Dissipative control for T-S fuzzy system with random time-delay and linear fractional uncertainties

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Abstract

In the paper, the problem of feedback dissipative control for T-S fuzzy system with random time-delay and linear fractional uncertainties is addressed. Considering the random time-delay factor and parameter uncertainties are considered with linear fractional transformation form. By implementing a proper novel Lyapunov functional together with linear matrix inequality approach, a set of delay-dependent conditions are obtained to guarantee the dissipativity of Takagi-Sugeno fuzzy system with time-delay and uncertainties that strictly stochastically dissipative of the closed-loop system. Further, the desired state feedback dissipative controller can be obtained by solving the linear matrix inequalities. Finally, an illustrative example based on the inverted pendulum model with random time-delay is provided to show the effectiveness of the proposed design technique.

Keywords

Fuzzy system; dissipative; random time-delay; linear fractional uncertainties; linear matrix inequality (LMI).

1. Introduction

During recent decades, T-S fuzzy systems [1] as a universal approximation has been widely and successfully studied. T-S fuzzy system model combines fuzzy logic and fuzzy reasoning with the mathematical model of linear system or nonlinear system, which provides a good theoretical framework for the stability analysis of fuzzy system and the design of the controller. As a consequence, a great number of important and successful results on analysis and synthesis problems (e.g., signal processing, pattern recognition, combinatorial optimization and commination) of T-S systems can be available [2-7]. Time delay is a common phenomenon in industrial systems, such as the internal time delay of the production object, the constant time delay of the digital system, and so on. It has been proved that the time delay will lead to the instability of the system and the poor performance of the system. Therefore, it has received considerable attention on T-S fuzzy time-delay system [7–9]. And no matter which system is investigated, the continuous-time singular system or the discrete one, results for both stability and stabilization problems are quite mature [8-10].

Uncertainty widely exists in the real system, the reasons mainly include the following two aspects: (1) the object is too complex to establish accurate mathematical model of the system caused by the uncertainty; (2) the environment changes, the measurement error, caused by element device aging and external interference factors uncertainty. Under normal circumstances, due to the impact of uncertain factors, the system performance will deteriorate, and even lead to the controlled system is not stable. The study of the robust control is developed under this background. The robustness of the control system, is refers to under the impact of uncertainty, to maintain certain performance according to the characteristics of [11]. Robustness requirements are different, can be divided into robust stability robustness and performance robustness. The design is a system controller for robust control of the target, so that in the uncertain under the influence of the property, the closed-loop system can

still keep stable or some performance index [12]. a new class of uncertainty in the linear fractional form is considered in [13].

On the other hand, compared with passivity [14] and H_∞ performance [15], dissipative is a more general criterion, which was first proposed by Willems in [16]. The dissipativity analysis is an important concept in system and control theory, it is in the system, circuit, network, control engineering and control theory has played an important role in the analysis on the stability of the system of nonlinear system and robust control has played an important role. Its essence is the existence of a nonnegative energy function, the energy loss within the system is always less than the external energy supply rate of H_∞ control and passive control are special cases of H_∞ control is the suppression of a dissipative control interference thought, while ensuring the stability of system that is the smallest degree of inhibition to the desired effect on the system output interference. When the supply rate of input and output product, become passive state problem. Passive system theory in many engineering problems, such as circuit Systems and thermal power systems have been widely used. At present, many important results have been obtained on the study of dissipative properties [17-21].

In the above reference, the stability analysis and synthesis of the nonlinear fuzzy system focus on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in the practical engineering applications, finite time control is of practical important. Finite time stability admits that the state does not exceed a certain bound during a fixed finite time interval. In many practical engineering applications, the finite time control is of practical significance, such as biochemistry reaction system, communication net- work system and robot control system. Recently, the study of finite time problem has received increasing attention, see for example [22-25]. Finite-time dissipative control [26] and passive control [27] were discussed respectively. In [28,29], the problem of finite time H1 filtering was solved for a class of discrete-time Markovian jump systems with switching transition probabilities and partly unknown transition probabilities respectively.

Motivated by the above discussions, the main objective of this paper is to study the problem on global dissipativity for uncertain T-S fuzzy system with random delay and linear fractional uncertainties. By employing appropriate Lyapunov-Krasovskii functional and LMI technique, sufficient condition of dissipativity converted to linear matrix inequalities. Finally, a fuzzy state feedback controller, which guarantees that the closed-loop system is dissipative, is designed based on the former conditions.

The remaining parts of this paper are organized as follows. Section 2 illustrates problem formulation and preliminaries of uncertain singular T–S fuzzy systems with time-varying delay and stochastic perturbation. Our main results are provided in Section 3. In Section 4, numerical simulation results are represented to illustrate the effectiveness of proposed methods. Finally, conclusions are drawn in Section 5.

Notations. The notations are quite standard. Throughout this paper, $\mathbb{R}^n$ denotes the $n-$ dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ represents the set of all real matrices. $P>0$ shows that P is a symmetric and positive definite matrix. The superscript $'-1'$ denotes the inverse of a matrix and $'T'$ denotes the transpose. $\mathrm{E}\{x\}$ denotes the expectation of x. In symmetric block matrices, the symbol $(*)$ is used as an ellipsis for terms induced by symmetry.

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^{\mathrm{T}} & C \end{bmatrix}$$

2. Problem formulation and preliminaries

Consider the following nonlinear systems, which can be presented as the following T-S fuzzy random delay model:

Plant rule i:

IF $\lambda_1(k)$ is χ_{i1} and $\lambda_2(k)$ is $\chi_{i2}\ldots\lambda_p(k)$ is χ_{ip}.

THEN

$$\begin{aligned}\dot{x}(k) &= (A_i + \Delta A_i(k))x(k) + (A_{di} + \Delta A_{di}(k))x(k-d(k)) \\ &\quad + (B_i + \Delta B_i(k))u(k) + (B_{wi} + \Delta B_{wi}(k))w(k) \\ z(k) &= (C_i + \Delta C_i(k))x(k) + (C_{di} + \Delta C_{di}(k))x(k-d(k)) \\ &\quad + (D_i + \Delta D_i(k))u(k) + (D_{wi} + \Delta D_{wi}(k))w(k) \\ x(k) &= \phi(k), i = 1,2,...,r\end{aligned} \tag{1}$$

where $\lambda_1(k),\cdots,\lambda_p(k)$ are the premise variables, $\chi_{ij}(j=1,2,...,p)$ is the fuzzy set, $x(k)\in R^n$ is the state vector, r is the number of IF-THEN rule, $u(k)\in R^m$ is the control input vector, $w(k)\in\mathbb{R}^q$ is the exogenous input which belongs to $L_2[0,\infty)$, $z(k)\in R^q$ is the control output vector, $A_i, A_{di}, B_i, B_{wi}, C_i, C_{di}, D_i$ and D_{wi} are known real constant matrices with appropriate dimensions. $\phi(k)$ is the initial function; Where $\Delta A_i(k)$, $\Delta A_{di}(k)$, $\Delta B_i(k)$, $\Delta B_{wi}(k)$, $\Delta C_i(k)$, $\Delta C_{di}(k)$, $\Delta D_i(k)$, $\Delta D_{wi}(k)$ are unknown matrices representing time varying parameter uncertainties described as:

$$\begin{aligned}&\left[\Delta A_i(k) \quad \Delta A_{di}(k) \quad \Delta B_i(k) \quad \Delta B_{wi}(k)\right] = M_i B_i(k)\left[H_{1i} \quad H_{2i} \quad H_{3i} \quad H_{4i}\right], \\ &\left[\Delta C_i(k) \quad \Delta C_{di}(k) \quad \Delta D_i(k) \quad \Delta D_{wi}(k)\right] = N_i B_i(k)\left[H_{1i} \quad H_{2i} \quad H_{3i} \quad H_{4i}\right], \\ &i = 1,2,...,r,\end{aligned} \tag{2}$$

where $M\in R^{n\times i}, H_1\in R^{j\times n}, H_2\in R^{j\times n}, H_3\in R^{j\times p}$ are known real constant matrices.

The linear fractional form uncertainty was proposed in [13], which can include the norm bounded uncertainties as a special case. The class of parameter uncertainties $B_i(k)$ that satisfy $B_i(k)=\left[I-F_i(k)J\right]^{-1}F_i(k),\ \ i=1,2,...,r.$ is said to be admissible, where J is also a known matrix satisfying $I-JJ^{\mathrm{T}}>0$ and $F_i(k)$ is an unknown matrix satisfying the following condition:

$F_i^{\mathrm{T}}(k)F_i(k)\le I$

Remark 1. The above structured linear fractional form includes the norm-bounded uncertainty as a special case when $J=0$.

The fuzzy basic function are given by

$$\xi_i(\lambda(k)) = \frac{\prod_{j=1}^{p}\chi_{ij}(\lambda_j(k))}{\sum_{i=1}^{r}\prod_{j=1}^{p}\chi_{ij}(\lambda_j(k))}, \qquad i\in S$$

with $\chi_{ij}(\lambda_j(k))$ representing the grade of membership of $\lambda_j(k)$ in χ_{ij} . For simplicity, $\xi_i(\lambda(k))$ take the place of ξ_i in the following some places. By definition, the fuzzy basic functions satisfy $0\le\xi_i(\lambda(k))\le 1,\ \ i\in S$, and $\sum_{i=1}^{r}\xi_i(\lambda(k))=1.$

It is assumed that the premise variables do not depend on input variable $u(k)$ explicitly. Then, the defuzzified system of the T–S fuzzy system in (1) can be represented as

$$\begin{aligned}\dot{x}(k) &= \sum_{i=1}^{r}\xi_i(\lambda(k))[(A_i + \Delta A_i(k))x(k) + (A_{di} + \Delta A_{di}(k))x(k-d(k)) + (B_i + \Delta B_i)u(k) \\ &\quad + (B_{wi} + \Delta B_{wi}(k))w(k)] \\ z(k) &= \sum_{i=1}^{r}\xi_i(\lambda(k))[(C_i + \Delta C_i)x(k) + (C_{di} + \Delta C_{di})x(k-d(k)) + (D_i + \Delta D_i)u(k) + \\ &\quad (D_{wi} + \Delta D_{wi})w(k)]\end{aligned} \tag{3}$$

We give the closed-loop system of (5) of (4) in a compact form

$$E\dot{x}(k) = \overline{A}(k)x(k) + \overline{A_{di}}(k))x(k-d(k)) + \overline{B}(k)u(k) + \overline{B_w}(k)w(k),$$
$$z(k) = \overline{C}(k)x(k) + \overline{C_d}(k)x(k-d(k)) + \overline{D}(k)u(k) + \overline{D_w}(k)w(k). \tag{4}$$

where

$\overline{A}(k) \triangleq \sum_{i-1}^{r} \xi_i(\lambda(k))(A_i + \Delta A_i(k))$, $\overline{A_d}(k) \triangleq \sum_{i=1}^{r} \xi_i(\lambda(k))(A_{di} + \Delta A_{di}(k))$

$\overline{B_w}(k) \triangleq \sum_{i=1}^{r} \xi_i(\lambda(k))(B_{wi} + \Delta B_{wi}(k))$, $\overline{C}(k) \triangleq \sum_{i-1}^{r} \xi_i(\lambda(k))(C_i + \Delta C_i(k))$

$\overline{C_d}(k) \triangleq \sum_{i=1}^{r} \xi_i(\lambda(k))(C_{di} + \Delta C_{di}(k))$, $\overline{D_w}(k) \triangleq \sum_{i-1}^{r} \xi_i(\lambda(k))(D_{wi} + \Delta D_{wi}(k))$

Now, consider

Plant rule i : if $\lambda_1(k)$ is χ_{i1} and $\lambda_2(k)$ is $\chi_{i2} \ldots \lambda_p(k)$ is χ_{ip} ,THEN

$u(k) = K_i x(k), \qquad i \in S$

where K_i is the gain matrix of the state-feedback controller in each rule; the state-feedback controller in (2) is given by

$$u(k) = \sum_{i=1}^{r} \xi_i(\lambda(k)) K_i x(k) \tag{5}$$

Under control law, the closed-loop system is obtained as

$$\begin{aligned} \dot{x}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i(\lambda(k))\xi_j(\lambda(k)) \big\{ (A_i + \Delta A_i(k)) + (B_i + \Delta B_i(k))K_j x(k) \\ &\quad + (A_{di} + \Delta A_{di}(k))x(k-d(k)) + (B_{wi} + \Delta B_{wi}(k))w(k) \big\}, \\ z(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i(\lambda(k))\xi_j(\lambda(k)) \big\{ (C_i + \Delta C_i(k)) + (D_i + \Delta D_i(k))K_j x(k) \\ &\quad + (C_{di} + \Delta C_{di}(k))x(k-d(k)) + (D_{wi} + \Delta D_{wi}(k))w(k) \big\}. \end{aligned} \tag{6}$$

The compact form of the closed-loop system (Σc) can be given as

$$\begin{aligned} \dot{x}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i(\lambda(k))\xi_j(\lambda(k)) \big\{ A(k)x(k) + A_d(k)x(k-d(k)) + B_w(k)w(k) \big\}, \\ z(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i(\lambda(k))\xi_j(\lambda(k)) \big\{ C(k)x(k) + C_d(k)x(k-d(k)) + D_w(k)w(k) \big\}. \end{aligned} \tag{7}$$

Where

$$\begin{aligned} A(k) &= (A_i + \Delta A_i(k) + (B_i + \Delta B_i(k))K_j), \\ A_d(k) &= A_{di} + \Delta A_{di}(k), \\ B_w(k) &= B_{wi} + \Delta B_{wi}(k), \\ C(k) &= (C_i + \Delta C_i(k)) + (D_i + \Delta D_i(k))K_j, \\ C_d(k) &= C_{di} + \Delta C_{di}(k), \\ D_w(k) &= D_{wi} + \Delta D_{wi}(k), \end{aligned}$$

Assumption (H1). The time-delay $d(k)$ is bounded, and its lower and upper bounds are $0 \le d_1 \le d(k) \le d_2$, its probability distribution can be observed, i.e., suppose $d(k)$ takes values in

$[d_1 : d_0]$ or $[d_0 : d_2]$ and $P\{d(k) \in [d_1 : d_0]\} = \tau_0$, where $d_1 \le d_0 < d_2$, and $0 \le \tau_0 \le 1$. In order to describe the probability distribution of the time delays, define the following two sets

$\Omega_1 = \{k | d(k) \in [d_1 : d_0]\}$ and $\Omega_2 = \{k | d(k) \in (d_0 : d_2]\}$

Moreover, we define two mapping functions as follows

$$d_1(k) = \begin{cases} d(k), k \in \Omega_1 \\ \overline{d_1}, \ else, \end{cases} \quad d_1(k) = \begin{cases} d(k), k \in \Omega_2 \\ \overline{d_0}, \ else. \end{cases}$$

where $\overline{d_0} = (d_0 : d_2]$. and $\overline{d_1} = [d_1 : d_0]$

Assumption (H2). Further, the time-varying delays $d_1(k)$ and $d_2(k)$ satisfying the following condition

$d_1 \le d_1(k) \le d_0, \dot{d}_1(k) \le \mu_1.$

$d_0 < d_2(k) \le d_2, \dot{d}_2(k) \le \mu_2.$

where τ_m, τ_0, τ_M, μ_1 and μ_2 are positive constants.

It follows from that $\Omega_1 \bigcup \Omega_2 = \mathbb{Z} > 0$ and $\Omega_1 \bigcap \Omega_2 = \Phi$. From it can be seen that $k \in \Omega_1$ implies the event $d(k) \in [d_1 : d_0]$ occurs and $k \in \Omega_2$ implies the event $d(k) \in (d_0 : d_2]$ occurs.

Defining a stochastic variable as

$$\tau(k) = \begin{cases} 1, & k \in \Omega_1 \\ 0, & else \end{cases}.$$

with

$P\{\tau(k) = 1\} = P\{\tau(k) \in [d_1 : d_0]\} = E[\tau(k)] = \tau_0,$

$P\{\tau(k) = 0\} = P\{\tau(k) \in (d_0 : d_2]\} = 1 - E[\tau(k)] = 1 - \tau_0.$

Further, it is easy to see that $E[\tau(k) - \tau_0] = 0$ and $E[(\tau(k) - \tau_0)^2] = \tau_0(1 - \tau_0)$

By introducing the random delay, then system can be equivalently rewritten in the following form

$$\begin{aligned} \dot{x}(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i(\lambda(k))\xi_j(\lambda(k)) \Big\{ & A(k)x(k) + \tau(k)A_d(k)x(k - d_1(k)) \\ & + (1 - \tau(k))A_d(k)x(k - d_2(k)) + B_w(k)w(k) \Big\}, \\ z(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i(\lambda(k))\xi_j(\lambda(k)) \Big\{ & C(k)x(k) + \tau(k)C_d(k)x(k - d_1(k)) \\ & + (1 - \tau(k))C_d(k)x(k - d_2(k)) + D_w(k)w(k) \Big\}. \end{aligned} \tag{8}$$

Before presenting the main results of this paper, we first introduce the following definitions for the fuzzy stochastic system (Σo), which will be essential for the derivations of following theorems. Throughout the paper, we will adopt the following definition and lemma.

The energy supply function of system(8)is defined as

$J(w, z, \tau) = \langle z, Qz \rangle_\tau + 2\langle z, Sw \rangle_\tau + \langle w, Rw \rangle_\tau, \forall \tau \ge 0$

where matrices Q, S and R real symmetric of appropriate dimensions, with $Q < 0$ and R being symmetric matrices, and

$\langle u, v \rangle_\tau \triangleq \int_0^\tau u^{\mathrm{T}}(k)v(k)d\tau$

Definition 1 Under zero initial condition, the system(8)is said to be stochastically (X, Y, Z)-dissipative if for all $w(k) \in L_2[0,\infty)$, the energy supply function satisfies

$$\mathrm{E}\{J(w,z,\tau)\} \geq 0, \quad \forall \tau > 0$$

Furthermore, the system(8)called strictly stochastically $(Q;S;R)-\alpha-$dissipative if for a sufficiently small scalar the energy supply function satisfies

$$\mathrm{E}\left\{J(w,z,\tau)-\alpha\langle w,w\rangle_\tau\right\} \geq 0, \text{ or } \mathrm{E}\left\{\int_0^\tau \left[z^{\mathrm{T}}(k)Qz(k)+2z^{\mathrm{T}}(k)Sw(k)+w^{\mathrm{T}}(k)(R-\alpha I)w(k)\right]d\tau\right\} \geq 0.$$

In addition, in order to prove the main results, the following lemma is very useful.

Lemma 1 (Schur complement theorem of matrix). Given constant matrices $M_{11}, M_{12}, M_{21}, M_{22}$, where $M_{11}=M_{11}^{\mathrm{T}}$ and $M_{22}=M_{22}^{\mathrm{T}}$, then $M=\begin{bmatrix} M_{11} & M_{12} \\ M_{12}^{\mathrm{T}} & M_{22} \end{bmatrix}$, where M_{11} is $r\times r$ dimensional, the following three conditions are equivalent:

(1) $M<0;$

(2) $M_{11}<0, M_{22}-M_{12}^{\mathrm{T}}M_{11}^{-1}M_{12}<0.$

(3) $M_{22}<0, M_{11}-M_{12}M_{22}^{-1}M_{12}^{\mathrm{T}}<0.$

Lemma 2 Given matrices P, Q, and R with $P^{\mathrm{T}}=P$, then

$$P+Q\Delta(k)R+R^{\mathrm{T}}\Delta^{\mathrm{T}}(k)Q^{\mathrm{T}}<0$$

holds for all $F(k)$ satisfying $F^{\mathrm{T}}(k)F(k)\leq I$, if and only if there exists a scalar $\varepsilon>0$ such that

$$\begin{bmatrix} P & Q & \varepsilon R^{\mathrm{T}} \\ Q^{\mathrm{T}} & -\varepsilon I & \varepsilon J^{\mathrm{T}} \\ \varepsilon R & \varepsilon J & -\varepsilon I \end{bmatrix}<0.$$

Lemma 3 Given any positive definite constant matrices $S\in R^{n\times n}$, $S=S^{\mathrm{T}}$, and scalars $0<d_1<d(k)<d_2$, for vector function $x(k):[k-d_2,k-d_1]\to R^n$, then

$$-\int_{k-d_2}^{k-d_1} x^{\mathrm{T}}(s)Sx(s)ds \leq -\frac{1}{d_2-d_1}\left[\int_{k-d_2}^{k-d_1} x(s)ds\right]^{\mathrm{T}} S\left[\int_{k-d_2}^{k-d_1} x(s)ds\right].$$

The objective of this paper is to study problem of $\alpha-$dissipativity control for singular T-S fuzzy system. By utilizing the Schur complement and some Lemma, we will derive some sufficient criteria in terms of LMIs so that singular system(8)is strictly $(Q;S;R)-\alpha-$dissipative.

3. Main results

Theorem 1. For system(8)with the zero initial condition, integer d_1 and d_2 satisfying $0<d_1\leq d(k)\leq d_2$, let a positive constant $\alpha>0$, then system is strictly $(Q,S,R)-\alpha-$dissipative if there exist a common nonsingular matrices $P\in\mathbb{R}^{n\times n}$, and such that the following set of LMIs hold:

$$\tilde{\Pi}_{ij}=\begin{bmatrix} \tilde{\Pi}_{(m\times n)ij} & \tilde{\Pi}_{i1}^{\mathrm{T}} \\ * & -I \end{bmatrix}<0, \quad m,n=1,2,...,13,14. \tag{9}$$

proof: Choose a basic-dependent Lyapunov-Krasovskii functional (LKF) for as

$$\begin{aligned} V(k,x(k)) \triangleq & V_1(k,x(k))+V_2(k,x(k))+V_3(k,x(k))+V_4(k,x(k)) \\ & +V_5(k,x(k))+V_6(k,x(k))+V_7(k,x(k)) \end{aligned}$$

$$V_1(k,x(k)) = x^{\mathrm{T}}(k)Px(k),$$

$$V_2(k,x(k)) = \int_{k-d_1(k)}^{k} x^{\mathrm{T}}(s)Q_1x(s)ds + \int_{k-d_0}^{k} x^{\mathrm{T}}(s)Q_2x(s)ds + \int_{k-d_1}^{k} x^{\mathrm{T}}(s)Q_3x(s)ds,$$

$$V_3(k,x(k)) = \int_{k-d_2(k)}^{k} x^{\mathrm{T}}(s)Q_4x(s)ds + \int_{k-d_2}^{k} x^{\mathrm{T}}(s)Q_5x(s)ds + \int_{k-d_0}^{k} x^{\mathrm{T}}(s)Q_6x(s)ds,$$

$$V_4(k,x(k)) = \int_{-d_0}^{-d_1}\int_{k+\theta}^{k} x^{\mathrm{T}}(s)R_1x(s)dsd\theta + \int_{-d_0}^{0}\int_{k+\theta}^{k} x^{\mathrm{T}}(s)R_2x(s)dsd\theta,$$

$$V_5(k,x(k)) = \int_{-d_2}^{-d_0}\int_{k+\theta}^{k} x^{\mathrm{T}}(s)R_3x(s)dsd\theta + \int_{-d_2}^{0}\int_{k+\theta}^{k} x^{\mathrm{T}}(s)R_4x(s)dsd\theta,$$

$$V_6(k,x(k)) = \int_{-d_0}^{-d_1}\int_{k+\theta}^{k} \dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)dsd\theta + \int_{-d_0}^{0}\int_{k+\theta}^{k} \dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)dsd\theta,$$

$$V_7(k,x(k)) = \int_{-d_2}^{-\tau_0}\int_{k+\theta}^{k} \dot{x}^{\mathrm{T}}(s)S_3\dot{x}(s)dsd\theta + \int_{-d_2}^{0}\int_{k+\theta}^{k} \dot{x}^{\mathrm{T}}(s)S_4\dot{x}(s)dsd\theta,$$

By calculating the derivatives $\dot{V}(k,x(k))$ along the trajectories of system(8)and taking the mathematical expectation, we can obtain

$$\mathrm{E}\left\{\dot{V}_1(k,x(k))\right\} = \mathrm{E}\left\{2x^{\mathrm{T}}(k)P\dot{x}(k)\right\},$$

$$\begin{aligned}\mathrm{E}\left\{\dot{V}_2(k,x(k))\right\} \le \mathrm{E}\{&x^{\mathrm{T}}(k)(Q_1+Q_2+Q_3)x(k) - (1-\mu_1)x^{\mathrm{T}}(k-d_1(k))Q_1x(k-d_1(k)) \\ &- x^{\mathrm{T}}(k-d_0)Q_2x(k-d_0) - x^{\mathrm{T}}(k-d_1)Q_3x(k-d_1)\},\end{aligned}$$

$$\begin{aligned}\mathrm{E}\left\{\dot{V}_3(k,x(k))\right\} \le \mathrm{E}\{&x^{\mathrm{T}}(k)(Q_4+Q_5+Q_6)x(k) - (1-\mu_2)x^{\mathrm{T}}(k-d_2(k))Q_4x(k-d_2(k)) \\ &- x^{\mathrm{T}}(k-d_2)Q_5x(k-d_2) - x^{\mathrm{T}}(k-d_0)Q_6x(k-d_0)\},\end{aligned}$$

$$\begin{aligned}\mathrm{E}\left\{\dot{V}_4(k,x(k))\right\} = \mathrm{E}\{&(d_0-d_1)x^{\mathrm{T}}(k)R_1x(k) - \int_{k-d_0}^{k-d_1} x^{\mathrm{T}}(s)R_1x(s)ds + d_0x^{\mathrm{T}}(k)R_2x(k) \\ &- \int_{k-d_0}^{k} x^{\mathrm{T}}(s)R_2x(s)ds\},\end{aligned}$$

$$\begin{aligned}\mathrm{E}\left\{\dot{V}_5(k,x(k))\right\} = \mathrm{E}\{&(d_2-d_0)x^{\mathrm{T}}(k)R_3x(k) - \int_{k-d_2}^{k-d_0} x^{\mathrm{T}}(s)R_3x(s)ds + d_2x^{\mathrm{T}}(k)R_4x(k) \\ &- \int_{k-d_2}^{k} x^{\mathrm{T}}(s)R_4x(s)ds\},\end{aligned}$$

$$\begin{aligned}\mathrm{E}\left\{\dot{V}_6(k,x(k))\right\} = \mathrm{E}\{&(d_0-d_1)\dot{x}^{\mathrm{T}}(k)S_1\dot{x}(k) - \int_{k-d_0}^{k-d_1} \dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)ds + d_0\dot{x}^{\mathrm{T}}(k)S_2\dot{x}(k) \\ &- \int_{k-d_0}^{k} \dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)ds\},\end{aligned}$$

$$\mathrm{E}\left\{\dot{V}_7(k,x(k))\right\} = \mathrm{E}\{(d_2-d_0)\dot{x}^{\mathrm{T}}(k)S_3\dot{x}(k) - \int_{k-d_2}^{k-d_0}\dot{x}^{\mathrm{T}}(s)S_3\dot{x}(s)ds + d_2\dot{x}^{\mathrm{T}}(k)S_4\dot{x}(k)$$
$$-\int_{k-d_2}^{k}\dot{x}^{\mathrm{T}}(s)S_4\dot{x}(s)ds\},$$

Also, it follows from that

$$\begin{aligned}\mathrm{E}\left\{\dot{V}(k,x(k))\right\} \le \mathrm{E}\{&2x^{\mathrm{T}}(k)P\dot{x}(k) + x^{\mathrm{T}}(k)(Q_1+Q_2+Q_3+Q_4+Q_5+Q_6+(d_0-d_1)R_1\\ &+d_0R_2+(d_2-d_0)R_3+d_2R_4)x(k) - (1-\mu_1)x^{\mathrm{T}}(k-d_1(k))Q_1x(k-d_1(k))\\ &-x^{\mathrm{T}}(k-d_0)(Q_2+Q_6)x(k-d_0) - x^{\mathrm{T}}(k-d_1)Q_3x(k-d_1)\\ &-(1-\mu_2)x^{\mathrm{T}}(k-d_2(k))Q_4x(k-d_2(k)) - x^{\mathrm{T}}(k-d_2)Q_5x(k-d_2)\\ &+\dot{x}^{\mathrm{T}}(k)\left((d_0-d_1)S_1+d_0S_2+(d_2-d_0)S_3+d_2S_4\right)\dot{x}(k)\\ &-\int_{k-d_0}^{k-d_1}x^{\mathrm{T}}(s)R_1x(s)ds - \int_{k-d_0}^{k}x^{\mathrm{T}}(s)R_2x(s)ds - \int_{k-d_2}^{k-d_0}x^{\mathrm{T}}(s)R_3x(s)ds\\ &-\int_{k-d_2}^{k}x^{\mathrm{T}}(s)R_4x(s)ds - \int_{k-d_0}^{k-d_1}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)ds - \int_{k-d_0}^{k}\dot{x}^{\mathrm{T}}(s)S_2\dot{x}(s)ds\\ &-\int_{k-d_2}^{k-d_0}\dot{x}^{\mathrm{T}}(s)S_3\dot{x}(s)ds - \int_{k-d_2}^{k}\dot{x}^{\mathrm{T}}(s)S_4\dot{x}(s)ds\}\end{aligned}$$

By using the vary-varying delay, the integrations can be written as

$$-\int_{k-d_0}^{k-d_1}x^{\mathrm{T}}(s)R_1x(s)ds = -\int_{k-d_0}^{k-d_1(k)}x^{\mathrm{T}}(s)R_1x(s)ds - \int_{k-d_1(k)}^{k-d_1}x^{\mathrm{T}}(s)R_1x(s)ds,$$
$$-\int_{k-d_2}^{k-d_0}x^{\mathrm{T}}(s)R_3x(s)ds = -\int_{k-d_2}^{k-d_2(k)}x^{\mathrm{T}}(s)R_3x(s)ds - \int_{k-d_2(k)}^{k-d_0}x^{\mathrm{T}}(s)R_3x(s)ds,$$
$$-\int_{k-d_0}^{k-d_1}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)ds = -\int_{k-d_0}^{k-d_1(k)}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)ds - \int_{k-d_1(k)}^{k-d_1}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)ds,$$
$$-\int_{k-d_2}^{k-d_0}\dot{x}^{\mathrm{T}}(s)S_3\dot{x}(s)ds = -\int_{k-d_2}^{k-d_2(k)}\dot{x}^{\mathrm{T}}(s)S_3\dot{x}(s)ds - \int_{k-d_2(k)}^{k-d_0}\dot{x}^{\mathrm{T}}(s)S_3\dot{x}(s)ds.$$

By applying Lemma 2 to each integrals in above equations, we can obtain the following inequalities;

$$-\int_{k-d_0}^{k-d_1(k)}x^{\mathrm{T}}(s)R_1x(s)ds \le -\frac{1}{d_0-d_1}\left[\int_{k-d_0}^{k-d_1(k)}x(s)ds\right]^{\mathrm{T}}R_1\left[\int_{k-d_0}^{k-d_1(k)}x(s)ds\right],$$
$$-\int_{k-d_1(k)}^{k-d_1}x^{\mathrm{T}}(s)R_1x(s)ds \le -\frac{1}{d_0-d_1}\left[\int_{k-d_1(k)}^{k-d_1}x(s)ds\right]^{\mathrm{T}}R_1\left[\int_{k-d_1(k)}^{k-d_1}x(s)ds\right],$$
$$-\int_{k-d_2}^{k-d_2(k)}x^{\mathrm{T}}(s)R_3x(s)ds \le -\frac{1}{d_2-d_0}\left[\int_{k-d_2}^{k-d_2(k)}x(s)ds\right]^{\mathrm{T}}R_3\left[\int_{k-d_2}^{k-d_2(k)}x(s)ds\right],$$
$$-\int_{k-d_2(k)}^{k-d_0}x^{\mathrm{T}}(s)R_3x(s)ds \le -\frac{1}{d_2-d_0}\left[\int_{k-d_2(k)}^{k-d_0}x(s)ds\right]^{\mathrm{T}}R_3\left[\int_{k-d_2(k)}^{k-d_0}x(s)ds\right],$$
$$-\int_{k-d_0}^{k-d_1(k)}\dot{x}^{\mathrm{T}}(s)S_1\dot{x}(s)ds \le -\frac{1}{d_0-d_1}\left[\int_{k-d_0}^{k-d_1(k)}\dot{x}(s)ds\right]^{\mathrm{T}}S_1\left[\int_{k-d_0}^{k-d_1(k)}\dot{x}(s)ds\right]$$

$$= -\frac{1}{d_0 - d_1}\left[x(k-d_1(k)) - x(k-d_0)\right]^{\mathrm{T}} S_1 \left[x(k-d_1(k)) - x(k-d_0)\right],$$

$$-\int_{k-d_1(k)}^{k-d_1} \dot{x}^{\mathrm{T}}(s) S_1 \dot{x}(s) ds \le -\frac{1}{d_0 - d_1}\left[\int_{k-d_1(k)}^{k-d_1} \dot{x}(s) ds\right]^{\mathrm{T}} S_1 \left[\int_{k-d_1(k)}^{k-d_1} \dot{x}(s) ds\right]$$

$$= -\frac{1}{d_0 - d_1}\left[x(k-d_1) - x(k-d_1(k))\right]^{\mathrm{T}} S_1 \left[x(k-d_1) - x(k-d_1(k))\right],$$

$$-\int_{k-d_2}^{k-d_2(k)} \dot{x}^{\mathrm{T}}(s) S_3 \dot{x}(s) ds \le -\frac{1}{d_2 - d_0}\left[\int_{k-d_2}^{k-d_2(k)} \dot{x}(s) ds\right]^{\mathrm{T}} S_3 \left[\int_{k-d_2}^{k-d_2(k)} \dot{x}(s) ds\right]$$

$$= -\frac{1}{d_2 - d_0}\left[x(k-d_2(k)) - x(k-d_2)\right]^{\mathrm{T}} S_3 \left[x(k-d_2(k)) - x(k-d_2)\right],$$

$$-\int_{k-d_2(k)}^{k-d_0} \dot{x}^{\mathrm{T}}(s) S_3 \dot{x}(s) ds \le -\frac{1}{d_2 - d_0}\left[\int_{k-d_2(k)}^{k-d_0} \dot{x}(s) ds\right]^{\mathrm{T}} S_3 \left[\int_{k-d_2(k)}^{k-d_0} \dot{x}(s) ds\right]$$

$$= -\frac{1}{d_2 - d_0}\left[x(k-d_0) - x(k-d_2(k))\right]^{\mathrm{T}} S_3 \left[x(k-d_0) - x(k-d_2(k))\right].$$

On the other hand, for any matrices P_1 of appropriate dimensions the following equalities hold,

$$\mathrm{E}\{\sum_{i=1}^{r}\sum_{j=1}^{r}\xi_i(\lambda(k))\xi_j(\lambda(k))\{2\dot{x}^{\mathrm{T}}(k)P_1[(A_i + B_i K_j)x(k) + \tau(k)A_d(k)x(k-d_1(k))$$
$$+(1-\tau(k))A_d(k)x(k-d_2(k)) + B_w(k)w(k) - \dot{x}(k)]\}\} = 0.$$

We obtain

$$J(k) = E\left\{\int_0^{\tau}\left[z^{\mathrm{T}}(k)Qz(k) + 2z^{\mathrm{T}}(k)Sw(k) + w^{\mathrm{T}}(k)(R-\alpha I)w(k)\right]d\tau\right\}$$
$$= E\left\{\int_0^{\tau}\left[\dot{V}(k,x(k)) + V(0,x(0)) - V(\tau,x(\tau)) - z^{\mathrm{T}}(k)Qz(k)\right.\right.$$
$$-2z^{\mathrm{T}}(k)Sw(k) - w^{\mathrm{T}}(k)(R-\alpha I)w(k)d\tau$$

Under the zero initial condition, using in gives

$$J(k) \le \mathrm{E}\left\{\int_0^{\tau}\left[\dot{V}(k,x(k)) - z^{\mathrm{T}}(k)Qz(k) - 2z^{\mathrm{T}}(k)Sw(k) - w^{\mathrm{T}}(k)(R-\alpha I)w(k)\right]d\tau\right\}$$

Calculating the increment of $V(k)$ along the trajectory of system and taking expectation, we have

$$\mathrm{E}\left\{\dot{V}(k,x(k))\right\} \le \mathrm{E}\{\sum_{i=1}^{r}\sum_{j=1}^{r}\xi_i\xi_j[2x^{\mathrm{T}}(k)\left(P + (A_i + B_i K_j)^{\mathrm{T}} P_1^{\mathrm{T}}\right)\dot{x}(k) + x^{\mathrm{T}}(k)(Q_1 + Q_2$$
$$+Q_3 + Q_4 + Q_5 + Q_6 + (d_0 - d_1)R_1 + (d_2 - d_0)R_3 + d_0 R_2 + d_2 R_4)x(k)$$
$$-(1-\mu_1)x^{\mathrm{T}}(k-d_1(k))Q_1\dot{x}(k-d_1(k)) - x^{\mathrm{T}}(k-d_0)(Q_2 + Q_6)x(k-d_0)$$
$$-(1-\mu_2)x^{\mathrm{T}}(k-d_2(k))Q_4 x(k-d_2(k)) - x^{\mathrm{T}}(k-d_1)Q_3 x(k-d_1)$$
$$+2\dot{x}^{\mathrm{T}}(k)P_1\tau(k)A_{di}x(k-d_1(k)) + 2\dot{x}^{\mathrm{T}}(k)P_1(1-\tau(k))A_{di}x(k-d_2(k))$$
$$+\dot{x}^{\mathrm{T}}(k)\left((d_0 - d_1)S_1 + d_0 S_2 + (d_2 - d_0)S_3 + d_2 S_4 - 2P_1\right)\dot{x}(k)$$

$$-\frac{1}{d_0-d_1}\left[\int_{k-d_1(k)}^{k-d_1} x(s)ds\right]^{\mathrm{T}} R_1\left[\int_{k-d_1(k)}^{k-d_1} x(s)ds\right]$$

$$-\frac{1}{d_2-d_0}\left[\int_{k-d_2(k)}^{k-d_0} x(s)ds\right]^{\mathrm{T}} R_3\left[\int_{k-d_2(k)}^{k-d_0} x(s)ds\right]$$

$$-\frac{1}{d_2-d_0}\left[\int_{k-d_2}^{k-d_2(k)} x(s)ds\right]^{\mathrm{T}} (R_3+R_4)\left[\int_{k-d_2}^{k-d_2(k)} x(s)ds\right]$$

$$-\left[\int_{k-d_2}^{k} \dot{x}(s)ds\right]^{\mathrm{T}} S_4\left[\int_{k-d_2}^{k} \dot{x}(s)ds\right]$$

$$+2\dot{x}^{\mathrm{T}}(k)P_1B_{wi}(k)w(k)-\frac{1}{d_0-d_1}\left[\int_{k-d_0}^{k-d_1(k)} x(s)ds\right]^{\mathrm{T}} (R_1+R_2)\left[\int_{k-d_0}^{k-d_1(k)} x(s)ds\right]$$

$$-\frac{1}{d_0}\left[\int_{k-d_1(k)}^{k} x(s)ds\right]^{\mathrm{T}} R_2\left[\int_{k-d_1(k)}^{k} x(s)ds\right]-\frac{1}{d_0}\left[\int_{k-d_0}^{k} \dot{x}(s)ds\right]^{\mathrm{T}} S_2\left[\int_{k-d_0}^{k} \dot{x}(s)ds\right]$$

$$-\frac{1}{d_2}\left[\int_{k-d_2}^{k} x(s)ds\right]^{\mathrm{T}} R_4\left[\int_{k-d_2}^{k} x(s)ds\right]$$

$$-\frac{1}{d_0-d_1}\left[x(k-d_1)-x(k-d_1(k))\right]^{\mathrm{T}}(S_1+S_2)\left[x(k-d_1)-x(k-d_1(k))\right]$$

$$-\frac{1}{d_0-d_1}\left[x(k-d_1(k))-x(k-d_0)\right]^{\mathrm{T}} S_1\left[x(k-d_1(k))-x(k-d_0)\right]$$

$$-\frac{1}{d_2-d_0}\left[x(k-d_2(k))-x(k-d_2)\right]^{\mathrm{T}}(S_3+S_4)\left[x(k-d_2(k))-x(k-d_2)\right]$$

$$-\frac{1}{d_2-d_0}\left[x(k-d_0)-x(k-d_2(k))\right]^{\mathrm{T}} S_3\left[x(k-d_0)-x(k-d_2(k))\right]\}$$

In order to study the dissipative performance index of the system, we introduce the following relation

$$J(w,z,\tau)=\mathrm{E}\left\{\int_0^{\tau}\left[-z^{\mathrm{T}}(k)Qz(k)-2z^{\mathrm{T}}(k)Sw(k)-w^{\mathrm{T}}(k)(R-\alpha I)w(k)\right]d\tau\right\},$$

$$=\mathrm{E}\{\int_0^{\tau}[\dot{V}(k,x(k))-z^{\mathrm{T}}(k)Qz(k)-2z^{\mathrm{T}}(k)Sw(k)-w^{\mathrm{T}}(k)(R-\alpha I)w(k)$$

$$+V(0,x(0))-V(\tau,x(\tau))]dk\}$$

$$\le \mathrm{E}\{\int_0^{\tau}[\dot{V}(k,x(k))-z^{\mathrm{T}}(k)Qz(k)-2z^{\mathrm{T}}(k)Sw(k)-w^{\mathrm{T}}(k)(R-\alpha I)w(k)]dk\}$$

Under the zero initial condition,

$$J(w,z,\tau)\le \mathrm{E}\{\sum_{i=1}^{r}\sum_{j=1}^{r}\xi_i(\lambda(k))\xi_j(\lambda(k))\{\int_0^{\tau}\zeta^{\mathrm{T}}(k)\Pi_{ij}\zeta(k)\}dk\}.$$

Where

$$\zeta^{\mathrm{T}}(k)=\Big[x^{\mathrm{T}}(k)\quad x^{\mathrm{T}}(k-d_1)\quad x^{\mathrm{T}}(k-d_1(k))\quad x^{\mathrm{T}}(k-d_2(k))\quad x^{\mathrm{T}}(k-d_0)\quad x^{\mathrm{T}}(k-d_2)$$
$$\dot{x}^{\mathrm{T}}(k)\quad \int_{k-d_0}^{k-d_1(k)} x^{\mathrm{T}}(s)ds\quad \int_{k-d_1(k)}^{k-d_1} x^{\mathrm{T}}(s)ds\quad \int_{k-d_1(k)}^{k} x^{\mathrm{T}}(s)ds\quad \int_{k-d_2}^{k-d_2(k)} x^{\mathrm{T}}(s)ds\quad \int_{k-d_2(k)}^{k-d_0} x^{\mathrm{T}}(s)ds$$
$$\int_{k-d_2(k)}^{k} x^{\mathrm{T}}(s)ds\quad w^{\mathrm{T}}(k)\Big]$$

and

$$\Pi_{ij}=\begin{bmatrix}\Pi_{(m\times n)ij} & \Pi_{i1}^{\mathrm{T}}\\ * & -I\end{bmatrix},\qquad m,n=1,2,\ldots,14. \tag{10}$$

with

$$\Pi_{(1,1)ij}=Q_1+Q_2+Q_3+Q_4+Q_5+Q_6+(d_0-d_1)R_1+(d_2-d_0)R_3+d_0R_2$$
$$+d_2R_4-2P_1-C^{\mathrm{T}}(k)QC(k),$$

$$\Pi_{(1,3)ij}=-\tau_0C^{\mathrm{T}}(k)QC_d(k),\quad \Pi_{(1,4)ij}=-(1-\tau_0)C^{\mathrm{T}}(k)QC_d(k),$$

$$\Pi_{(1,14)ij}=-C^{\mathrm{T}}(k)QD_w(k)-2C^{\mathrm{T}}(k)S,\quad \Pi_{(2,2)ij}=-(Q_3+\frac{1}{d_0-d_1}S_1),$$

$$\Pi_{(2,3)ij}=\frac{1}{d_0-d_1}S_1,\quad \Pi_{(3,3)ij}=(1-\mu_1)Q_1-\tau_0^2C_d^{\mathrm{T}}(k)QC_d(k),$$

$$\Pi_{(3,5)ij}=\tau_0(1-\tau_0)C_d^{\mathrm{T}}(k)QC_d(k),\quad \Pi_{(6,5)ij}=(1-\tau_0)P_1A_d(k),$$

$$\Pi_{(5,5)ij}=-(1-\mu_2)Q_4-(1-\tau_0)^2C_d^{\mathrm{T}}(k)QC_d(k),\quad \Pi_{(6,3)ij}=\tau_0P_1A_d(k),$$

$$\Pi_{(6,6)ij}=-Q_5,\quad \Pi_{(6,13)ij}=P_1B_w(k),\quad \Pi_{(7,7)ij}=-\frac{1}{d_0-d_1}R_1,\quad \Pi_{(8,8)ij}=-\frac{1}{d_0-d_1}R_1,$$

$$\Pi_{(9,9)ij}=-\frac{1}{d_2-d_0}R_3,\quad \Pi_{(10,10)ij}=-\frac{1}{d_2-d_0}R_3,\quad \Pi_{(11,11)ij}=-\frac{1}{d_0}R_2,\quad \Pi_{(12,12)ij}=-\frac{1}{d_2}R_2,$$

$$\Pi_{(13,1)ij}=D_w(k)QC(k)-C^{\mathrm{T}}(k)S,\quad \Pi_{(13,3)ij}=\tau_0D_w^{\mathrm{T}}(k)QC_d(k)-\tau_0C_d^{\mathrm{T}}(k)S,$$

$$\Pi_{(13,5)ij}=(1-\tau_0)D_w^{\mathrm{T}}(k)QC_d(k)-(1-\tau_0)C_d^{\mathrm{T}}(k)S,$$

$$\Pi_{(13,13)ij}=D_w^{\mathrm{T}}(k)QD_w(k)-2D_w(k)S-R+\alpha I.$$

In order to obtain the state feedback controller gain matrix, take $P_1=\lambda P$, where λ is the designing parameter and let pre- and post-multiply(10) by diag $\{\underbrace{X,\ldots,X}_{14},I\}$, and denote $\tilde{Q}_i=XQ_iX, i=1,2,\ldots,6, \tilde{R}_i=XR_iX, i=1,2,\ldots,4,$ and $K_j=Y_jX^{-1}$.

4. Numerical example

In this part, we will present two following example simulations to prove the validity and flexibility of the results developing in this paper.

Example 1: Considering the system (1) with the number of IF-THEN rules $r=2$ and the system model is represented as follows:

Plant rule1: if $x_1(k)$ is $h_1(x_1(k))$, THEN

$$\dot{x}(k) = A_1 x(k) + A_{d1} x(k-d(k)) + B_1 u(k) + B_{w1} w(k)$$
$$z(k) = C_1 x(k) + C_{d1} x(k-d(k)) + D_1 u(k) + D_{w1} w(k)$$

Plant rule2: if $x_1(k)$ is h_2, THEN

$$\dot{x}(k) = A_2 x(k) + A_{d2} x(k-d(k)) + B_2 u(k) + B_{w2} w(k)$$
$$z(k) = C_2 x(k) + C_{d2} x(k-d(k)) + D_2 u(k) + D_{w2} w(k)$$

Where $x(k)=\left[x_1(k),\ x_2(k),\ x_3(k)\right]^{\mathrm{T}}$, $x_i(k) \in \mathbb{R}^1, i=1,2,3$. The membership functions may be chosen as

$$h_1(x_1(k)) = \frac{1+\cos(x_1(t))}{2}, h_2(x_1(k)) = \frac{1-\cos(x_1(t))}{2}.$$

Where

$$A_1 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix},\quad A_2 = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix},\quad A_{d1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},\quad A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},\quad C_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix},\quad C_{d1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix},\quad C_{d2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 0.5 \\ 0.8 & 1 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 1.8 \\ 1.1 & 2.1 \end{bmatrix},\quad D_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.5 \end{bmatrix},\quad D_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.4 \end{bmatrix}$$

$$B_{w1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\quad B_{w2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},\quad D_{w1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},\quad D_{w2} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix},$$

$$M_1 = M_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\quad H_{1i} = H_{2i} = H_{3i} = H_{4i} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},\quad i=1,2.$$

Choose

$$Q = \begin{bmatrix} -0.01 & 0 \\ 0 & -0.01 \end{bmatrix},\quad S = \begin{bmatrix} 2 & 1 \\ 1 & 1.5 \end{bmatrix},\quad R = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix}$$

By solving linear matrix inequalities(11)、(12), controller gain matrix are as follows

$$K_1 = \begin{bmatrix} 0.6606 & -0.4008 \\ -0.1787 & 1.3856 \end{bmatrix},\quad K_2 = \begin{bmatrix} 1.3674 & -0.4747 \\ -0.3877 & 0.1627 \end{bmatrix}$$

5. Conclusion

In this paper, the problem of a class of T-S fuzzy system has been investigated with random input time-delay and linear fractional uncertainties. Sufficient conditions has been acquired to ensure the closed-loop system is robustly asymptotically stable with a given dissipative performance. It should be mentioned that the obtained dissipative performance conditions are established in terms of LMIs which can be easily solved via MATLAB LMI tool box. Finally, a numerical example based on truck-trailer model has been given to illustrate the effectiveness of the proposed state feedback controller. Further, the discrete-time fuzzy systems with random delay governed by Poisson process or Levy noise is an untreated topic. Our future work will be improving the existing techniques and finding new methods to deal the above issues.

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