The Influence of Winding Channel on the Turbulent Flow in Numerical Simulation

Xiaolong Zhang

School of Chongqing Jiaotong University, Chongqing 400000, China.

xl291218@outlook.com

Abstract

In this paper, orthogonal curvilinear coordinate system by establishing a track toward the river. The instantaneous velocity and pressure term is decomposed into elementary stream term and coherent volutes disturbance. Results obtained dimensionless perturbation O-S by calculating the equation, and then the multi-modal scale turbulence calculated. Calculate Re = 30000 and Re = 60000, the river bend in different degrees, curve circular frequency and wave number of the growth rate. And were obtained when Re = 30000 and Re = 60000, the growth rate of perturbations with the river bend changes. By bending on Turbulent flow of the river to study, it can be applied to practical engineering simulation.

Keywords

Turbulent flow, eynolds number, Meandering river, numerical simulation.

1. Introduction

Curved Inland is a very common phenomenon in Yangtze River valley and the study of turbulent flow in natural curved channel has become one of the most popular research topics in recent years. The turbulent flow of curved rivers play a decisive influence on the cause of riverbed and the movement of silt, meanwhile, turbulent flow also plays an important role in the stability of the river bank and the development trend of the channel. Many domestic and overseas researchers have made some careful studies about the velocity distribution and achieved ideal results[4-6]. With the improvement of the reliability in free surface and the further study of the turbulent flow model, three-dimensional flow pattern simulation in natural channel can be realized[7-9].Although the numerical simulation technique is mature enough, the application in natural open channel is limited by the stability of hyper-concentration flow and the evolution feature of river bed. In recent years, some significant progress has been made in the related areas[10].This paper focus on the turbulent flow Vortex structure in open channels based on the others researchers' theoretical method. Turbulent flow in curved areas is consisted of varies Vortex structures[11],according to the Fluid mechanics research results in recent years[12-15],This thesis put quasi order Vortex structures as a kind of turbulence and try to figure out the evolution of multi-scale turbulent flow structure in curved channels based on the theoretical analysis method, thus laying foundations for finding ways to control the form of river and the bed forming vortex from the theoretical point of view.

2. Theory Model

The research about theturbulent flow of two-dimensional meandering streams is based on the theory model. In order to reflect the evolution of turbulent flow nearby the winding river bank, coordinate transformation method will be used to fit the wall curved boundary.

2.1 Basic Variation and Governing Equations.

Abscissa : y=y(x) River center —s coordinate, y-coordinate—n,building the orthogonal curvilinear coordinate .

2.2 Evolution Analysis of Quasi Order Disturbance Quantity.

Instantaneous speed and Pressure term can be resolved into basic flow pattern and quasi order vortex disturbance term.

$$\begin{bmatrix} u_s \\ u_n \\ P \end{bmatrix} = \begin{bmatrix} \bar{u}_s \\ \bar{u}_n \\ \bar{P} \end{bmatrix} + \begin{bmatrix} u_{s\frac{1}{R}} \\ u_{n\frac{1}{R}} \\ P_{\frac{1}{R}} \end{bmatrix} \tag{1}$$

Basic flow： $\bar{u}_s$, $\bar{u}_n$ .Disturbance of the vortex on the flow in the curved river: $\bar{P}$. $u_{s\frac{1}{R}}$, $u_{n\frac{1}{R}}$, $P_{\frac{1}{R}}$.

2.3 Basic Flow Equation.

$$\frac{\partial \bar{u}_s}{\partial s} + \frac{\partial \bar{u}_n}{\partial n} = 0 \tag{2}$$

$$\frac{\partial \bar{u}_s}{\partial t} + \bar{u}_s \frac{\partial \bar{u}_s}{\partial s} + \bar{u}_n \frac{\partial \bar{u}_s}{\partial n} = f_s - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial s} + \frac{2}{\rho}\frac{\partial}{\partial s}\left[\mu\left(\frac{\partial \bar{u}_s}{\partial s}\right)\right] + \frac{1}{\rho}\frac{\partial}{\partial n}\left[\mu\left(\frac{\partial \bar{u}_n}{\partial s} + \frac{\partial \bar{u}_s}{\partial n}\right)\right] \tag{3}$$

$$\frac{\partial \bar{u}_n}{\partial t} + \bar{u}_s \frac{\partial \bar{u}_n}{\partial s} + \bar{u}_n \frac{\partial \bar{u}_n}{\partial n} = f_s - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial n} + \frac{2}{\rho}\frac{\partial}{\partial n}\left[\mu\left(\frac{\partial \bar{u}_n}{\partial s}\right)\right] + \frac{1}{\rho}\frac{\partial}{\partial s}\left[\mu\left(\frac{\partial \bar{u}_n}{\partial s} + \frac{\partial \bar{u}_s}{\partial n}\right)\right] \tag{4}$$

2.4 Quasi Order Disturbance Quantity Equation.

$$\frac{1}{1-N}\frac{\partial u_{s\frac{1}{R}}}{\partial s} - \frac{u_{n\frac{1}{R}}}{(1-N)R} + \frac{\partial u_{n\frac{1}{R}}}{\partial n} = 0 \tag{5}$$

$$\begin{aligned} &\frac{\partial u_{s\frac{1}{R}}}{\partial t} + \frac{1}{1-N}\bar{u}_s \frac{\partial u_{s\frac{1}{R}}}{\partial s} + u_{n\frac{1}{R}}\frac{\partial \bar{u}_s}{\partial n} - \frac{u_{n\frac{1}{R}}\bar{u}_s}{(1-N)R} = \frac{1}{(1-N)\rho}\frac{\partial P_{\frac{1}{R}}}{\partial s} + \frac{\mu}{(1-N)^2\rho}\frac{\partial^2 u_{s\frac{1}{R}}}{\partial s^2} \\ &-\frac{2\mu}{\mathrm{R}(1-N)^2\rho}\frac{\partial u_{n\frac{1}{R}}}{\partial s} + \frac{\mu}{\rho}\frac{\partial^2 u_{s\frac{1}{R}}}{\partial n^2} - \frac{\mu}{\mathrm{R}(1-N)\rho}\frac{\partial u_{s\frac{1}{R}}}{\partial n} - \frac{\mu}{\mathrm{R}^2(1-N)^2\rho}u_{s\frac{1}{R}} \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{\partial u_{n\frac{1}{R}}}{\partial t} + \frac{1}{1-N}\bar{u}_s \frac{\partial u_{n\frac{1}{R}}}{\partial s} + \frac{2u_{s\frac{1}{R}}\bar{u}_s}{(1-N)R} = -\frac{1}{\rho}\frac{\partial P_{\frac{1}{R}}}{\partial n} + \frac{\mu}{(1-N)^2\rho}\frac{\partial^2 u_{n\frac{1}{R}}}{\partial s^2} + \frac{2\mu}{\mathrm{R}(1-N)^2\rho}\frac{\partial u_{s\frac{1}{R}}}{\partial s} \\ &+\frac{\mu}{\rho}\frac{\partial^2 u_{n\frac{1}{R}}}{\partial n^2} - \frac{\mu}{\mathrm{R}^2(1-N)^2\rho}u_{n\frac{1}{R}} - \frac{\mu}{\mathrm{R}\ (1-N)\rho}\frac{\partial u_{n\frac{1}{R}}}{\partial n} \end{aligned} \tag{7}$$

3. The Solution Method of Theoretical Model Equation

3.1 The theoretical solution of basic flow equation.

For the convenience of theoretical analysis, the turbulence index flow rate formula to replace Fig.(2) to Fig.(7)：

$$\bar{u}_s = \bar{u}_{sc}\left(\frac{b-n}{b}\right)^{1/7} \tag{8}$$

Maximum flow speed $\bar{u}_{sc}$,half wide of the river b, the equation is as below：

$$\bar{u}_s = (1-n)^{1/7} \tag{9}$$

3.2 The Solution Method of Quasi Order Disturbance Quantity Equation.

Quasi Order Disturbance Quantity Equation:

$$\begin{bmatrix} u_{s\frac{1}{R}} \\ u_{n\frac{1}{R}} \\ P_{\frac{1}{R}} \end{bmatrix} = \begin{bmatrix} \hat{u}_{s\frac{1}{R}} \\ \hat{u}_{n\frac{1}{R}} \\ \hat{P}_{\frac{1}{R}} \end{bmatrix} e^{i(\alpha s-\omega t)} \tag{10}$$

$D=\frac{d}{dn}$, $D^2=\frac{d^2}{dn^2}$ put (10) into (7) the Quasi Order Disturbance Quantity Equation as follows:

$$\frac{i\alpha}{1-N}\hat{u}_{s\frac{1}{R}} = [\frac{1}{(1-N)R} - D]\hat{u}_{n\frac{1}{R}} \tag{11}$$

$$\left[\nu\left(D^2 - \frac{\alpha^2}{(1-N)^2} - \frac{D}{(1-N)R} - \frac{1}{(1-N)^2R^2}\right) - \left(\frac{i\alpha}{(1-N)}\bar{u}_s - i\omega\right)\right]\hat{u}_{s\frac{1}{R}} = \frac{i\alpha}{\rho(1-N)}\hat{P}_{\frac{1}{R}} + \left[\left(D\bar{u}_s - \frac{\bar{u}_s}{(1-N)R}\right) + \frac{2i\alpha}{R(1-N)^2}\nu\right]\hat{u}_{n\frac{1}{R}} \tag{12}$$

$$\left[\nu\left(D^2 - \frac{\alpha^2}{(1-N)^2} - \frac{D}{(1-N)R} - \frac{1}{(1-N)^2R^2}\right) - \left(\frac{i\alpha}{(1-N)}\bar{u}_s - i\omega\right)\right]\hat{u}_{n\frac{1}{R}} = \frac{1}{\rho}D\hat{P}_{\frac{1}{R}} + \left[\frac{2\bar{u}_s}{(1-N)R} - \frac{2i\alpha}{R(1-N)^2}\nu\right]\hat{u}_{s\frac{1}{R}} \tag{13}$$

Calculating $\frac{i\alpha}{1-N}\left[D(12) - \frac{i\alpha}{(1-N)}(13)\right]$,Quasi Order Disturbance Quantity Equation:

$$\begin{aligned} &\left\{\left[D^2 - \frac{\alpha^2}{(1-N)^2}\right]^2 - \frac{1}{\nu}\left(\frac{i\alpha}{(1-N)}\bar{u}_s - i\omega\right)\left(D^2 - \frac{\alpha^2}{(1-N)^2}\right) + \frac{1}{\nu}\frac{i\alpha}{(1-N)}D^2\bar{u}_s\right\}\hat{u}_{n\frac{1}{R}} \\ &+\left\{-\frac{1}{(1-N)R}\left[2D^3 - \frac{D}{(1-N)^2R^2} - \frac{D}{(1-N)^2R} - \frac{D^2}{(1-N)}\right] - \frac{1}{\nu}\frac{i}{(1-N)\mathrm{R}}\left[\omega - \frac{\alpha}{(1-N)}\cdot\bar{u}_s\right]D\right. \\ &\left.-\left[\frac{1}{(1-N)^4R^4} - \frac{1}{(1-N)^4R^3} + \frac{2\alpha^2}{(1-N)^4R^2}\right] - \frac{i}{\nu(1-N)^2R^2}\left[\frac{\alpha}{(1-N)}\bar{u}_s + \omega\right]\right\}\hat{u}_{n\frac{1}{R}} \\ &= 0 \end{aligned} \tag{14}$$

With the velocity of the basic flow as the reference velocity and the reference length of the half river width B is dimensionless. $\frac{\bar{u}_{sc}\cdot b}{\nu} = \mathrm{Re}$,Dimensionless disturbance equation(O-S):

$$\begin{aligned} &\left\{\left[D^2 - \frac{\alpha^2}{(1-N)^2}\right]^2 - \mathrm{Re}\left(\frac{i\alpha}{(1-N)}\bar{u}_s - i\omega\right)\left(D^2 - \frac{\alpha^2}{(1-N)^2}\right)\bar{u}_s + \mathrm{Re}\frac{i\alpha}{(1-N)}D^2\right\}\hat{u}_{n\frac{1}{R}} \\ &+\left\{-i\,\mathrm{Re}\frac{1}{(1-N)R^2}\left[\frac{\alpha}{(1-N)}\bar{u}_s + \omega\right] - \frac{i\,\mathrm{Re}}{(1-N)\mathrm{R}}\left[\omega - \frac{\alpha}{(1-N)}\cdot\bar{u}_s\right] - \left[\frac{1-R+2\alpha^2}{(1-N)^4R^2}\right]\right. \\ &\left.-\frac{1}{(1-N)R}\left[2D^3 + \frac{D^2}{(1-N)} + \frac{D}{(1-N)^2R^2} - \frac{D}{(1-N)^2R}\right]\right\}\hat{u}_{n\frac{1}{R}} = 0 \end{aligned} \tag{15}$$

In the process of time-mode instability evolution, α is the wave number of the disturbance wave. $\omega = \alpha\cdot\mathrm{c}$ (c wave speed) is the frequency of disturbance wave(plural), the imaginary part (α,ω) represents the growth rate of disturbance amount. The equation is as below:

$$F(\alpha, \mathrm{Re}, \omega, \mathrm{R}) = 0 \tag{16}$$

Equation(15) represents theOrr-Sommerfeldin the quasi order turbulent flow evolution feature of curved channels. The equation shows that due to the corrugation of the wall, there are additional parameters of the river radius of curvature into the equation.

4. Conclusion of the calculation and analysisof multi-scale turbulent flowin curved channels

4.1 The Calculation of Multi-scale Turbulent Flow Mode.

(1) There maybe multiple disturbances in the turbulence of the curved river. The figure has calculated the most stable modal when Re=30000, $\varepsilon_{1/R}=0$ and mode-2 always on the dominant place.

紊流顺直河道的模态计算

	第一模态		第二模态	
a	*wr*	*wi*	*wr*	*wi*
0.2	0.0580650	−0.0226228	0.1952280	−0.0033296
0.3	0.0802210	−0.0140156	0.2938590	−0.0042289
0.4	0.1013670	−0.0109796	0.3926590	−0.0050091
0.5	0.1212310	−0.0098603	0.4915740	−0.0057073
0.6	0.1393910	−0.0094340	0.5905750	−0.0063428
0.7	0.1558020	−0.0091013	0.6896470	−0.0069274
0.8	0.1710310	−0.0087223	0.7887760	−0.0074691
0.9	0.1854970	−0.0083494	0.8879560	−0.0079736
1	0.1993530	−0.0080054	0.9871800	−0.0084450
2	0.3180590	−0.0058455	1.9811030	−0.0119100
3	0.4160710	−0.0047633	2.9768560	−0.0141636
4	0.5025760	−0.0040886	3.9735000	−0.0160050
5	0.5812550	−0.0036218	4.9705980	−0.0177380
6	0.6540870	−0.0032704	5.9679560	−0.0194632
7	0.7223110	−0.0030067	6.9654760	−0.0212134
8	0.7867570	−0.0027901	7.9631180	−0.0230000
9	0.8480160	−0.0026112	8.9608500	−0.0248281
10	0.9652420	−0.0026194	9.9586500	−0.0266990

(2) to express more clearly about the variation of two modes, when calculating respectivelyRe= 30000 and Re= 60000 under different bending degree of channel, the changing curve of circular frequency and growth rate are as below(Fig.1 to Fig.6).

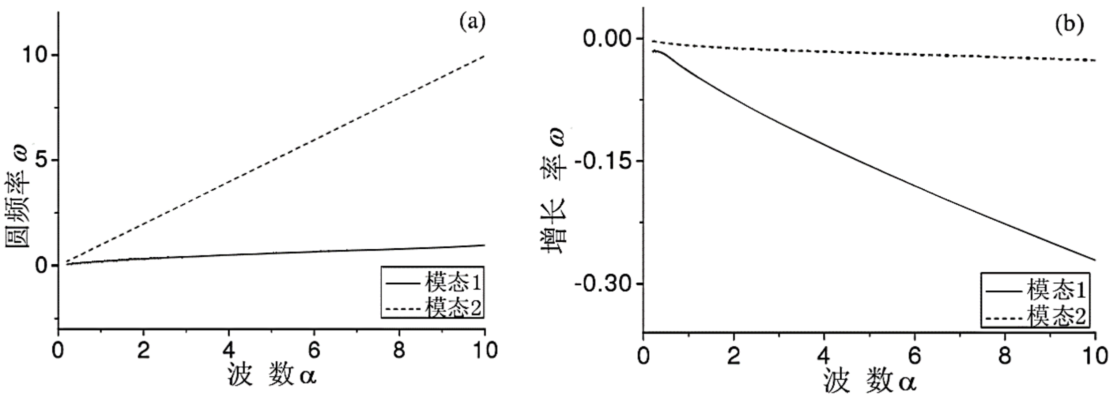

Fig.1 modal calculation of the circular frequency (a) and growth rate($\varepsilon_{1/R}=0$, Re=30000)

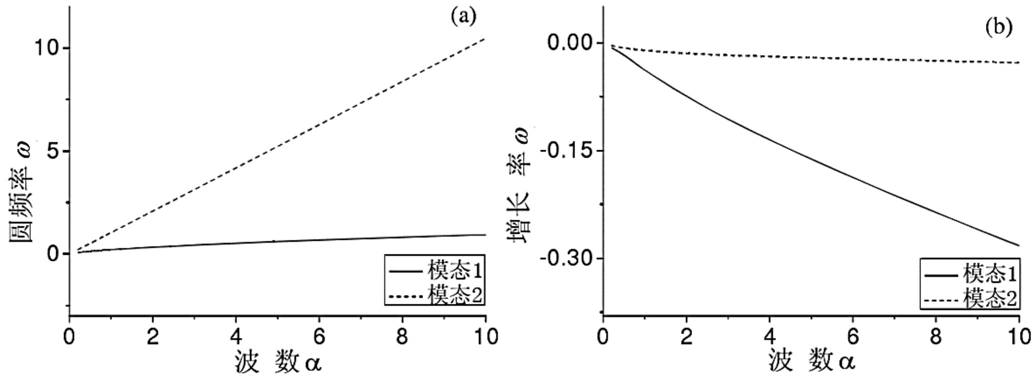

Fig.2 modal calculation of the circular frequency(a) and growth rate($\varepsilon_{1/R} = 0.05$, Re= 30000)

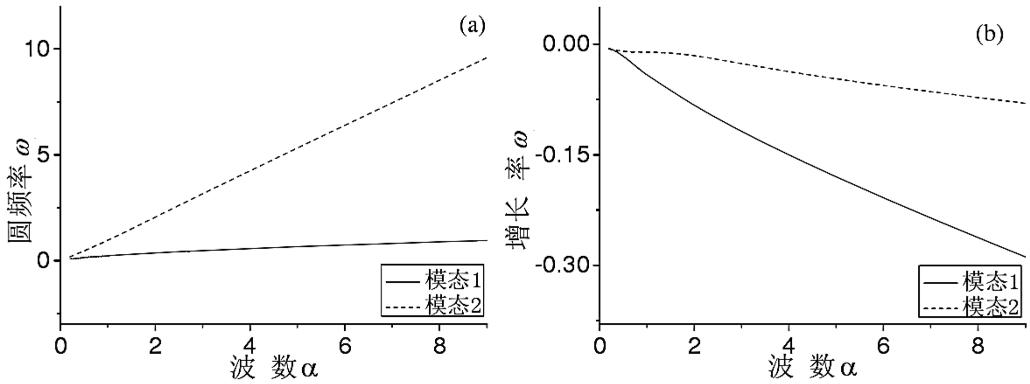

Fig.3 modal calculation of the circular frequency(a) and growth rate($\varepsilon_{1/R} = 0.167$, Re=30000)

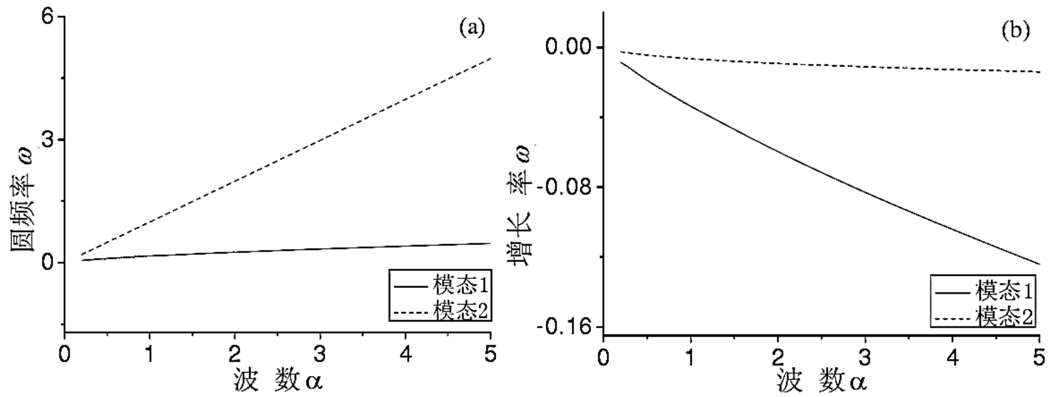

Fig.4 modal calculation of the circular frequency(a)and growth rate(b)($\varepsilon_{1/R} = 0$, Re=60000)

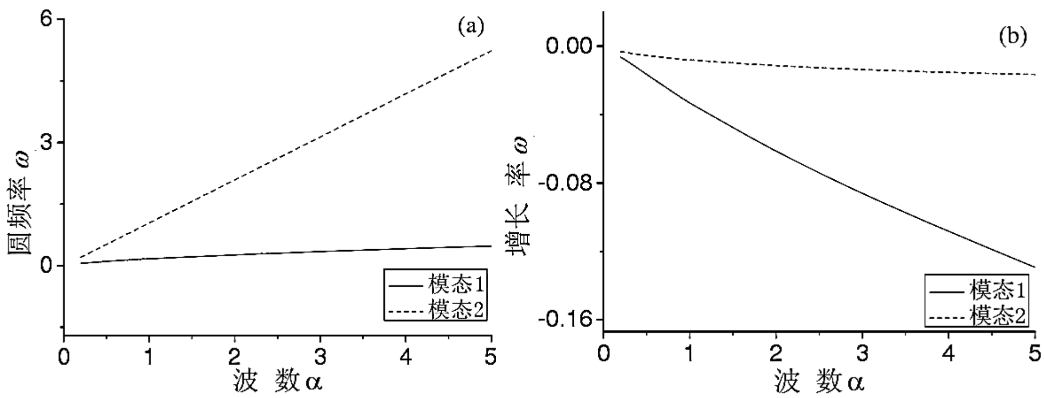

Fig.5 modal calculation of the circular frequency(a)and growth rate(b)($\varepsilon_{1/R} = 0.05$, Re= 60000)

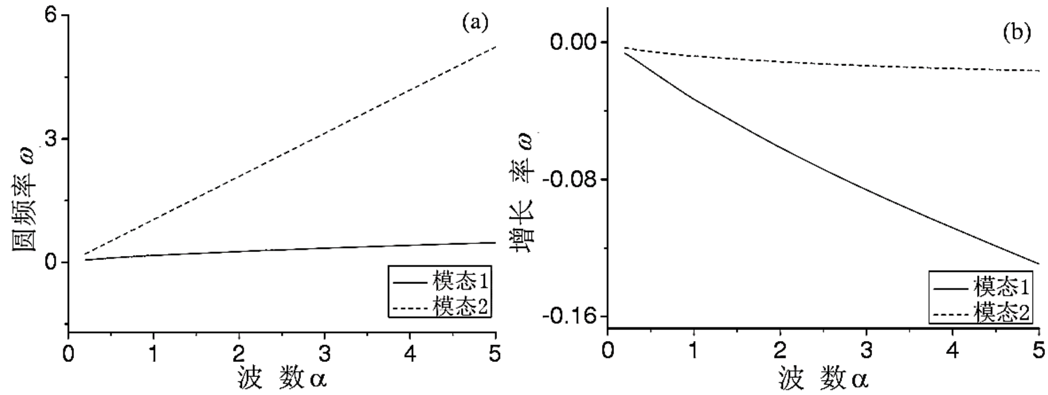

Fig.6 modal calculation of the circular frequency(a)and growth rate(b)($\varepsilon_{1/R} = 0.167$, Re= 60000)

4.2 The Influence of the Degree of Bending On the Characteristics of Multi-scale Turbulence in Open Channel.

The winding river bank as a kind of wavelike boundary which has additional turbulence on the flow.There is a significant difference in flow characteristic when compared the curvature turbulent flow of the river bank to the straight one,The turbulent energy near the convex bank will be reduced while the turbulent energy near the concave bank will increase[16].When calculating respectivelyRe= 30000 and Re= 60000 under different bending degree of channel, the changing curve of turbulent growth rate ω_i are as below(Fig.7 to Fig.10).

(1) Re=30000, the turbulent growth rate with the degree of channel bending

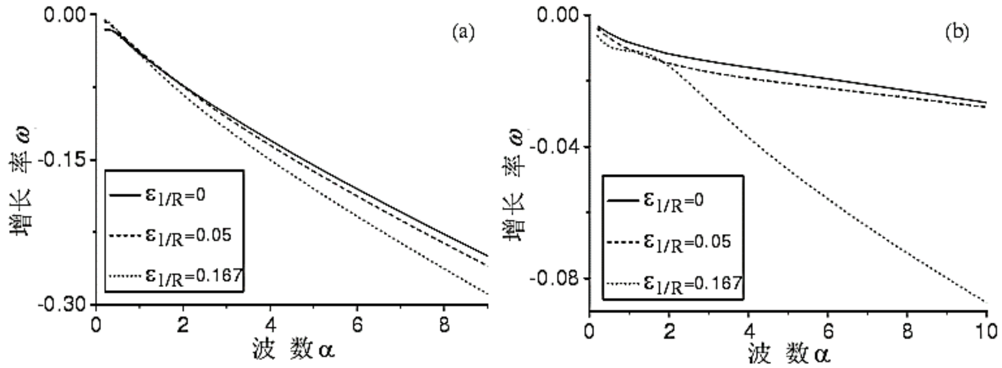

Fig.7 mode1(a) and mode2(b) The growth rate with the degree of channel bending

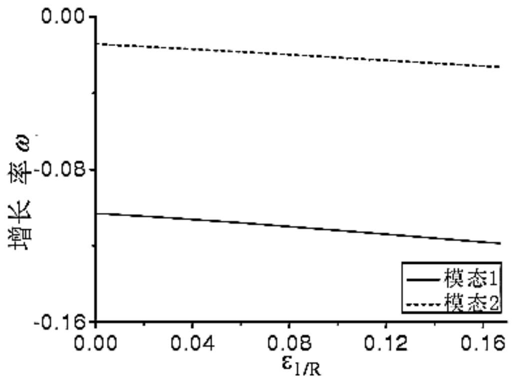

Fig.8 The growth rate with the degree of channel bending (α =3)

(2) Re =60000, the turbulent growth rate with the degree of channel bending

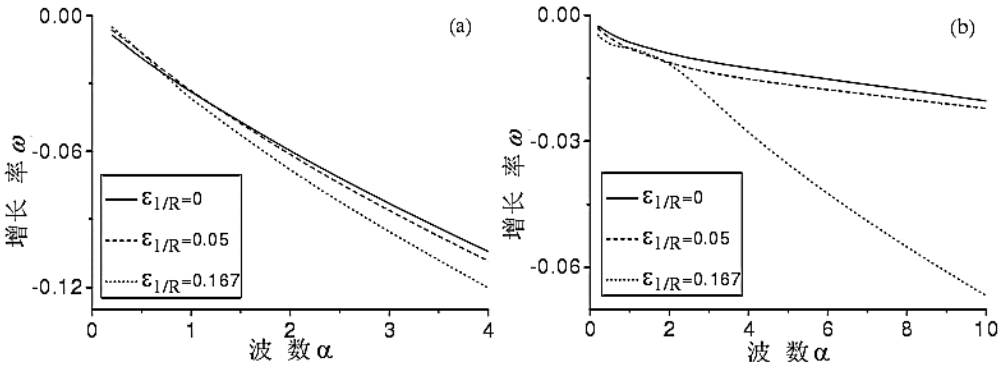

Fig.9 mode1(a) and mode2(b) The growth rate with the degree of channel bending

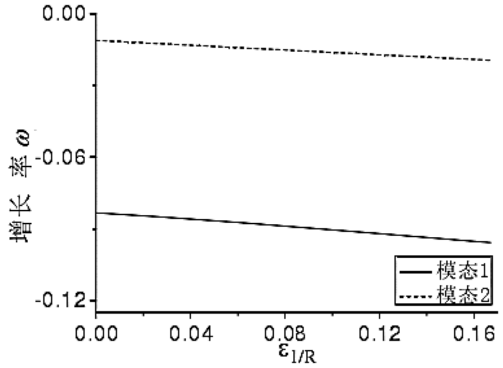

Fig.10 The growth rate with the degree of bending(α =3)

5. Conclusion

From diagram one to six, the turbulent of circular frequency of mode-one with wave number changed a little while the mode-two increased a lot, however, the variationtrend of turbulent growth rate decreases in both two models. The decay rate of mode-one decreases more when compared to mode-two.

From diagram seven to ten, a gradual increase in river bend will bring more internal boundary turbulence which will raise the imaginary absolute value of eigenvalue.By theoreticalresearch, Quasi order wave parameter can be selected in different ways and there is a limited range. When wave number $\alpha < 2$,the growth rate changed a little to the degree of bending but changed a lot when the wave number is big with the increase of the degree of bending, the imaginary absolute value of eigenvalue grow rapidly, The variation trend of growth rate to the degree of bending in mode-one and mode-two are the same, but the latter varies more remarkable.

References

[1] shigaki T., Shiono K., Rameshwaran P., PIV and LDA measurements of secpndary flow in a meandering channel for overbank flow, Journal of Visualization, 2002, 5(2): 153~159

[2] KanhuCharan Patra, Srijib K. Kar and Amartya K. Bhattacharya., Flow and Velocity Distribution in Meandering Compound Channels, Journal of Hydraulic Engineering, 2004, 130(5): 398~411

[3] Du Heqing, Miwa Hajime, Experimental study on the relation of bed morphology with surface flow in meander channels, International Journal of Sediment Research, 2006, 21(1): 1~12

[4] Xia Junqiang, Wang Guangqian, Wu Baosheng, evolution of wandering river and its numerical simulation. Beijing: China Water Conservancy and Hydropower Press, 2005

[5] Wu Xiuguang, Shen Yongming, Zheng Yonghong, et al. SIMPLEC algorithm for two-dimensional flow calculation in nonorthogonal curvilinear coordinates. Journal of hydraulic engineering, 2:25~37

[6] Xu Dong. Study on the dynamic process of meandering river evolution: [Ph.D. Dissertation], 2008

[7] Li Yanhong, Zhou Huajun, three dimensional numerical simulation of meandering river, 2004, 19:856~864

[8] C. A. M. E. Wilson, J. B. Boxall, I. Guymer, and N. R. B. Olsen., Validation of a Three-Dimensional Numerical Code in the Simulation of Pseudo-Natural Meandering Flows, Journal of Hydraulic Engineering, 2003, 129(10): 758~768

[9] Nils Reidar B. Olsen., Three-Dimensional CFD Modeling of Self-Forming Meandering Channel, Journal of Hydraulic Engineering, 2003, 129(5): 366~372

[10] Bai Chuan, Xu Haijue. Study on the flow stability characteristics of hyper concentrated flow. Chinese Science (G), 2008, 38 (2): 135~155

[11] Yalin M. S., Mechanics of sediment transport, 2nd Edition, Pergamon Press, 1977, 259

[12] Zhou Heng, Lu Changgen, Luo Jisheng. Simulation of single coherent structures in the near wall region of turbulent boundary layer. Chinese Science (A), 1999, 29 (4): 366~372

[13] Zhang Dongming, Luo Jisheng, Zhou Heng. A dynamic model of coherent structures in the near wall region of turbulent boundary layer. Chinese Science (G), 2003, 33 (1): 76~82

[14] Zhang Dongming, Luo Jisheng, Zhou Heng. A mechanism for the coherent structure of the near wall region in turbulent boundary layer perturbation. Applied Mathematics and mechanics, 2005, 26 (4): 379~385

[15] Luo Jisheng, Zhou Heng. A theoretical model and experimental comparison of large-scale coherent structures in the outer boundary layer of turbulent boundary layer. Chinese Science (A), 1993, 23 (1): 56~62

[16] Luo J, and Lakshminarayana B, Prediction of strongly curved turbulent duct flows with Reynolds stress model, AIAA Journal, 1997, 35(1): 91~98