

# Research on AC / DC System Flow Optimization Method with VSC - HVDC

Mingchuan Liu, Jin Ma

School of Electric and Electronic Engineering, North China Electric Power University,  
Beijing 102206, China  
Daldom@foxmail.com

---

## Abstract

With the development of VSC-HVDC transmission system and the popularity of the new power network structure, the analysis of its economics operation becomes more and more important. In this paper, it is considered that the power flow optimization method of the AC/DC hybrid system with VSC-HVDC. In the comparison of the existing domestic and foreign algorithms, the power flow optimization of the network is selected by the NSGA-II (Non-dominated Sorting Genetic Algorithm- II) and the primal dual interior point method. The algorithm of NSGA-II and interior point method is introduced in detail. Then the characteristics of VSC-HVDC are analyzed, including relevant state variables and constraint conditions of mathematical modeling. This paper analyzes the characteristics of the power system containing VSC-HVDC .And the related objective function and constraint conditions are established according to the evaluation index. Finally, the practical utility of the optimization algorithm is demonstrated by a numerical example. The results show that the optimized algorithm can quickly and effectively optimize the operation parameters by using the alternate algorithm.

## Keywords

VSC-HVDC, AC/DC, OPF, NSGA-II, Kannarkar.

---

## 1. Introduction

In the history of electric power development, because AC (alternating current) is easy to realize voltage conversion through electromagnetic coupling relationship, AC power transmission has always been the mainstream of power system transmission. However, with the development of power electronics technology, DC (direct current) transmission gradually occupy a place in the power grid. The economy and flexibility of the DC transmission line are of great concern to the industry, but the technical limitations of the thyristor itself limit the large-scale application of DC transmission. DC transmission is more used to connect two asynchronous power grids. Full-controlled power electronic devices are leading the DC transmission into the development of a new era. High voltage DC (HVDC) transmission technology based on the voltage source converter (hereinafter referred to as VSC-HVDC) makes AC-DC hybrid transmission increasingly become the standard form of the power system.

The core of the VSC-HVDC is the voltage source converter (VSC), which replaces the thyristor converter in a conventional HVDC with a full-controlled device. VSC is controlled by pulse width modulation (PWM) technology. VSC-HVDC can flexibly and efficiently control the flow, improve voltage stability and reduce voltage fluctuations. The power flow optimization is essential in the power system operation. Since the power loss is proportional to the transmission distance, choosing an optimal power flow in a power network allows the energy to be delivered from the supply side to the

demand side with the minimal consumption, which can save a lot of money, protect the environment and improve the reliability of the power system.

This paper introduces the new transmission mode mentioned above and discusses its power flow calculation method and optimization algorithm. The rest of this paper is organized as follows. Section 2 gives a detailed introduction of the interior point method. The main elements of the simple genetic algorithm, as well as the improvement of NSGA (Non-dominated Sorting Genetic Algorithm) and NSGA-II on simple algorithm are elaborated. Meanwhile, an iterative algorithm based on these two algorithms is proposed. In Section 3, the model of AC/DC power flow optimal calculation with VSC-HVDC is carried out. Some discussion of the power flow calculation is made. Section 4 takes IEEE 14 nodes system as an example to verify the feasibility of the algorithm. Section 5 is devoted to the conclusions.

## 2. Power Flow Optimal Algorithm

In a previous study [1], the power flow optimization based on Kannarkar interior point method is used. The interior point method is used to find the optimal solution directly within the feasible region, which avoids the disadvantages of the simplex method along the edge, greatly improving the computational efficiency. Another study [2] detailed introduced the improved algorithm of interior point method: the primal-dual interior point algorithm and the predictor-corrector interior point method. In the case of maintaining the primal and dual feasibility, the primal-dual interior point method is to find the optimal solution by the primal-dual path. The interior and external point method is proposed in the literature [3], which combines the interior point method and the external point method. The globally optimal solution is founded through the interior point method, and the next iterative computation is made by external point method.

In another study [4], considering the opening condition of the regional electricity market, a new objective function based on daily load curve is proposed. An optimal power flow problem is proposed in [5], which combines the optimal load reduction and active power restart scheduling problem. And then the power flow algorithm based on interior point method is optimized. In [6], an improvement measure of the power flow algorithm is proposed. The reactive genetic algorithm is improved by using the variable convergence method, which makes the iteration speed faster. In study [7], a VSC-HVDC steady-state model for PDIPM (primal-dual interior-point method) and PCPDIPM (predictive-corrector PDIPM) to solve the optimal power flow is proposed. In another study [8], the classical problem of reactive power optimization is discussed as well as the concrete steps of reactive power optimization. Another author [9] combined the multi-objective fuzzy optimization technique and the genetic algorithm to realize the improvement of the efficiency of the genetic algorithm in the selection of offspring. Study [10] discussed the improved NSGA and NSGA-II. In study [11], a method of differential local search is used to improve the NSGA-II.

## 3. Flow Optimization Method

### 3.1 Characteristics of Power System Flow Optimization Problem

Power system flow optimization needs to configure the parameters of generator output, transformer tap, SVC (static var compensator) and other power components, including discrete variables and continuous variables, which raises a new requirement for the flow optimization algorithm. The traditional linear programming algorithm mostly optimized for continuous variables, the result is usually smooth and continuous, is not conducive to quickly get the best flow. The linear programming method, which is based on the point method, is suitable for solving the optimization problem of continuous function. While the artificial intelligence algorithm is good at dealing with nonlinear problems and discrete problems. In order to speed up the convergence, this paper takes the two kinds of algorithms alternately.

### 3.2 Interior Point Method

The interior point method is based on the fact that all the available solutions are within the feasible region. From the initial interior point, use the gradient method to find a fastest descending direction from the starting point. So that each step is the optimal, directly from the feasible domain to the optimal solution. The basic idea of interior point method is to convert inequality constraints to equality constraints. The equality constraint is added to the objective function by the Laplace operator.

### 3.3 Non-dominated Sorting Genetic Algorithm-II (NSGA-II)

#### 3.3.1 NSGA

Unlike the single-objective optimization problem, the multi-objective optimization problem needs to satisfy the optimal form of multiple targets at the same time. That is, to obtain the Pareto optimal solution set in the solution space. The NSGA developed by Deb can Pareto order the breeding individual by changing the operator selection, then assign a common fitness to each population to rateably choose the degree of fitness. This optimal solution set, also known as non-dominated solution, its characteristic is that when raising the value of any one of the objective functions, it is not satisfied that the other target function not reducing at the same time. NSGA has a significant advantage over traditional genetic algorithms. In addition to the operator selection before calculation and the re-layer according to the individual's dominance, the other makes no difference with traditional genetic algorithm.

#### 3.3.2 NSGA-II

NSGA application has a certain limit. Among them, when the population is large, the calculation time is too long, the complexity is too high. Also, NSGA does not have the elite strategy, which makes the better parent may be abandoned. And it is needed to specify the range of solutions which can share the fitness. To this end, Deb proposed the improved algorithm NSGA-II which has been improved in the above three aspects. For the complexity of the algorithm, a fast non-dominated sorting method is proposed to reduce the cubic complexity of the original algorithm to the square complexity. The elite strategy is added to ensure that the outstanding father is not eliminated. As for the shared radius problem, with the crowding degree and crowding comparison operator to replace, the sort of superior descendants to retain. This allows the Pareto concentration of individuals to extend to the entire Pareto domain.

### 3.4 NSGA-II and Prime-dual Interior Point Method Alternate Iteration Algorithm

Refer to the computational strategy of the iterative algorithm of genetic algorithm and interior point method mentioned in [12], the two method both have their own calculation shortcomings. In this paper, an improved genetic algorithm and interior point method are used to achieve the best results.

When using a hybrid strategy, the problem is usually divided into discrete part and continuous part. Firstly, the discrete variables are ignored and the nonlinear programming problem is solved by the interior point method to complete the initial population generation. Then, in the continuous part, the discrete variables in the optimal power flow (OPF) model are taken as constant values, and the generator-side voltage and the output of the power plants are calculated using the original-dual interior point method. In the discrete part, the continuous variables in the OPF model are taken as constant values, and the discrete amounts are optimized using NSGA-II. After completing an iteration, the Pareto ordering mechanism is used to sort the populations, form the Pareto prospective optimal solution set, and the elite strategy is used to retain the excellent parent, and then continue the iteration. The above is the iterative process of embedding the interior point method into the genetic algorithm.

## 4. Power Flow Optimization Mathematical Model

### 4.1 VSC-HVDC Model

In the AC / DC power flow calculation, VSC will be modeled using the following assumptions.

- (1) VSC bus three-phase AC voltage is phase 120 ° symmetrical sine wave.
- (2) Set the rated capacity of the converter as the reference value.
- (3) The losses of the converter (including the converter inside and the commutator) are simulated by the resistor R.
- (4) VSC is fully symmetrical when loaded.

The basic operating rules of the VSC station are as follows: Properly adjusting the amplitude and phase angle of the VSC, the AC current can be adjusted by the phase reactors to control the active power and reactive power on the AC-DC co-coupled bus (PCC) power. Thus, the active and reactive power (or bus voltage) of the PCC bus is known as a known amount in the conventional AC power flow calculation, which is the same as the way of handling the injected active power and the generator terminal voltage.

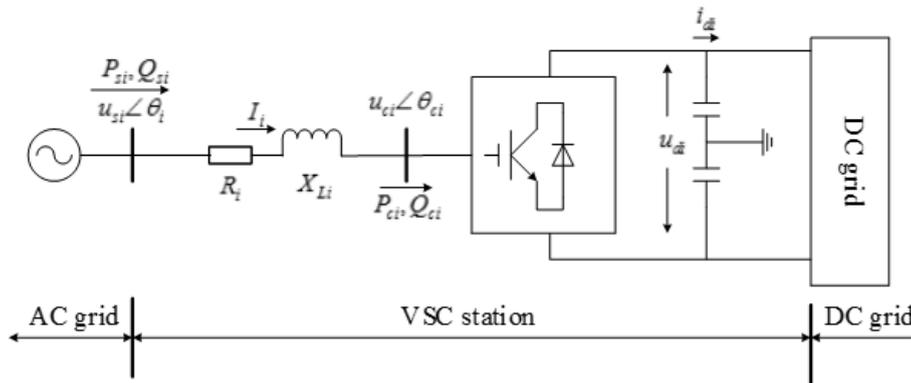


Fig. 1 VSC station typical disposition

The VSC station model in steady state includes the following three parts:

- (1) The Relationship between power and voltage of AC / DC power network.

Assuming that the power loss is ignored, the VSC power exchange amount is defined as follows:

$$P_c = -P_{cdc} \tag{1}$$

In order to avoid the overmodulation harmonics of the VSC, it is assumed that the phase voltage peak of the converter AC-side bus is lower than the corresponding converter DC-side bus [13]. Thus, the voltage relationship between the VSC AC and DC bus can be expressed using the real values shown in equation (2):

$$\bar{V}_c = (\sqrt{3} / \sqrt{2}) V_{cdc} \tag{2}$$

It can also be expressed as a pure unit value as shown in equation (3):

$$\bar{V}_c = k_v V_{cdc} \tag{3}$$

If the standard unit of DC and AC is set to 1p.u., the voltage relationship factor,  $k_v$ , is defined by the maximum value allowed by the AC bus.  $k_v$  can be modified based on the lower bounds of the two constraints if the maximum AC bus voltage generated by the VSC does not coincide with the maximum allowable voltage of AC bus for isolated purposes. The VSC AC-side bus voltage is defined by equation (4).

$$V_c \leq \bar{V}_c \tag{4}$$

- (2) VSC capacity limits

The normal operation of the VSC is mainly limited by the maximum current and maximum DC voltage flowing through the VSC valve. The former determines the maximum VSC apparent power limit, the latter determines the output limits of the VSC reactive power. Normally, VSC constraints are applied to the PCC bus [14]. In this OPF model, the VSC is limited to the AC side bus of the converter because the power exchange of the VSC on the AC side of the converter is set as the control variable.

a) The maximum constraint of apparent power

For a given maximum valve current, the apparent power of the VSC can be defined as:

$$|S_c| \leq |V_c \bar{I}_v| \tag{5}$$

b) The maximum constraint of reactive power

The maximum reactive power limit of the VSC is defined by equation (6), assuming that the conductance of the reactor is much less than its susceptance.

$$\bar{Q}_c = -B_r \left( \bar{V}_c^2 - \bar{V}_c \bar{V}_f e^{j(\delta_c - \delta_f)} \right) \tag{6}$$

It can be simplified as shown in equation (7).

$$\bar{Q}_c = -B_r \left( \bar{V}_c^2 - \bar{V}_c \bar{V}_f \right) \tag{7}$$

c) Reactive power minimum constraint

$$Q_c = -k_Q S_{nom} \tag{8}$$

The constraint factor  $K_Q$  is determined by the actual project.

(3) VSC power loss

The active power loss of the VSC can be expressed as a quadratic function of the VSC valve phase current [15], as shown in equation (9), where the phase current of the VSC valve can be calculated using equation (10).

$$P_{vscloss} = \sum_{i=1}^M \left( A_i + B_i I_{vi} + C_i I_{vi}^2 \right) \tag{9}$$

$$\left( I_{vi} V_{ci} \right)^2 = P_{ci}^2 + Q_{ci}^2 \forall i \in M \tag{10}$$

## 4.2 OPF model of AC / DC system with VSC-HVDC

For AC / DC hybrid system, AC / DC system with VSC-HVDC consists of three parts, namely, AC network, converter station and DC network. In practical applications, VSC-HVDC control objectives are generally the following variables: AC system port voltage, DC network voltage, the converter station injection active and reactive power. For each VSC, select two of the above four control variables. The four frequently-used combinations are:

(1)  $P_s$  and  $Q_s$  constant. (2)  $Q_s$  and  $u_d$  constant. (3)  $P_s$  and  $u_s$  constant. (4)  $u_s$  and  $u_d$  constant.

For common two-port VSC-HVDC, the common control compound modes are (2) (1), (1) (4), (2) (3) and (3) (4). For multi-port VSC-HVDC, the number of control combinations is more.

### 4.2.1 Objective function of OPF model

$$\min \sum_{i \in S_c} \left( a_{2i} P_{Gi}^2 + a_{1i} P_{Gi} + a_{0i} \right) \tag{11}$$

### 4.2.2 The basic constraints of OPF model

(1) Equality constraint is the power equation of each node:

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^n V_j \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) = 0 \tag{12}$$

$$Q_{Gi} - Q_{Di} + V_i \sum_{j=1}^n V_j \left( G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij} \right) = 0 \tag{13}$$

(2) Inequality constraints:

Reactive constraints:

$$\begin{cases} \underline{P}_{Gi} \leq P_{Gi} \leq \overline{P}_{Gi} \forall i \in NG \\ \underline{Q}_{Gi} \leq Q_{Gi} \leq \overline{Q}_{Gi} \forall i \in NG \end{cases} \quad (14)$$

AC bus voltage constraint:

$$\underline{V}_i \leq V_i \leq \overline{V}_i \forall i \in N \quad (15)$$

AC power line capacity constraints:

$$\underline{S}_{ij} \leq S_{ij} \leq \overline{S}_{ij} \quad (16)$$

### 4.2.3 VSC-MTDC system constraints

In this paper, the balanced bipolar VSC configuration was used as the standard operating mode. In this mode of operation, the DC load current equation can be expressed as:

$$P_{i_{dc}} = 2V_{i_{dc}} \sum_{j_{dc}=1}^M G_{i_{dc}j_{dc}} V_{j_{dc}} \forall i_{dc} \in M \quad (17)$$

DC bus voltage constraint:

$$\underline{V}_{i_{dc}} \leq V_{i_{dc}} \leq \overline{V}_{i_{dc}} \forall i_{dc} \in M \quad (18)$$

DC transmission line flow constraint:

$$\underline{P}_{i_{dc}j_{dc}} \leq P_{i_{dc}j_{dc}} \leq \overline{P}_{i_{dc}j_{dc}} \forall i_{dc} \in M, \forall j_{dc} \in M \quad (19)$$

### 4.3 Power Flow Calculation of Flow Optimization

For AC / DC hybrid system calculation, it is generally based on the traditional AC power flow calculation for some correction. In the AC / DC hybrid system, the nodes are divided into the AC and DC nodes according to whether they are connected with the converter, and the two are processed separately to realize the unified calculation.

## 5. Case Study

In this paper, we use IEEE 14 nodes system for simulation, the detailed data used are as follows:

Table 1 Improved IEEE 14 nodes data

node	type	Active load	Reactive load	Initial voltage	Initial phase angle	Max voltage	Min voltage
1	3	0	0	1.06	0	1.06	0.94
2	2	21.7	12.7	1.045	-4.9	1.06	0.94
3	2	94.2	19	1.01	-12.7	1.06	0.94
4	1	47.8	-3.9	1.019	-10.3	1.06	0.94
5	1	7.6	1.6	1.02	-8.7	1.06	0.94
6	2	11.2	7.5	1.07	-14.2	1.06	0.94
7	1	0	0	1.062	-13.3	1.06	0.94
8	2	0	0	1.09	-13.3	1.06	0.94
9	1	29.5	16.6	1.056	-14.9	1.06	0.94
10	1	9	5.8	1.051	-15.1	1.06	0.94
11	1	3.5	1.8	1.057	-14.8	1.06	0.94
12	1	6.1	1.6	1.055	-15.1	1.06	0.94
13	1	13.5	5.8	1.05	-15.2	1.06	0.94
14	1	14.9	5	1.036	-16.1	1.06	0.94

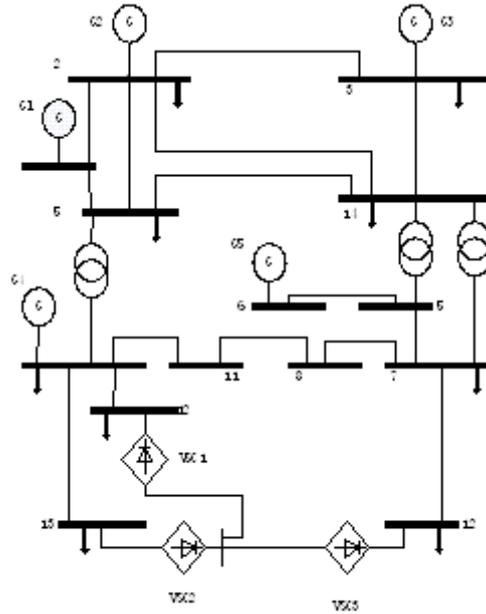


Fig. 2 Modified IEEE 14 nodes AC / DC hybrid system

Set the voltage range between [0.94, 1.06]. The range of the on-load transformer is set between [0.9, 1.1] and the adjustment step of the on-load transformer is set to 0.012. Node 9 is set with reactive power compensation device whose capacity is 0.5, the reference capacity is set as 100MVA, adjust step is set as 0.01. VSC-HVDC modulation degree of DC system is [0, 0.1], active transmission capacity is [-2.0, 2.0], reactive power transmission capacity is [-1.0, 1.0], DC voltage between [1.8, 2.2]. Assume that the control schemes of the converter stations VSC1, VSC2 and VSC3 are (1), (3), (2). The maximum number of iterations of the iterative algorithm is set to 10, and the population size of the NSGA-II part in each iteration is 40, and the maximum number of iterations is 60.

The modified IEEE 14 nodes AC-DC hybrid system optimization results are as follows, compared with the interior point method. Among them, the optimization results of the generator, transformer, reactive power compensation device and DC voltage are shown in the following table.

In the table below, IPM represents the interior point method. AIM represents the alternating iterative method.

Table 2 Generator optimization data

algorithm	$P_{G1} / pu$	$P_{G2} / pu$	$P_{G3} / pu$	$P_{G4} / pu$	$P_{G5} / pu$
IPM	1.682	0.360	0.150	0.329	0.143
AIM	1.680	0.359	0.150	0.329	0.142

Table 3 Transformer optimization data

algorithm	$K_1 / pu$	$K_2 / pu$	$K_3 / pu$
IPM	0.978	0.969	0.933
AIM	0.962	1.100	1.038

Table 4 Reactive power and DC voltage optimization data

algorithm	Reactive power compensation		DC voltage	
	$Q_R / MVar$	$U_{d1} / pu$	$U_{d2} / pu$	$U_{d3} / pu$
IPM	0.190	2.202	2.199	2.198
AIM	0.001	2.200	2.199	2.197

Table 5 DC optimization data I

algorithm	DC current			Control angle		
	$I_{d1} / pu$	$I_{d2} / pu$	$I_{d3} / pu$	$\delta_1 / (^\circ)$	$\delta_2 / (^\circ)$	$\delta_3 / (^\circ)$
IPM	0.021	0.031	-0.052	0.369	0.531	-0.879
AIM	0.020	0.029	-0.049	0.359	0.522	0.849

Table 6 DC optimization data II

algorithm	Modulation degree			DC active power		
	$M_1$	$M_2$	$M_3$	$P_{s1} / pu$	$P_{s2} / pu$	$P_{s3} / pu$
IPM	0.780	0.784	0.778	0.047	0.067	-0.113
AIM	0.776	0.780	0.776	0.045	0.063	-0.110

Table 7 DC optimization data III

algorithm	DC reactive power			Power generation cost	calculation time
	$Q_{s1} / pu$	$Q_{s2} / pu$	$Q_{s3} / pu$	$f_{cost} / \$$	$t / s$
IPM	-0.008	-0.043	-0.062	844.503	0.350
AIM	-0.017	-0.058	-0.058	824.997	49.29

It can be seen from the above optimization results that although the calculation time of alternating iteration method is larger than that of interior point method, the optimization effect is better than that of pure interior point method. The reason for the more calculation time of iterative method is that it is necessary to repeat iterations. In each iteration, it is necessary to perform the iteration calculation of NSGA-II. In this regard, some improvements are needed for the alternating iterative method.

## 6. Conclusion

For the AC / DC hybrid with VSC-HVDC power flow optimization, that is, facing a multi-objective multi-variable nonlinear programming problem. There are many algorithms for solving nonlinear programming. After looking up the advanced algorithm at home and abroad, this paper chooses the improved genetic algorithm NSGA-II and the original-dual interior point method to solve the AC / DC hybrid system with VSC-HVDC. Since the system contains discrete variables and continuous variables, this alternating iterative algorithm can be used to solve this nonlinear multivariate optimization problem. NSGA-II improved the defect of NSGA, making the calculation time reduced. On the issue of multi-variable and multi-constraint, the NSGA-II performs better with the increase of dimension.

It can be seen from the case study, although the optimization result of the alternating iterative algorithm is better than the pure interior point method, it is still inferior in speed. For the optimization problem of low dimension, the alternating iterative method is not as good as the pure interior point method. This is where this research needs to be further improved. The number of iterations can be changed by determining the dimension. Using the improved method of NSGA-II mentioned in [8] is also helpful. These are the directions that can be made in the next step of this study.

## References

- [1] Y.G. Hao, G.Y. Liu, E.J. Yu. A New OPF Algorithm Based on Karmarkar' Interior Point Method, Proceedings of the CSEE, vol. 16 (1998), p.409-412.
- [2] L.S. Yang, X. Yang, J.C. Gu. The application of primal-dual IPM and predictor-corrector IPM on optimal power flow, Journal of Electric Power University, vol. 39 (2012), p.29-34.
- [3] H.S. Zhao, B. Chen. Interior-Exterior Point Method for Optimal Power Flow, Transactions of China Electrotechnical Society, vol. 25 (2010), p.129-134.
- [4] X.M. Chang. Research on Optimization of Coupling Pair Power Flow Based on Interior Point Method. Harbin Institute of Technology (2007).
- [5] Y.G. Qi. Primal-dual Interior Point Method Based Optimal Power Flow for Reliability Evaluation. South China University of Technology (2011).
- [6] Y. Wu. Improvement of power flow algorithm in reactive power optimal genetic algorithm, East China Electric Power, vol. 5 (2003), p.10-12.
- [7] Z.N. Wei, C. Ji, G.Q. Sun, C. Wang, W.Z. Sun. Interior-point Optimal Power Flow of AC-DC System With VSC-HVDC. Proceedings of the CSEE, vol. 32 (2012), p.89-95.
- [8] L.Y. Wang. Study on reactive power optimization based on the improved genetic algorithm. Chongqing University (2007).
- [9] J.M. Hao, Z.S. Ji, Y.S. Dai, Y. Lin. Genetic Algorithms Based on Fuzzy Evaluation for Multicriterion function Optimization, SYSTEMS ENGINEERING- THEORY METHODOLOGY APPLICATIONS, vol. 8 (1999), p.39-43.
- [10] Y. Gao. Non-dominated Sorting Genetic Algorithm and Its Applications. Zhejiang University (2006).
- [11] C.W. Xie, K. Li, G.Y. Liao. Improved NSGA2 Algorithm with Differential Evolution Local Search, Computer Science, vol. 40 (2013), p. 235-238.
- [12] Z.H. Li, X.Z. Duan. Comparison and Analysis of Multiobjective Evolutionary Algorithm for Reactive Power Optimization, Proceedings of the CSEE, vol. 30 (2010), p. 57-65.
- [13] Z.N. Wei, W.W. Hu, G.Q. Sun, et al. Transient Stability Constrained Optimal Power Flow Considering of VSC-HVDC Integration, Proceedings of the CSEE, vol.33 (2013), p. 50-58.
- [14] Pizano-Martinez, Fuerte-Esquivel, C.R, Ambriz-Perez, H. Modeling of VSC-Based HVDC Systems for a Newton-Raphson OPF Algorithm, Power Systems, vol. 22 (2007), p. 1794-1803.
- [15] Z.N. Wei, C. Ji, Y.P. Zheng, et al. Optimal Power Flow of AC-DC Systems With VSC-HVDC Based on a Novel Unified Hybrid Algorithm, Proceedings of the CSEE, vol.34 (2014), p. 635-643.