Multiple attribute decision making of interval probabilistic hesitant fuzzy information

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Abstract

Hesitant fuzzy set (HFS), which can be accurately and perfectly described in terms of the opinions of decision makers, is considered as a powerful tool to express uncertain information in the process of multi-attribute decision making (MADM) problems. The core of the HFS is the hesitant fuzzy element (HFE).This paper developed a method, which using the interval probabilistic hesitant fuzzy element (IP-HFE) express the decision information, to deal with multi-criteria decision making (MCDM) problems. First, a concept of interval probabilistic hesitant fuzzy element (IP-HFE) is defined and its normalized method and score and the deviation degree formula are developed. Then, a method based on IP-HFE is developed to handle MCDM problems. Finally, an application study is conducted to indicate the feasibility, reasonable and applicability of the proposed methodology.

Keywords

multi-criteria decision making; interval probabilistic hesitant fuzzy element; score; deviation degree.

1. Introduction

In the decision-making process, because of the complexity and uncertainty of objective things and the fuzziness of human thinking, it is hard for experts to express preference information with precise numbers. In order to deal with this problem, Zadeh [3] first introduced the theory of fuzzy sets as an effective approach to deal with such vagueness and ambiguity in the decision making process. Then, many scholars have studied some extended forms of fuzzy set, such as intuitionistic fuzzy set (IFS) [4,5], type-2 fuzzy set[6], type-n fuzzy set[6], fuzzy multiset[7], and hesitant fuzzy set(HFS) [8]. In real-life decision situation in which the decision makers (DMs) may hesitate among several values to asses an indicator, alternative, variable, etc., the HFS [8] is widely applied. The HFS can be able to express the hesitancy of human beings efficiently, especially when two or more sources of vagueness appear simultaneously.

The core of the HFS is the hesitant fuzzy element (HFE) [9]. The MCDM with HFEs is called the hesitant fuzzy MCDM. Now, there are many achievements about hesitant fuzzy MCDM. Xu and Zhang [10] developed a novel approach based on TOPSIS and the maximizing deviation method for solving MADM problems, in which the evaluation information provided by the decision maker is expressed in hesitant fuzzy elements. Zhang etc. [11] and Liao etc. [12] extended the VIKOR method for solving the MCDM problem with hesitant fuzzy set information. Wang etc. [13] proposed an outranking approach for multi-criteria decision-making problems with hesitant fuzzy sets. Zhang etc. [14] proposed an interval programming approach that directly deals with MAGDM problems with hesitant fuzzy elements. Based on different types of aggregation operators, several decision methods [9,15,16,17] have been developed. What's more, the method, which based on the hesitant preference relations provided by the DMs through pair-wise comparisons of alternatives, has been certain research. Zhu etc. [18] explored the ranking methods with hesitant fuzzy preference relations in the group decision making environments. Zhang etc. [19] proposed the concept of hesitant multiplicative preference relations and developed a consistency and consensus-based decision support model for group decision making with hesitant multiplicative preference relations. However, those methods are based on a hypothesis, which the support degree of elements of HFEs is same. For example, for a HFE {0.3,0.5,0.6}, 0.3,0.5 and 0.6 have the same support degree. But, in practice, decision makers have different support for different evaluation information. Therefore, Zhu [1] proposed the concept of probabilistic hesitant fuzzy set (P-HFS). Zhang [20] studied some operations of P-HFSs and gave their application to multi criteria decision making. Zhang etc. [21] improve and further perfect the P-HFS. In this paper, inspired by those academic achievements, we define a concept of interval probabilistic hesitant fuzzy element (IP-HFE), propose a method to normalize the IP-HFEs and develop the score and the deviation degree of a IP-HFE. And develop a method for handling the MCDM problems under interval probabilistic hesitant fuzzy environment.

The rest of this paper is organized as follows. In Section 2, we define a concept of interval probabilistic hesitant fuzzy element (IP-HFE) at first. Then, the method of normalizing the IP-HFEs is proposed. Finally, the score and the deviation degree of a IP-HFE is developed. In Section 3, a method, which using the IP-HFE express the decision information, is proposed to handle the MCDM problems. In Section 4, an application study of the proposed method on selecting an appropriate resource planning (ERP) system is showed. In Section 6, the conclusions of the study are presented.

2. Interval Probabilistic Hesitant Fuzzy Elements

2.1 The Concept of Interval Probabilistic Hesitant Fuzzy Elements

Definition 2.1 [1]. Let X be a fixed set, a P-HFS on X is in terms of a function that when applied to X returns to a subset of [0,1], which is expressed as $H_n=\{\langle x,h_x(p_{nx})\,|\,x\in X\rangle\}$, where both h_x and p_{nx} are two sets of some values in [0,1]. h_x denotes the possible membership degrees of the element $x\in X$ to the set E ; p_{nx} is a set of probabilities associated with h_x . $h_x(p_{nx})$ is called the probabilistic hesitant fuzzy element(P-HFE).

For convenience, let the P-HFE $h_x(p_{nx})$ as $h(p_n)$, and a generic P-HFE is expressed as $h(p_n)=\{\gamma_l(p_{nl})\,|\,l=1,2,\cdots,|h(p_n)|\}$, where p_{nl} is the probability of the membership degree γ_l , $\gamma_l(p_{nl})$ is called a term of the P-HFE, $|h(p_n)|$ is the number of all different membership degrees, and $\sum_{l=1}^{|h(p_n)|}p_{nl}=1$.

Obviously, the P-HFE can retain much more evaluation information from the DMs than the HFE, and using P-HFE instead of HFE to express the DMs' preference is more reliable and reasonable. However, the P-HFE $\gamma_l(p_{nl})$ only use a certain number of value in [0,1] to express the probability of the membership degree γ_l . Although this method fully expresses the fuzzy degree of experts in the decision-making process, it ignores the degree of hesitation of expert evaluation information. Therefore, we define a novel concept, named interval probabilistic hesitant fuzzy element, to express the DMs' preference information.

Definition 2.2. Let X be a fixed set, a interval probabilistic hesitant fuzzy element (IP-HFS) on X is defined as

$$H=\{\langle x,h_x(p_x)\,|\,x\in X\rangle\}$$

Where both h_x is a set of some values in [0, 1], p_x is a set of some interval values in [0,1]. h_x Denotes the possible membership degrees of the element $x\in X$ to the set E ; p_x is a set of probabilities associated with h_x . $h_x(p_x)$ is called the interval probabilistic hesitant fuzzy element(IP-HFE).

For convenience, let the IP-HFE $h_x(p_x)$ as $h(p)$, and a generic IP-HFE is expressed as $h(p)=\{\gamma_l(p_l)\,|\,l=1,2,\cdots,|h(p)|\}$, where p_l is the probability of the membership degree γ_l and $p_l=\left[p_l^l,p_l^u\right]$, $\gamma_l(p_l)$ is called a term of the IP-HFE, $|h(p)|$ is the number of all different membership degrees.

2.2 The Normalization of a Set Of IP-HFE

In order to effectively deal with information, we always hope that a set of general IP-HFEs satisfies the following two properties: (1) all IP-HFEs in it have complete probabilistic information; (2) all IP-HFEs in it have the same length. If two properties hold, the calculation of the IP-HFEs in it and the distance of every two IP-HFEs will be easier, and the result of decision making will be more accurate. However, the probabilistic information of a set of general IP-HFEs may be not complete or has redundant data. So, we propose following method to normalize the IP-HFEs.

Definition 2.3. If a IP-HFE $h(p)$ is given by $p_l^l\le p_l^u$ and $\sum_{l=1}^{|h(p)|}p_l^u<1$, then, its associated IP-HFE $h(p)$ is defined as $h(p)=\{\gamma_l(\bar{p}_l)\,|\,l=1,2,\cdots,|h(p)|\}$, where $\bar{p}_l=\left[p_l^l,\bar{p}_l^u\right]$ and $\bar{p}_l^u=p_l^u\big/\sum_{l=1}^{|h(p)|}p_l^u$.

Definition 2.4 [2]. If a IP-HFE $h(p)$ is given by $p_l^l\le p_l^u$ and $\sum_{l=1}^{|h(p)|}p_l^l\le 1\le\sum_{l=1}^{|h(p)|}p_l^u$, then, its associated IP-HFE $h(p)$ is defined as $h(p)=\{\gamma_l(\bar{p}_l)\,|\,l=1,2,\cdots,|h(p)|\}$, where $\bar{p}_l=\left[\bar{p}_l^l,\bar{p}_l^u\right]$ and

$$\begin{cases}\bar{p}_l^l=\max(p_l^l,1-\sum_{\substack{i=1\\i\ne l}}^{|h(p)|}p_i^u)\\ \bar{p}_l^u=\min(p_l^u,1-\sum_{\substack{i=1\\i\ne l}}^{|h(p)|}p_i^l)\end{cases}.$$

Example1.let $h_1(p)$, $h_2(p)$ and $h_3(p)$ be three IP-HFEs, and $h_1(p)=\{0.9([0.5,0.6]),0.7([0.2,0.3])\}$, $h_2(p)=\{0.9([0.5,0.6]),0.6([0.4,0.7])\}$, $h_3(p)=\{0.8([0.5,0.8]),0.7([0.2,0.5])\}$. Using definition 2.3 and definition 2.4, the normalized IP-HFEs can be obtained as follows:

$h_1'(p)=\{0.9([0.67,0.67]),0.7([0.33,0.33])\}$, $h_2'(p)=\{0.9([0.5,0.6]),0.6([0.4,0.5])\}$,

$h_3'(p)=\{0.8([0.5,0.8]),0.7([0.2,0.5])\}$.

2.3 The Ranking of the IP-Hfes

In this section, we define the score and the deviation degree of a IP-HFE.

Definition 2.5. For a IP-HFE $h(p)$, its score is defined as:

$$s(h(p))=\left[\min\left(\sum_{l=1}^{|h(p)|}\gamma_l\dot{p}_l\right),\max\left(\sum_{l=1}^{|h(p)|}\gamma_l\dot{p}_l\right)\right],$$

where $\dot{p}_l$ is a real number and satisfies $\sum_{l=0}^{|h(p)|}\dot{p}_l=1$ and $p_l^l\le\dot{p}_l\le p_l^u$. $s(h(p))$ is an interval number.

For two IP-HFEs $h_1(p)$ and $h_2(p)$, if $s(h_1(p))>s(h_2(p))$, then we consider that $h_1(p)$ is superior to $h_2(p)$, denoted as $h_1(p)>h_2(p)$. if $s(h_1(p))<s(h_2(p))$, then we consider that $h_2(p)$ is superior to $h_1(p)$, denoted as $h_1(p)<h_2(p)$. However, if $s(h_1(p))=s(h_2(p))$, then, the two IP-HFEs can't be ranked anymore. Therefore, the deviation degree of IP-HFEs is defined as follows.

Definition 2.6. For a IP-HFE $h(p)$ and its score $s(h(p))$ be denoted by $\bar{\gamma}$, then the deviation degree of $h(p)$ is:

$$d(h(p))=\sum_{l=1}^{|h(p)|}(\gamma_l p_l-\bar{\gamma})^2.$$

Thus, a method can be proposed to compare two IP-HFEs $h_1(p)$ and $h_2(p)$:

(1) If $s(h_1(p))>s(h_2(p))$, then $h_1(p)>h_2(p)$;

(2) If $s\left(h_1(p)\right)=s\left(h_2(p)\right)$ and $d\left(h_1(p)\right)<d\left(h_2(p)\right)$, then $h_1(p)>h_2(p)$;

If $s\left(h_1(p)\right)=s\left(h_2(p)\right)$ and $d\left(h_1(p)\right)>d\left(h_2(p)\right)$, then $h_1(p)<h_2(p)$;

If $s\left(h_1(p)\right)=s\left(h_2(p)\right)$ and $d\left(h_1(p)\right)=d\left(h_2(p)\right)$, then $h_1(p)$ is equivalent to $h_2(p)$, denoted as $h_1(p)\sim h_2(p)$.

Example2. (Example 1 continues) According to definition 2.5 and definition 2.6, we obtain:

$s\left(h_1(p)\right)=[0.9\times0.67+0.7\times0.33,0.9\times0.67+0.7\times0.33]=[0.834,0.834]$,

$s\left(h_2(p)\right)=[0.9\times0.5+0.6\times0.5,0.9\times0.6+0.6\times0.4]=[0.75,0.78]$,

$s\left(h_3(p)\right)=[0.8\times0.5+0.7\times0.5,0.8\times0.8+0.7\times0.2]=[0.75,0.78]$.

$d\left(h_1(p)\right)=\left(([0.603,0.603]-[0.834,0.834])^2+([0.231,0.231])-[0.834,0.834])\right)^2=0.417$,

$d\left(h_2(p)\right)=\left(([0.45,0.54]-[0.75,0.78])^2+([0.24,0.3])-[0.75,0.78])\right)^2=0.318$,

$d\left(h_2(p)\right)=\left(([0.4,0.64]-[0.75,0.78])^2+([0.14,0.35])-[0.75,0.78])\right)^2=0.33$.

According to the calculation result above, $h_1(p)>h_2(p)>h_3(p)$ is obtained.

3. The Proposed Method

In this section, we develop a method for handling the MCDM problems under interval probabilistic hesitant fuzzy environment.

Suppose there exist $A=\{A_1,A_2,\cdots,A_m\}$ be the set of possible alternatives must choose on the basis of the set of criteria $C=\{c_1,c_2,\cdots,c_n\}$. And denote the criteria weighting vector by $w=\{w_1,w_2,\cdots,w_n\}$. The evaluation information of the alternatives under each criterion is given by using IP-HFEs. By the Definition 2.2, we get a judgment matrix D with IP-HFEs.

$$D=\begin{bmatrix} IP_{11} & IP_{12} & \cdots & IP_{1n} \\ IP_{21} & IP_{22} & \cdots & IP_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ IP_{m1} & IP_{m2} & \cdots & IP_{mn} \end{bmatrix}.$$

The method can be summarized as:

Step1. Normalize the IP-HFEs in D and get the normalized matrix D_N.

Step2. Calculate the score of all IP-HFEs in D_N and add the criteria weight, get the score matrix S_w with the criteria weight**.**

Step3. Calculate comprehensive score evaluation E_i of S_w by using the formula $E_i=\left[\sum_{j=1}^{n}w_j s_{ij}^l,\sum_{j=1}^{n}w_j s_{ij}^u\right]$, where $s_{ij}=\left[s_{ij}^l,s_{ij}^u\right]$ is the score of IP_{ij} $(i=1,2,\cdots,m)(j=1,2,\cdots,n)$.

Step4. According to the value of comprehensive score evaluation of A_i, ranking the alternatives. If there is a unique maximum, the best alternative can be got. The best choice is the alternative of the max value of comprehensive score evaluation. Otherwise, turn step 5.

Step5. Calculate the deviation degree of D_N and add the criteria weight, get the deviation degree matrix G_w with the criteria weight**.**

Step6. Calculate comprehensive deviation degree evaluation d_i of G_i by using the formula $d_i=\sum_{j=1}^{n}w_j d_{ij}$, where d_{ij} is the deviation degree of IP_{ij} $(i=1,2,\cdots,m)(j=1,2,\cdots,n)$.

Step7. When the value of comprehensive score evaluation of two alternatives is same, compare the value of comprehensive deviation degree evaluation. The best choice is the alternative of the min value of comprehensive deviation degree evaluation.

4. Illustrative Example

In this section, we give an example on how to select an appropriate resource planning (ERP) system. The example demonstrates the effective of our method.

The problem is described as follows: there are four candidates, which consist of the decision alternative set $A=\{A_1,A_2,A_3,A_4\}$, and there are four criteria related to the four candidates, which consist of vendor's reputation (c_1), strategic fitness (c_2), vendor's ability (c_3), technology (c_4). And denote the criteria weighting vector by $w=\{0.2,0.3,0.2,0.3\}$.Suppose that decision maker gives evaluations of the four alternatives under each criterion by using IP-HFEs. The evaluation information of A is described in table 1.

Table 1. The evaluation information of A

	c_1	c_2	c_3	c_4
A_1	$\{0.5([0.3,0.5]),0.8([0.2,0.6])\}$	$\{0.7([0.6,0.8]),0.9([0.1,0.4])\}$	$\{0.2([0.4,0.5]),0.3([0.3,0.4])\}$	$\{0.4([0.4,0.5]),0.5([0.5,0.6])\}$
A_2	$\{0.2([0.5,0.6]),0.4([0.3,0.4])\}$	$\{0.8([0.5,0.7]),0.9([0.3,0.6])\}$	$\{0.6([0.3,0.5]),0.7([0.2,0.6])\}$	$\{0.5([1,1])\}$
A_3	$\{0.6([1,1])\}$	$\{0.2([0.5,0.6]),0.3([0.3,0.4])\}$	$\{0.8([0.6,0.7]),0.9([0.3,0.6])\}$	$\{0.5([0.3,0.5]),0.6([0.5,0.8])\}$
A_4	$\{0.8([0.3,0.5]),0.9([0.7,0.9])\}$	$\{0.5([0.4,0.5]),0.6([0.5,0.6])\}$	$\{0.3([1,1])\}$	$\{0.7([0.3,0.5]),0.8([0.5,0.6])\}$

Step1. Normalize the IP-HFEs in table 1 by using definition 2.3 and definition 2.4. The result is shown by table 2.

Table 2. The normalized evaluation information of A

	c_1	c_2	c_3	c_4
A_1	$\{0.5([0.4,0.5]),0.8([0.5,0.6])\}$	$\{0.7([0.6,0.8]),0.9([0.2,0.4])\}$	$\{0.2([0.56,0.56]),0.3([0.44,0.44])\}$	$\{0.4([0.4,0.5]),0.5([0.5,0.6])\}$
A_2	$\{0.2([0.6,0.6]),0.4([0.4,0.4])\}$	$\{0.8([0.5,0.7]),0.9([0.3,0.5])\}$	$\{0.6([0.4,0.5]),0.7([0.5,0.6])\}$	$\{0.5([1,1])\}$
A_3	$\{0.6([1,1])\}$	$\{0.2([0.6,0.6]),0.3([0.4,0.4])\}$	$\{0.8([0.6,0.7]),0.9([0.3,0.4])\}$	$\{0.5([0.3,0.5]),0.6([0.5,0.7])\}$
A_4	$\{0.8([0.3,0.3]),0.9([0.7,0.7])\}$	$\{0.5([0.4,0.5]),0.6([0.5,0.6])\}$	$\{0.3([1,1])\}$	$\{0.7([0.4,0.5]),0.8([0.5,0.6])\}$

Step2. Calculate the score of all IP-HFEs in table 2 and add the criteria weight. The result is shown by table 3.

Table 3. The score of all IP-HFEs and add the criteria weight

	c_1	c_2	c_3	c_4
A_1	$\{0.2([0.65,0.68])\}$	$\{0.3([0.74,0.78])\}$	$\{0.2([0.244,0.244])\}$	$\{0.3([0.45,0.46])\}$
A_2	$\{0.2([0.48,0.48])\}$	$\{0.3([0.83,0.85])\}$	$\{0.2([0.65,0.66])\}$	$\{0.3([0.5,0.5])\}$
A_3	$\{0.2([0.6,0.6])\}$	$\{0.3([0.24,0.24])\}$	$\{0.2([0.83,0.84])\}$	$\{0.3([0.55,0.57])\}$
A_4	$\{0.2([0.87,0.87])\}$	$\{0.3([0.55,0.56])\}$	$\{0.2([0.3,0.3])\}$	$\{0.3([0.75,0.76])\}$

Step3. Calculate comprehensive evaluation of $A_i\,(i=1,2,3,4)$.

$E_1=[0.5358,0.5568]$; $E_2=[0.625,0.633]$; $E_3=[0.523,0.531]$; $E_4=[0.624,0.63]$.

Step4. According to the value of comprehensive evaluation of $A_i\,(i=1,2,3,4)$, $A_2 \succ A_4 \succ A_1 \succ A_3$ can be obtained. So, the best alternative is A_2.

5. Conclusion

In this paper, we define a concept of interval probabilistic hesitant fuzzy element (IP-HFE). And use IP-HFEs to express the experts' preference information, which make the information is preserved as much as possible. Then, a method of normalizing the IP-HFEs is proposed and the score and the deviation degree of a IP-HFE is developed to rank the IP-HFEs. Based on above works, we further propose a method based on IP-HFEs to handle MCDM problems. Finally, an illustrative example is given to verify the feasibility of the developed method.

This paper makes three significant contributions to the exiting literature on solving MCDM problems with IP-HFEs. First, we first propose the concept of interval probabilistic hesitant fuzzy element (IP-HFE) and further develop its score and the deviation degree formula. Second, we give a method to normalize the IP-HFEs. Finally, we use the IP-HFEs to deal with the MCDM problems under interval probabilistic hesitant fuzzy environment.

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