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# An Analysis on the Separability of the Energy Features Based on Wavelet Packet in Low Altitude Acoustic Target

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## Abstract

The object of target recognition is to extract and recognize the information about the target identify. To distinguish the acoustic signal features of low-altitude flying targets accurately, this paper proposes a method to measure feature separability based on the standard Euclidean distance. Firstly, the signal is decomposed by 3-layer wavelet packet. Then the features are extracted by calculating the rate of band energy to total signal energy and the separability of different targets feature is calculated by the proposed method. Finally, recognize the targets by SVM classifier on the features whose features separability value are high between wavelet and wavelet packet. The recognition result shows that the separability of the wavelet packet features is higher than that of the wavelet. So as the classification rate. So the proposed method can measure the feature separability effectively.

## Keywords

Low-altitude acoustic target; Feature extraction, Wavelet packet; Separability.

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## 1. Introduction

Acoustic signals emitted from the low altitude target can be used to identify targets effectively, and the target sound can't be "hidden" [1], it overcomes the poor detection performance of Radar in low altitude. It is a hot topic to study how to extract the features effectively from the target signal. Target sound contains numerous discriminative features that can be used to classify target. Different approaches and various kind of acoustic features were proposed with varying success rates. The features can be extracted either directly from the time domain signal or from a transformation domain depending upon the choice of the signal analysis approach, such as short-time Fourier transform [2], Wigner-Vill distribution [3] and wavelet transform [4]. The above methods are studied on the basis of Fourier transform. By using Fourier transform we might be able to determine all the frequencies present in a signal but we do not know when they are present. It causes Gibbs effect and aliasing. Another problem associated with Fourier transform is that it can't deal with non-stationary signal. While the acoustic signal is non-stationary mostly. In recent years, due to the good time-frequency characteristics and the ability to deal with non-linear and non-stationary signal, the wavelet packet (WP) analysis raises more and more attention. Wavelet packet analysis decomposes signal into different bands of the original signal in detailed distribution. And Mallat fast algorithm is developed to speed up the operation [5]. Although the wavelet packet analysis can effectively extract the energy features, but the separability of the extracted feature is still unknown. We cannot distinguish the difference of different classes' targets. For the extracted feature, features can be defined as a minimal unit, which distinguishes maximally close classes. It is important that good features must have wide variation from class to class, should be insensitive to the irrelevant variations and should have low correlation with other features. The quality of a good feature is that it gives maximum information about the class

within much smaller dimension. Further, these features are important in deciding the overall recognition performance of the target recognition system [6]. At present, there is no specific quantification criterion for the feature separability [7]. In [8], a class separability criterion based on class distance is proposed, but it only defines the criteria of judgment between different classes. A separability criterion based on class distance is proposed in [9], which overcomes the problem that the separability criterion based on class distance. It is insufficient and better to define the separability. The separability between features measured by the standard Euclidean distance is proposed in this paper. It overcomes the distance calculation error caused by different features statistics distribution via standardizing them. At last, the experimental results show the separability of different target calculated by standard Euclidean distance is in significant difference. The validity of the wavelet packet energy feature is verified by the simulation of the field acquisition signal, and it is proved that the evaluation of the extracted features have reached the classification requirement by implementing wavelet packet. It is helpful to improve the real time of classification.

## 2. Wavelet packet analysis

### 2.1 Definition of wavelet packet analysis[10]

The subspace, which is projected into a set of orthogonal wavelet basis functions, is the basis of multiresolution analysis. Different frequencies characteristics can be obtained by the expanses of the signal at different scales. Although multi-resolution analysis is an effective time-frequency analysis method, but it is only the low-frequency part of the signal analysis, high-frequency part of the fixed. And its poor in high frequency resolution. The wavelet packet decomposition can decompose the high frequency part and the low frequency part of the signal at the same time, and adaptively determine the resolution of the signal at different frequencies [11]. Because there existing the double scale relation between the scale function  $\varphi(x)$  and the wavelet function  $\psi(x)$ :

$$\begin{cases} \mu_0 = \sum_{k \in \mathbb{Z}} h(k) \mu_0(2x - k) \\ \mu_1 = \sum_{k \in \mathbb{Z}} g(k) \mu_0(2x - k) \end{cases} \quad (1)$$

Where  $\mu_0 = \varphi(x)$ ,  $\mu_1 = \psi(x)$ . Then

$$\begin{cases} \mu_{2l} = \sum_{k \in \mathbb{Z}} h(k) \mu_l(2x - k) \\ \mu_{2l+1} = \sum_{k \in \mathbb{Z}} g(k) \mu_l(2x - k) \end{cases} \quad (2)$$

Definition  $\mu_n(x)$  is the wavelet packet of the orthogonal function  $\mu_0(x) = \varphi(x)$ , where  $n = 2l$  or  $n = 2l + 1$ ,  $l = 0, 1, \dots, L$ . According the multi-resolution analysis, we get the function.

$$\begin{cases} c_{2l}(n) = \sqrt{2} \sum_{k \in \mathbb{Z}} h(k) c_l(2n - k) \\ c_{2l+1}(n) = \sqrt{2} \sum_{k \in \mathbb{Z}} g(k) c_l(2n - k) \end{cases} \quad (3)$$

The above equation is the wavelet packet decomposition of the signal. The result of the wavelet packet decomposition is two sequences, and the two sequences are further decomposed, that is to say. The decomposition process is not only about the low frequency part, but also the high frequency part of the signal. So that it improved the resolution of the high frequency part of the signal.

### 2.2 Wavelet Packet Decomposition

Wavelet packet decomposition, also known as sub-band tree, not only decomposes the low-frequency part, but also decomposes the high-frequency part. The wavelet packet decomposition is shown in Fig.1. Each layer decomposition through a high-pass filter and a low-pass filter, as shown in Fig.2.

The signal is decomposed into high frequency and low frequency, the decomposed signal goes through the above processing.  $2^n$  nodes are got by n-layer wavelet packet decomposition finally.

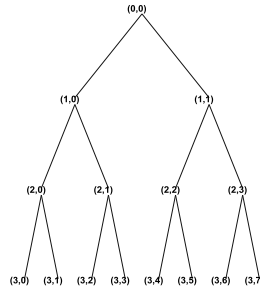


Fig.1 Diagram of 3-layer wavelet packet decomposition

In the figure,  $(i, j)$  represents the  $j$ -th node of the  $i$ -th layer, the decomposition coefficient is denoted by  $S_{ij}$ , and the characteristic signal is represented by  $S_{ij}$ , and the original acoustic signal  $S_0$  is represented by  $(0,0)$ .

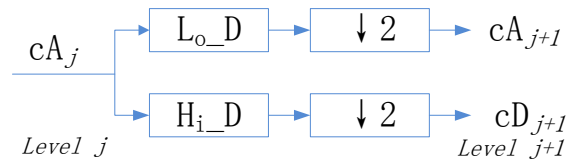


Fig.2 Decomposition Step of Wavelet Packet

Initialization  $cA_0=S_0$ , where  $L_o\_D$  and  $H_i\_D$  denote convolve with  $L_o\_D$  or  $H_i\_D$ .  $\downarrow 2$  denotes downsample. The original signal  $S_0$  passes through the high-pass filter and the low-pass filter, respectively, and then down-sampling to obtain the high-frequency detail factor  $X_{11}(cD_1)$  at the node  $(1,1)$  and the low-frequency approximation coefficients  $X_{10}(cA_1)$  at the node  $(1,0)$ . Repeat the above steps to get all the decomposition coefficient. The reconstructed signal  $X_{30}$  is represented by  $S_{30}$ ,  $X_{31}$  is represented by  $S_{31}$ , and so on. Finally  $S_0 = S_{30} + S_{31} + S_{32} + S_{33} + S_{34} + S_{35} + S_{36} + S_{37}$ .

### 3. Features extraction based on wavelet packet

#### 3.1 The separability of extracted features

Feature extraction is the key step of target recognition. The main purpose is to extract the features vectors which can distinguish the various types of targets. The extracted feature minimizes the probability of identifying errors and is a basic criterion for determining the feature separability. However, in practice, even if the conditional probability density of the known class is known, the calculation of the recognition error probability is also very complicated. Moreover, in most cases, the probability distribution is unknown. Therefore, this paper presents a simple and effective separability criteria based on the standard Euclidean distance. Euclidean distance refers to the distance between two points in the space of n-dimensional Euclidean space, it is also called  $L_2$  distance. Target categories are  $\omega_1, \omega_2, \dots, \omega_m$ , a total of  $m$  class, the corresponding features vectors are  $\mathbf{T}_i (i = 1, 2, \dots, m)$ . Then the Euclidean distance within classes is calculated as follows:

$$d_E(\omega_i, \omega_j) = \sqrt{(\mathbf{T}_i - \mathbf{T}_j)(\mathbf{T}_i - \mathbf{T}_j)'} \tag{4}$$

Due to inconsistencies in the statistical distribution feature vector of each type of target, it may occur error in calculating the Euclidean distance. Therefore the each feature vector are "normalized" to zero mean and unit variance. Assume that the mean feature vector is  $\mu$ , the variance is  $\sigma$ . The standardization process is as follows:

$$\mathbf{T}^* = \frac{(\mathbf{T} - \mu)}{\sigma} \tag{5}$$

So the standard Euclidean distance is as follows:

$$d_{SE}(\omega_i, \omega_j) = \sqrt{(\mathbf{T}_i^* - \mathbf{T}_j^*)(\mathbf{T}_i^* - \mathbf{T}_j^*)'} \quad (6)$$

The standardized Euclidean distance is also known as the weighted Euclidean distance. The feature separability based on the standard Euclidean distance is proposed. The greater the separability is, the greater the difference between the different features is.

$$S(\omega_i, \omega_j) = \begin{cases} \frac{d_{SE}(\omega_i, \omega_j)}{d_{SE}(\omega_i, \omega_i) + d_{SE}(\omega_j, \omega_j)}, & \omega_i \neq \omega_j \\ d_{SE}(\omega_i, \omega_j), & \omega_i = \omega_j \end{cases} \quad (7)$$

### 3.2 Wavelet packet band-energy feature extraction

Wavelet packet analysis is a high-precision analysis of the signal method, the signal is decomposed into  $2^n$  frequency band, and the features can be extracted from calculating the rate of each band energy to the total energy of the signal effectively. The energy is expressed in different frequency bands.  $E_{ij}$  represents the energy of the  $j$ -th band of the  $i$ -th layer decomposition.

$$E_{ij} = \int |S_{ij}(t)|^2 dt = \sum_{k=1}^{N_j} |x_{jk}|^2 \quad (8)$$

Where  $x_{jk}$  ( $k = 0, 1, \dots, N_j$ ),  $i$  is the number of decomposed layers and  $S_{ij}$  is the discrete point amplitude of the reconstructed signal. When the energy is large,  $E_{ij}$  is usually a large value and not conducive to the calculation. So  $E_{ij}$  is normalized:

$$E_{ij}^* = E_{ij} / \sqrt{\sum_{j=0}^{2^i} |E_{ij}|^2} \quad (9)$$

Then the band-energy features vector is expressed as:

$$\mathbf{T} = [E_{i0}^*, E_{i1}^*, \dots, E_{i(2^i-1)}^*] \quad (10)$$

## 4. Simulation and results analysis

The acoustic signals used in the study include three categories, which are decomposed by three layers of wavelet packets to extract the features vectors. In the experiment, db1, which is Harr wavelet too, is used in the Daubenchies wavelet function family. The acoustic signal used is a field-acquired signal with a sampling rate of 44.100 KHz and a quantization depth of 16 bits, in the format of wav, stereo, and signal length  $T = 10240$ . The simulations are performed using MATLAB R2016a a, running on an Inter (R) Core (TM) i5-7200, 2.50 GHz processor with 8 GB of memory, under Windows 10 OS.

Figure 3 (a) (c) (e) is the time-domain waveform of target, (b) (d) (f) is the corresponding spectrum. The frequency is mainly below 300 Hz, and the energy is mainly concentrated near 100 Hz of A from (b). The frequency of target B is shown in Fig. 3 (d), which is mainly distributed below 600 Hz and exhibits a distinct characteristic of the discrete spectrum in the low-frequency part. The main difference is that the spectral shape and the spectral distribution from A. The frequency of the target C is mainly within 500 Hz, and there is a strong harmonic component at about 100 Hz. Compared with the spectrum of the three types of signals, it can be found that the frequency domain is obviously discrete, but the energy distribute is different, which provide a good foundation to extract good performance features. Wavelet packet decomposition of the three categories of targets acoustic signal, as can be seen from Fig. 4. Class A target energy is mainly concentrated in the 0 ~ 0.25 unit frequency, and Fig.3 shows that the energy of A is mainly concentrated within 300Hz consistent. The decomposition results of class B and class C are also consistent with the results of spectrum analysis.

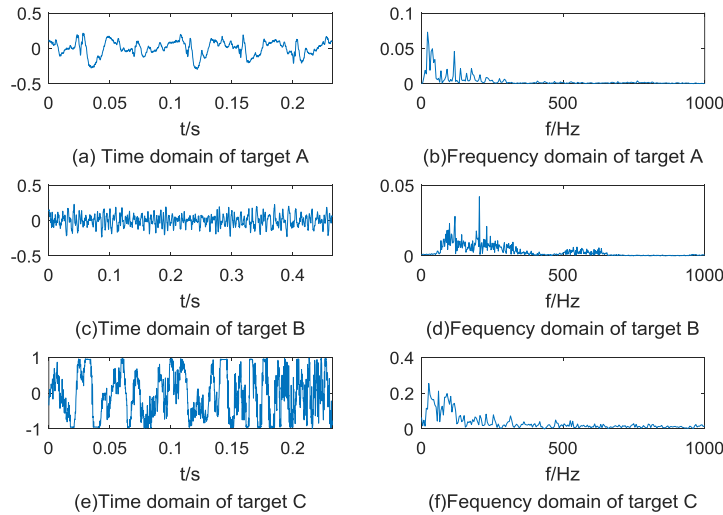


Fig.3 Time domain waveform and spectrum of target

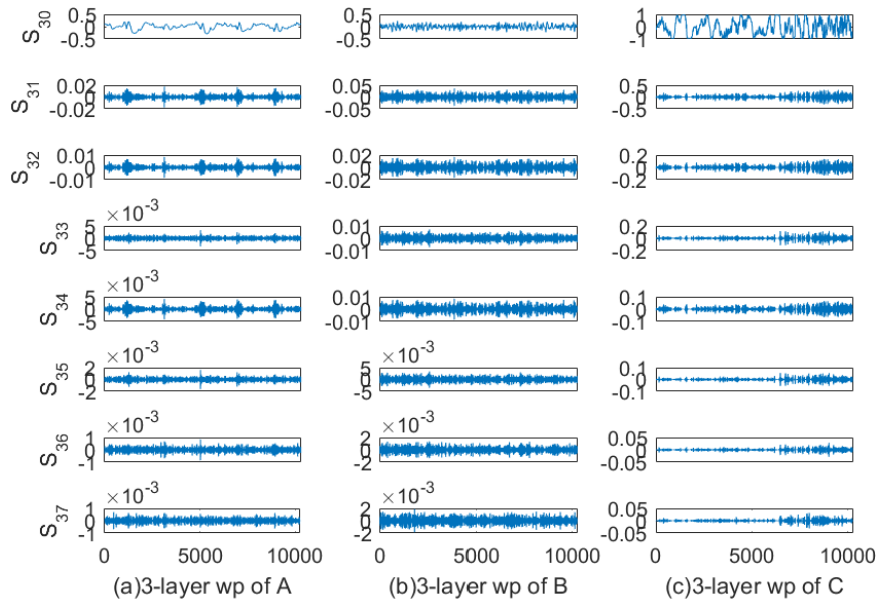


Fig.4 Wavelet decomposition results of target samples

The 3-layer wavelet packet decomposition is implemented in acoustic signal of 3 targets at the same time. The 8-dimensional vector obtained by extracting the rate of band-energy to total signal energy after wavelet decomposition is shown in Fig. 5. The energy mainly concentrated in the low frequency, but the energy distribution difference is obvious. Compared with the three types of low-altitude flying targets, the distribution of the characteristics of the wavelet packet has obvious difference, which indicates that the extracted feature contains the spectral distribution characteristics of the signal.

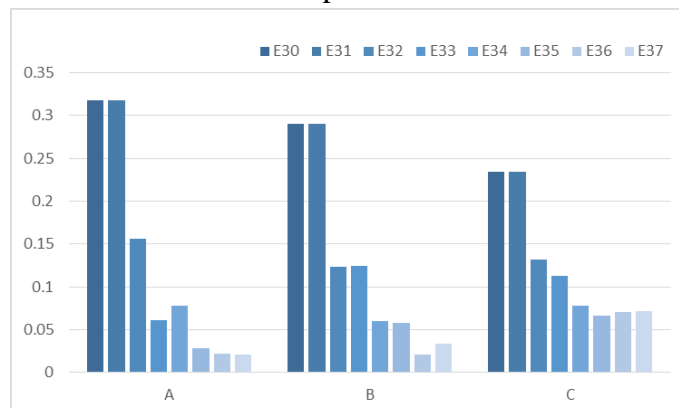


Fig.5 Feature sequence of 3 target with 3-layer wavelet packet decomposition

In order to verify the separability of the wavelet packet energy feature of the low-altitude flying target, this paper calculates the standard Euclidean distance between the targets firstly, and randomly compares the samples from each of the 900 samples for 3 samples as Table 1 shows.

Table 1. Comparison of different classes features distance

$d_{SE}(\omega_i, \omega_j)$	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>
A <sub>1</sub>	0								
A <sub>2</sub>	0.103	0							
A <sub>3</sub>	0.299	0.218	0						
B <sub>1</sub>	4.285	4.296	4.425	0					
B <sub>2</sub>	3.914	3.923	4.054	0.401	0				
B <sub>3</sub>	4.203	4.214	4.344	0.088	0.315	0			
C <sub>1</sub>	5.132	5.187	5.375	3.741	3.668	3.721	0		
C <sub>2</sub>	5.323	5.376	5.554	4.405	4.303	4.378	0.986	0	
C <sub>3</sub>	5.677	5.730	5.917	3.945	3.921	3.935	0.588	1.147	0

From the distance between the features of the above table, it can be clearly seen that the distance between the same classes is significantly smaller than the distance between the different classes. In order to accurately quantify the separability between the features, we use the Eq. (7) to calculate the separability between the target feature vectors. The results are shown in Table 2 As shown.

Table 2. The separability analysis

$S(\omega_i, \omega_j)$	Wavelet packet features			Wavelet features		
	A	B	C	A	B	C
A	0.206			0.415		
B	8.827	0.268		6.254	0.489	
C	4.918	3.405	0.907	3.587	2.989	1.397

The larger  $S(\omega_i, \omega_j)$  is, the better separability between  $\omega_i$  and  $\omega_j$  is. The experimental data in Table 2 show that the energy features, extracted from the low altitude flying target decomposed by wavelet packet, whose  $S(\omega_i, \omega_j)$  is small within classes, and  $S(\omega_i, \omega_j)$  is obviously different between classes. Therefore, the three categories of targets can be classified under this method, and the average separability is 5.716. Compared with the wavelet feature, the wavelet packet feature separability is obviously improved. To further verify the proposed separability method based on the standard Euclidean distance effectiveness. A total of 100 samples of three types of low-altitude flying targets were taken as training experimental data. 100 samples were taken as test data. SVM classifiers were used to obtain the recognition results. The result is given in Table 3.

Table 3. Recognition rate result

target features	A	B	C	average
wavelet packet	92%	94%	93%	93%
wavelet	88%	87%	89%	88%

It can be seen from the table that the recognition rate of the wavelet packet feature with high separability is high, and the three kinds of targets can be correctly identified. So the energy features of the wavelet packet can reflect the characteristics of the target. And it can identify target easily with prosed extracted feature. And this method can accurately determine the separability between classes.

### 5. Conclusion

In this paper, the wavelet packet analysis of the actual low altitude acoustic target is carried out, and the feature vector based on the wavelet packet is extracted. The separability between the features is determined by the separability measure based on the standard Euclidean distance. By SVM

classification experiment, the recognition rate of the wavelet packet feature is higher than that of the traditional wavelet feature. Compared with wavelet, wavelet packet analysis can obtain more local information, more suitable for low-altitude target feature extraction. The separability based on the standard Euclidean distance is simple and effective. The recognition burden of the classifier is reduced and the real time of low altitude battlefield classification is improved by measure the separability in advance with the proposed method.

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