A Study on the Formula of Calculating the Volume of Rotating Bodies with Arbitrary Cross Sections

Yun Xu [1, a], Zhirong Li [1,b] and Qi Li [2,c]

[1] Sichuan University of Science & Engineering, Zigong Sichuan 643000, China;

[2]Fujian Weino CNC Co.,LTD, Putian Fujian 351100,China

[a]xuyunzigong@163.com, [b]yoursos@126.com, [c]13778523122@163.com

Abstract

This paper proposes a new formula which can calculate the volume of rotating body with arbitrary cross sections by making an inference and development on the known formula of calculating the rotating bodies, and it also verifies the validity of this formula by the method of induction. The authors put forward one way to quickly calculate the volume of rotating bodies with arbitrary cross sections combining the common CAD drawing tools with the new formula of volume. This way makes a breakthrough in the common practice of calculating the volume of rotating bodies with its simple calculating procedure and can be also helpful to improve the design quality and standard.

Keywords

Arbitrary cross section; rotating body; volume calculation formula.

1. Introduction

Volume calculation plays an important role in mechanical design. The calculation of part volume is often needed in the whole production process, such as the parts mass calculation, cost estimation, lifting, transportation and so on. Because the appearance of parts is different, the calculation can only be achieved by summing up the volume data of cylindrical, rectangular, prism and other regular rotators divided from the parts. But it is very difficult to get the volume of rotating body with arbitrary cross sections without uniform formula. So, how to quickly and accurately calculate the volume of revolving solid with arbitrary cross sections is the key to solve the problem in the case of known material density.

2. A new Formula Conjecture for Calculating the Volume of Rotating Bodies

In Figure 1, the shaded area is the rotating section of cylinder. The calculation formula of cylinder volume is known as $V=\pi r^2 h$, form which the following formula can be deduced.

$$V=\pi r^2 h=\pi\cdot r\cdot r\cdot h=2(\pi\cdot\frac{r}{2})\cdot(r\cdot h)$$

Let $2(\pi\cdot\frac{r}{2})=L$, $r\cdot h=S$, the calculation formula of cylinder volume can be written as:

$$V=SL \quad (1)$$ [1]

What is completely different between the new calculation formula of cylinder volume and the conventional method of calculating volume lies in using neither bottom area, nor the use of cylindrical high in the calculation of cylinder volume.

L is the circumference of the circle whose radius is from the center of gravity of the rotating section to the axis of rotation, and S is the rotating section area of cylinder. In other words, the volume of rotating body can be expressed as product of the rotating section area and the circumference of the circle whose radius is from the section center to the axis of rotation.

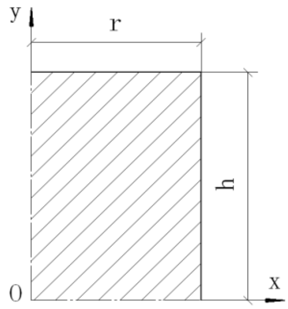

Fig1 The cross section of cylinder

3. The Mathematical Rroof for the New Calculation Formula of Volume

Formula (1) has been developed from the calculation formula of cylinder volume, but there is a question as whether Formula (1) is general and also applicable to the volume calculation of rotating bodies with arbitrary cross sections. The study found that Formula (1) can be proved to be general with a mathematical method.

A revolving solid with an arbitrary cross section is needed first for the following proof. The arbitrary cross section is constructed through removing a section with arbitrary curve from a rectangle with a random length and width. The rectangular area is marked as S1, the area of removed section is marked as -Sk, and the area of remained section is marked as Sk+1. As shown in the Figure 2, the two shaded sections are the constructed arbitrary cross sections. Now, the Formula (1) can be proved with the induction method[1].

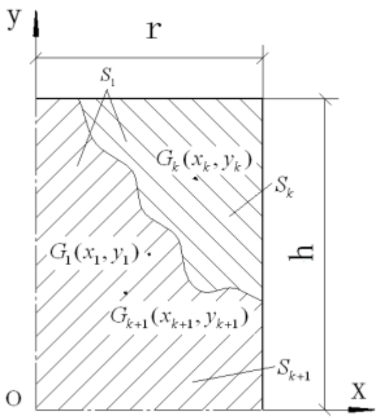

Fig2 The redefined cross-section with arbitrary cross sections

According to the above reasoning process of the volume formula conjecture about the rotating body, Formula (1) is correct when the rotating body is generated by a rectangle. That is if n=1, the following formula is workable:

$$V1=S1L1 \tag{2}$$

Let Sk be the arbitrary cross-section area made up of k small rectangles, S k+1 be the arbitrary cross-section area made up of k+1 rectangles, then there is an equation as following:

$$S1=Sk+Sk+1 \tag{3}$$

In order to make it convenient to prove the following formula, it can be assumed that Vk（Vk+1）is the summed volume of k（k＋1）small rectangle rotating bodies; Sk（Sk+1）is the area of k(k+1) small rectangles; Lk（Lk+1）is the perimeter of the arc from the core of k(k+1) small rectangles to the axis of rotation. Assume that if n=k, that is to say if it is workable that the cross sections of rotating bodies with arbitrary sections are constructed by k small rectangles, then

$$V_k = S_k L_k \tag{4}$$

Now we need to verify that if n=k+1, then Formula (1) is workable, and that is also to verify Vk+1=Sk+1Lk+1. Assume that the core coordinate of the arbitrary cross sections of k small rectangles is $G_k(x_k, y_k)$ and the core coordinate of the cross sections of the rotating bodies constructed by k+1 small rectangles is $G_{k+1}(x_{k+1}, y_{k+1})$ as shown in the Diagram 2.

Let us now work at the relations of three cores: G1, G kand G k+1. As the relation of formula (3) on area is verified, the new core formed by the combined area Sk+Sk+1 is G1, and the ligature connecting the core G k and core G k+1 must cross G1. According to the core quality of " torque can keep balance everywhere if the support is the core" we can get the torque balance formula with G1 as the core:

$$|\overline{G_1G_{k+1}}| \cdot S_{k+1} \cdot \rho = |\overline{G_1G_k}| \cdot S_k \cdot \rho \tag{5}$$

ρin the formula is the surface density of the cross section.
The geometric relations of coordinate points in Formula (3) and Diagram 2 are the following:

$$\frac{S_{k+1}}{S_k} = \frac{|\overline{G_1G_k}|}{|\overline{G_1G_{k+1}}|} = \frac{|x_k - x_1|}{|x_1 - x_{k+1}|} = \frac{x_k - x_1}{x_1 - x_{k+1}} \tag{6}$$

Formula (6) can be modified as

$$S_{k+1} \cdot x_{k+1} = (S_k + S_{k+1}) \cdot x_1 - S_k \cdot x_k \tag{7}$$

Fit Formula (3) in it, the Formula (7) can be finally modified as

$$S_{k+1} \cdot x_{k+1} = S_1 \cdot x_1 - S_k \cdot x_k \tag{8}$$

Multiply both the left and right side of equation (8) by 2π, it can be modified as:

$$(2\pi x_{k+1}) \cdot S_{k+1} = (2\pi x_1) \cdot S_1 - (2\pi x_k) \cdot S_k \tag{9}$$

According to the definition of L_1,L_k and L_{k+1} in Formula (1):

$$\begin{aligned} L_1 &= 2\pi x_1 \\ L_k &= 2\pi x_k \\ L_{k+1} &= 2\pi x_{k+1} \end{aligned} \tag{10}$$

Fit Formula(10)in Formula(9):

$$L_{k+1} \cdot S_{k+1} = L_1 \cdot S_1 - L_k \cdot S_k \tag{11}$$

Fit Formula(2) and(4)in(11):

$$L_{k+1} \cdot S_{k+1} = V_1 - V_k \tag{12}$$

Taking consideration of Diagram 2, let's work at the geometric significance of the right side of Formula（12）. It shows that if Vk is cut off from the volume of V1, then the remained volume is Vk+1, the relational volume formula can be written as：

$$V_1 - V_k = V_{k+1} \tag{13}$$

Fit Formula（13）in（12）:

$$L_{k+1} \cdot S_{k+1} = V_{k+1} \tag{14}$$

That is to say, if n=k+1, then Formula (1) is still workable.

Until now, the formula (1) is verified. This formula can be applicable to the calculation of volume of the rotating bodies with arbitrary cross sections.

4. The practical value of new volume formula

In an engineering practice, the new volume formula of rotating body will not be used if the cross section area of rotating body and the barycentric coordinate of the section are unknown. Fortunately, the area and barycentric coordinate of the cross section can be obtained through the current domestic and international CAD software. For example, the domestic CAD software CAXA has the area query function and barycenter query function in the tool menu of CAXA2013[2]. The new volume formula of rotating body in this paper will be applied in the practical work with the two basic elements.

The international general CAD software has the similar query functions for area and barycenter of arbitrary cross section. In the case of autoCAD2014, we can determine the region to calculate first, and then query the area and barycenter of the defined region in the domain/quality characteristics menu under the query menu bar[3]. The center of mass is the center of gravity for the homogeneous material[4]. So, the new volume formula of rotating body in this paper can be supported and used in AutoCAD.

With the support of the query function for area and barycenter in CAD software, the new volume formula of rotating body can offer great help for the designers to calculate the volume of rotating body quickly, and the practical value of new volume formula will be reflected.

5. Conclusion

The volume calculation of rotating body with arbitrary cross section is a difficult problem in the engineering design. In this paper, the method to calculate the mass of rotating body with arbitrary cross section, through the query functions for area and barycenter in CAD software, is simple, accurate and rapid. The method has a lower level requirement for the user’s knowledge and capacity, and is very convenient to use.

Acknowledgements

This work was financially supported by Ministry of Industry and Information Technology (MIIT Package (2016) No.213) and key research and development project of Sichuan provincial science and Technology Department (No. 2017GZ0142, 2017GZ0340).

References

[1] Li zhi-rong, Wang ya-ping CAXA Based Rapid Mass Calculation for Rotating Objects[J]. Construction Machinery and Equipment,2006,37(12):3-7(in Chinese).
[2] Autodesk, Inc. User Documentation [CP/DK]. AutoCAD 2014 help document.
[3] CAXA, Inc. User Documentation [CP/DK]. CAXA 2013 help document.
[4] Zhang wen-jian, Wang shi-en,The Relation of the Center of Gravty Defination Between the Center of Mass[J]. Journal of Mengzi Teathers’ College,1999, 06(63-64) (in Chinese).