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# Application of Game Theory in Joint Beamforming and Power Allocation Optimization

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## Abstract

With the growing mobile communication network service requirements and the growing shortage of spectrum resources, D2D emerges as the times require. Due to the short transmission distance, introducing D2D into the cellular mobile communication network can receive fewer base station loads, higher transmission rates, lower latency and less transmit power. The D2D users share the radio resources of the cellular users by multiplexing. As a result of that, the spectral efficiency is improved. Firstly, this article mainly analyzes the problem of beamforming and power distribution when the D2D users and the cellular users exist in the MIMO cellular system simultaneously. Secondly, in order to enable each D2D user to be assigned the optimal beamforming vector and power distribution ratio, furthermore, maximize the effectiveness function of the D2D users, joint optimization of beamforming and power allocation based on Game Theory is provided according to the perfect CSI feedback of D2D users. Finally, it is proved that the effectiveness function satisfies the Nash equilibrium with the perfect CSI feedback through the established Game model.

## Keywords

D2D; Beamforming; Power Allocation; Perfect CSI.

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## 1. Introduction

Because of multimedia services needs and the number of mobile communication network equipment are increasing dramatically with the development of Mobile Internet (MI), shortage of spectrum resources has become the challenge of mobile communication. In order to improve spectrum utilization, Device-to-Device (D2D) technology emerges as the times require. D2D was first proposed by Qualcomm Corp in 2008 [1]. Currently D2D has become the key technology of the fifth generation mobile communication (5G) system [2].

D2D communication refers to under the control of the base station, geographically adjacent terminal equipments transfer data over a direct link instead of forwarding through the base station [1]. With the growth of localized mobile data traffic, there are more and more traffic data generated by close terminal equipments. But in conventional mobile cellular network, direct communication between terminal equipments is not allowed. This tends to lack flexibility and difficult to ensure real-time and reliability between different services [3]. The D2D communication between mobile devices solves this problem effectively. In the In-band D2D mode, D2D users multiplexes the cell resources to communicate under the control of the base station. So that it is necessary to allocate the radio resources

effectively and control interference and power strictly. For mobile cellular MIMO systems, this article discusses the D2D power and resource allocation scheme based on non-cooperative game theory.

In a cellular multi-user MIMO system, the interference between cellular users and D2D users can be reduced by beamforming and power allocation, further, system capacity can be effectively improved. The base station can perform beamforming and power allocation only if obtaining the Channel State Information (CSI) [4-6]. Assume that the D2D users cooperates with the base station, D2D users feeds the perfect CSI to the base station, and the base station reduces the interference to the D2D users by adjusting the beamforming vector and the power allocation ratio [7].

This article considers in the case of perfect CSI, the optimal beamforming vector and power allocation ratio is assigned to each D2D user for D2D users reach their maximum effectiveness. It is assumed that the constraints of the cellular mobile communication system include the following three points: (a) The interference threshold of the D2D user to the cellular user is less than the interference threshold of the cellular user; (b) The total transmit power of the cellular MIMO system is less than the maximum transmit power of the base station; (c) To ensure the quality of service (QoS) and fairness D2D users, the signal to interference plus noise ratio (SINR) of each D2D user is greater than the pre-set interference threshold.

In the case of CSI perfect feedback, it is the focus of this article that beamforming and power allocation in cellular MIMO systems with game theory. First, the game model is established, and the game utility function with CSI perfect feedback is given. Then, it is proved that the effectiveness function satisfies the Nash equilibrium with the perfect CSI feedback.

## 2. System Model

MIMO cellular system model shown in Fig. 1. This system includes a Base Station (BS),  $K$  D2D users ( $D2D_1, \dots, D2D_k$ ) and a cellular user ( $CUE_1$ ). The number of antennas of the BS is  $N_t$ , each D2D user and cellular user  $CUE_1$  have only one antenna.

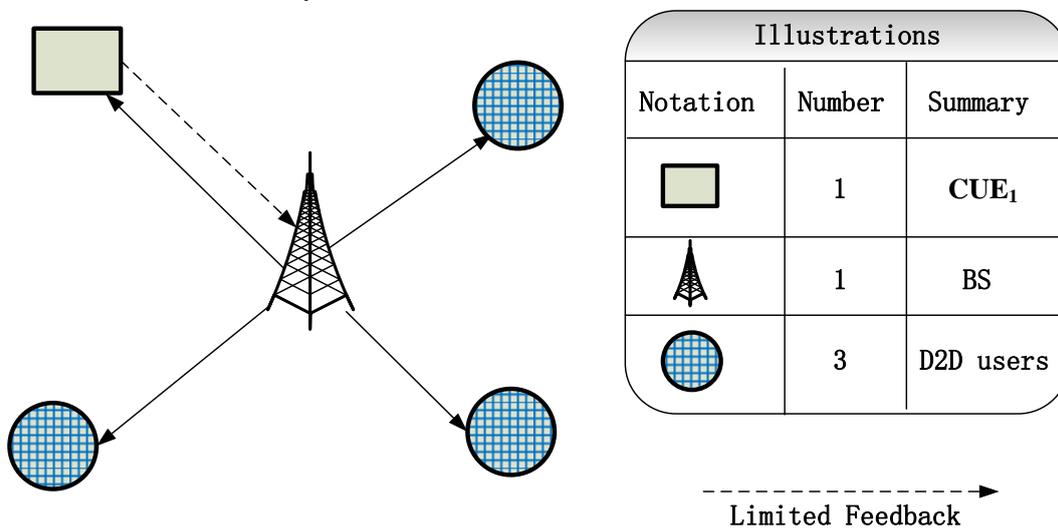


Fig. 1 Cellular MIMO system model

BS has the right to use limited spectrum and strong computing power. So it can eliminate the interference by pretreatment. Assuming that all users know CSI, and D2D users can feed their perfect CSI to the BS. The transmit signal of BS is

$$\mathbf{X} = \mathbf{F}\mathbf{S} \tag{1}$$

Where  $\mathbf{S} = [s_1, s_2, \dots, s_K]^H$  is the transmitted signal vector and  $s_k$  is the signal sent by the BS to  $D2D_k$ ;  $\mathbf{F} = [f_1, f_2, \dots, f_k] \in \mathbb{C}^{N_t \times K}$  is the beamforming matrix and  $f_k$  is the beamforming vector of  $D2D_k$ . The

power distribution matrix of the transmitted signal is  $\mathbf{P} = \text{diag} \{ \sqrt{p_1}, \dots, \sqrt{p_k} \}$ . Assuming that Channel fading that all users experiencing is independent and identically distributed.

The received signal of D2D<sub>k</sub> is

$$\mathbf{y}_k = h_k \mathbf{X} + n_k = \sqrt{p_k} h_k f_k s_k + h_k \sum_{i=1, i \neq k}^K \sqrt{p_i} f_i s_i + n_k. \tag{2}$$

Where the channel coefficient form BS to D2D<sub>k</sub> is  $h_k \in \mathbb{C}^{1 \times N_t}$ . Assume that the channel obeys the independent and identically Rayleigh distribution with mean 0 and variance 1.  $n_k$  is the additive white Gaussian noise with mean 0 and variance  $\sigma_k^2$ .

The received signal of CUE<sub>1</sub> is

$$\mathbf{y}_p = s_p + h_p \sum_{i=1}^K \sqrt{p_i} f_i s_i + n_p. \tag{3}$$

Where the channel coefficient form BS to CUE<sub>1</sub> is  $h_p \in \mathbb{C}^{1 \times N_t}$ .  $s_p$  is effective signal received by CUE<sub>1</sub> with mean 0 and variance  $\sqrt{p_n}$ .  $n_p$  is the additive white Gaussian noise with mean 0 and variance  $\sigma_n^2$ .

The signal to noise ratio of D2D<sub>k</sub> is  $SINR_k$  and the signal to noise ratio of CUE<sub>1</sub> is  $SINR_p$ , which are

$$\left\{ \begin{aligned} SINR_k &= \frac{p_k |h_k f_k|^2}{\sum_{i=1, i \neq k}^K p_i |h_k f_i|^2 + \sigma_k^2} \\ SINR_p &= \frac{p_p}{\sum_{i=1}^K |h_p f_i|^2 + \sigma_p^2} \end{aligned} \right. \tag{4}$$

In order to ensure the QoS of all D2D users, the SINR received by each D2D user should higher than the set threshold  $\gamma_k$ .

$$SINR_k \geq \gamma_k \tag{5}$$

In addition, in order to ensure the QoS of all cellular users, the interference received by the cellular users should less than the set threshold  $I_{th-obit}$ .

$$\sum_{k=1}^K p_k |h_p f_k|^2 \leq I_{th-obit} \tag{6}$$

### 3. Game Model

Optimization problems can be competitive with the knowledge of game theory to analyze and solve. In a wireless MIMO system, D2D users compete with each other to share the resources of cellular users, but at the same time also increased the interference between each other. Each D2D user is both rational and selfish. In order to maximize their own interests, they will not consider the interference to other D2D users. Therefore, a cost function is needed to ensure that the utility function reaches the Nash equilibrium. A non-cooperative game model can be expressed as

$$G = \{ \Omega, \{ p_k \}_{k \in \Omega}, \{ u_k \}_{k \in \Omega} \} \tag{7}$$

Where game participants are all  $K$  D2D users, which is  $\Omega = \{ D2D_1, \dots, D2D_K \}$ , and policy space is the power allocated  $p_k$  for each D2D user.

Gain function of the SINR, which is received by the  $k$ th D2D user is defined as

$$u_k = SINR_k \quad (8)$$

According to game theory point of view, we believe that the participants have selfishness. D2D users will increase its transmission power as much as possible, to maximize the benefits of their own. Therefore, here are one kind of utility function having penalties to prevent D2D user selfish behavior, which is

$$u_k = SINR_k - p_k^2 |h_p f_k|^2 \quad (9)$$

According to the constraints of the cellular mobile communication system we mentioned in the first part, the non-cooperative game problem is expressed as

$$\begin{aligned} \max \quad & u_k, \quad \forall k \in \Omega \\ \text{s.t.} \quad & \sum_{k=1}^K p_k |h_p f_k|^2 \leq I_{th-obit} \\ & \sum_{k=1}^K p_k \leq P_T \\ & SINR_k \geq \gamma_k \end{aligned} \quad (10)$$

Where  $p_k$  is the policy space of  $k$ th D2D user and  $P_T$  is the maximum transmit power of BS. Under the premise of satisfying three conditions, D2D users can choose the optimal transmission power to maximize the utility function.

#### 4. Analysis of Nash Equilibrium

The steady state achieved by the non-cooperative game process is called Nash Equilibrium. Satisfy the following condition is determined the Nash equilibrium is reached [8, 9]:

- Participants are limited collections.
- The strategy set is convex, closed and bounded.
- The utility function is quasi-concave function and continuous in space.

There are  $K$  D2D users participating in the game, so participants is a finite set and condition (a) is satisfied. Policy set is transmitted power  $p_k$ . The power of the cellular system is limited and the transmit power of BS is bounded, so the policy set is convex and closed and condition (b) is satisfied. Because of utility function is a simple function of a set of policies, the utility function is a continuously derivable function. Let  $f_k$  unchanged, there is

$$\frac{\partial^2 u_k}{\partial p_k^2} = -2 |h_p f_k|^2 \leq 0 \quad (11)$$

So the utility function is a convex function and condition (c) is satisfied. From the above analysis shows that Nash equilibrium of the game model given in this paper exists with perfect CSI feedback and given a beamforming vector.

#### 5. Conclusion

With the development of mobile Internet, the total amount of mobile devices has increased dramatically. D2D is introduced into the wireless communication system due to its communication flexibility. D2D as a key technology of 5G can effectively reduce the burden on the base station, and get higher transmission rates, lower latency and smaller transmit power. D2D communication shares the resources of the cellular user by multiplexing, thus improving the system's spectrum utilization. In this paper, game theory is applied to solve the problem of beamforming and power allocation in cellular MIMO systems with perfect CSI feedback. An utility function having penalties is presented and its Nash equilibrium is proved to exist, through the establishment of the game model.

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