small SNR source with smaller coherent signal source and incident angle spacing adopts MUSIC method and cannot be distinguished, noise subspace weight is proposed, And W-NMUSIC methods with centrally symmetric non-uniform linear array is used, thus under the low SNR conditions, strong correlation signal sources with relatively smaller interval get a higher ability to distinguish. When SNR = 0db, it still can distinguish two coherent signals with 1° interval, which significantly improves the direction-finding performance of algorithm.

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