Based on the Short Span Beam Bridge and the Theoretical Analysis and Experimental Tests of Its Vibration Characteristics

Da Xu [a], Baohong Han [b], Zhaogang Cheng [c], Wanheng He [d]

Ordnance Engineering College, Shijiazhuang 050003, China

[a]18663510780@163.com, [b]13393981@qq.com, [c]624441609@qq.com, [d]jxxyczg@126.com

Abstract

The calculation of the natural frequency of the transverse vibration of the beam with the cross section and continuous structure, is great significance in practical engineering. According to the structure of the short span beam bridge, simplified the model, and the model was simplified as a simply supported beam. Analysis of the natural frequency of the system, to avoid the occurrence of resonance phenomena in practical applications. The natural frequency of the simply supported beam that was got by frequency sweep method, compared with the theoretical value of continuum system and discrete system, the result are roughly same. The error was within 4%, and further analyzed error sources. At the same time by hammering method to get the first two order modal shape. Starting from the natural frequency of the simply supported beam, it provided a reference method for analyzing the vibration of complex structures in engineering applications.

Keywords

Beam bridge, Simply supported beam, Natural frequency, Vibration analysis.

1. Introduction

With the development of science and technology, the quality, accuracy and reliability of the equipment are put forward higher requirements in engineering. In modern engineering design, more and more dynamic analysis is considered, and the vibration analysis is one of the main aspects of dynamic analysis. So vibration analysis has become an indispensable part of the project. The engineering application of beam as a common structure form, its vibration characteristics is the most common in mechanical vibration, bridge, ship's keel, aircraft wings, can be simplified as a beam to explore. So the research on the vibration characteristics has very important practical significance.

In this paper, the short span beam bridge was simplified to a simple supported beam model, and then studied the vibration characteristics of a simple supported beam system. Firstly, the simple beam system was used as a continuum to get the modal function expression of the beam, and then the natural frequency was solved through the specific boundary conditions. Then, the simple supported beam was discretized to three degrees of freedom, and the first order natural frequency of the structure was obtained by the lumped mass method. By using the method of theoretical analysis and laboratory test, studied the dynamic analysis, found the natural frequency of simple supported beam system vibration, and compare the theoretical calculation and experimental results, analyzed of the first three natural frequencies, to provide a reference for the vibration analysis of complex mechanism.

2. Model Simplified Parameter Setting for Small Span Beam Bridge

In Fig.1, is the schematic diagram of small span beam bridge, which was simplified as a simple supported beam, in order to analyze the transverse vibration of the beam bridge. In the course of

vibration, the axial displacement of the beam was neglected, the cross section was neglected, and the cross section of the simply supported beam was always perpendicular to the axis of the beam.

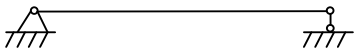

Fig. 1 Constant cross-sectional simple beams structure

Measured by measuring equipment:
L =640mm, b =56mm, h =8mm,
Cross-sectional area: $A = bh$,
The material for the beam is plain carbon steel, check list:
Material density: $\rho = 7.8\times10^{3}\,\mathrm{kg/m^3}$,
Elastic modulus: $E = 2.1\times10^{11}\,N/\mathrm{m}^2$,
Center principal moment of inertia: $I = \frac{1}{12}bh^3$
The simply supported beam quality: $M = \rho AL$

3. Theoretical Analysis of Simply Supported Beam Continuum Model

The length of a simply supported beam is L, $y(x,t)$ indicates lateral displacement, it is two element function of the section position x and the time t. The transverse force acting on the unit length of the beam by $q(x,t)$. The parameters of the system are the unit volume quality $\rho(x)$, the cross-sectional area $A(x)$, the flexural rigidity $EI(x)$. Take the micro section dx, using $Q(x,t)$ to express the shear force, $M(x,t)$ expresses the bending moment, as shown in figure 2.

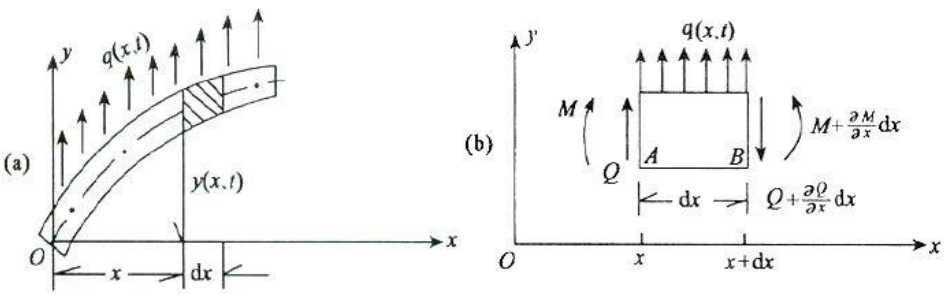

Fig.2 Bending vibration mode of beams

The motion equation of the Y direction of the lead is:

$$\left(M+\frac{\partial M}{\partial x}dx\right)-M-\left(Q+\frac{\partial Q}{\partial x}dx\right)dx+q(x,t)\,dx\frac{dx}{2}=0 \tag{1}$$

Ignoring the influence of section rotation, rotation equation of micro segment for

$$\left(M+\frac{\partial M}{\partial x}dx\right)-M-\left(Q+\frac{\partial Q}{\partial x}dx\right)dx+q(x,t)\,dx\frac{dx}{2}=0 \tag{2}$$

Neglecting high order quantities combining the relation of material mechanics, the bending moment and the deflection of the beam to get the partial differential equation of vibration

$$\rho(x)A(x)dx\frac{\partial^2 y(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right]=q(x,t) \tag{3}$$

The fixed end displacement and rotation angle are equal to 0 if $q(x,t)$=0, i.e., the partial differential equation for the free vibration of the beam is

$$\rho A dx \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right]=0 \tag{4}$$

The main vibration of the beam can be assumed as

$$y(x,t)=Y(x)F(x) \tag{5}$$

Two differential equations are obtained

$$\frac{\partial^2 F(t)}{\partial t^2} + \omega^2 F(t) = 0 \tag{6}$$

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2 Y(x,\mathrm{t})}{dx^2}\right] - \omega^2 \rho(x) A(x) Y(x) = 0 \quad (0 < x < L) \tag{7}$$

Then the general solution of (6) equation

$$F(t) = A\sin\omega t + B\cos\omega t = C\sin(\omega t + \varphi) \tag{8}$$

A, B is an integral constant, which is determined by two initial conditions.

By equation (8), we can get the general expression of the modal function, and the modal function $Y(x)$ must satisfy the boundary conditions.

If $\rho(x)$, $A(x)$ for the constant, the moment of inertia of the central principal axis $I(x)$ is constant, then the equation (7) is simplified as

$$\frac{d^4 Y(x)}{dx^4} - \beta^4 Y(x) = 0 \tag{9}$$

among

$$\beta^4 = \frac{\omega^2 \rho A}{EI} \quad \backslash$$

Solution of four order linear ordinary differential equations with constant coefficients:

$$Y(x) = C_1 \sin\beta x + C_2 \cos\beta x + C_3 sh\beta x + C_4 ch\beta x \tag{10}$$

This is the vibration mode function of the beam, in which, C_1, C_2, C_3, C_4 for the integral constant, four boundary conditions can be used to determine the 3 integral constants and the characteristic equation, so as to determine the natural frequency ω of the beam bending vibration. The boundary conditions of simply supported beam

$$Y(0)=0, \quad \frac{d^2 Y(0)}{dx^2} = 0, \quad Y(L)=0, \quad \frac{d^2 Y(L)}{dx^2}=0 \tag{11}$$

get the natural frequency

$$\omega_r = \frac{r^2 \pi^2}{L^2}\sqrt{\frac{EI}{\rho A}}, (r = 1, 2, 3\cdots) \tag{12}$$

The natural frequency of simply supported beam of continuum system can be calculated by the relevant parameters.

4. Simple Beam Discrete System Theory Analysis

Continuum system with infinite degrees of freedom, so the research on the vibration characteristics of the continuum systems is very difficult, in most cases, it is very difficult to obtain the analytic solution, so

some simple models can be obtained based on the idea of discretization. In this experiment, used the lumped mass method, the experimental beams were divided into three sections, simplified to three degrees of freedom vibration system model, as shown in Fig 3, established the three mutually coupled ordinary differential equations, solved the eigenvalue of the equation, obtained the natural frequency. The total mass of the beam is M ,and the quality of the beam is focused on three mass blocks, that is $m = M/3$, $F(t)$ is the exciting force generated by the vibration exciter.

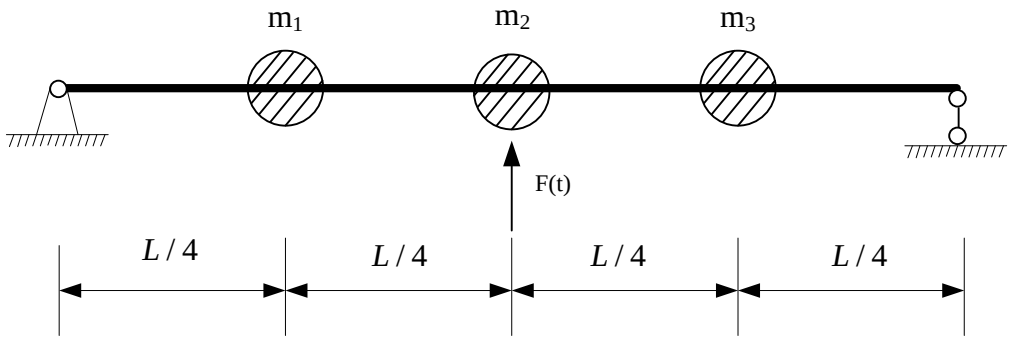

Fig.3 Simplified simple supported beam with three degrees of freedom

In the dynamic problem, the matrix form of the motion differential equation of n degrees of freedom linear system is

$$M\ddot{x} + C\dot{x} + Kx = F \tag{13}$$

M , C , K respectively as mass matrix, damping matrix and stiffness matrix of the system, and the damping matrix is not considered.

Through structural mechanics, for inverse matrix of the stiffness matrix, the flexibility matrix can be drawn on the stiffness matrix, the experimental simply supported beam is uniform beam, through the reciprocal theorem in structural mechanics

$$\delta_{ij} = \delta_{ji} \tag{14}$$

The flexibility matrix can be obtained by graph multiplication

$$\delta = \frac{l^3}{768EI}\begin{bmatrix} 9 & 11 & 7 \\ 11 & 16 & 11 \\ 7 & 11 & 9 \end{bmatrix}$$

$q' = \dfrac{l^3}{768EI}$, the stiffness matrix $K = \delta^{-1}$:

$$K = \frac{1}{28q'}\begin{bmatrix} 23 & -22 & 9 \\ -22 & 32 & -22 \\ 9 & -22 & 23 \end{bmatrix}$$

also

$$M = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The instantaneous displacement of m1, m2, m3 mass blocks are x_1, x_2, x_3, so the motion differential equations of system is:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \frac{1}{28q'} \begin{bmatrix} 23 & -22 & 9 \\ -22 & 32 & -22 \\ 9 & -22 & 23 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ F(t) \\ 0 \end{bmatrix} \tag{15}$$

The natural frequency of the simply supported beam was obtained by solving the characteristic equation of equation(15) by MATLAB, and compared with the theoretical analysis of the continuum system.

5. Experimental Test

5.1 Experimental Principle

From the theoretical analysis, it is known that the simply supported beam is a continuum system, so there are infinite number of natural frequencies and natural modes. If the excitation force is applied to a simply supported beam, the frequency of the exciting force is close or exactly equal to the natural frequency of a simply supported beam, resonance will occur.

5.2 Laboratory Equipment

The test system was composed of a signal generator, a power amplifier, a vibration exciter, a simple supported beam, an acceleration sensor, a INV3018 acquisition system and a computer, as shown in figure 4.

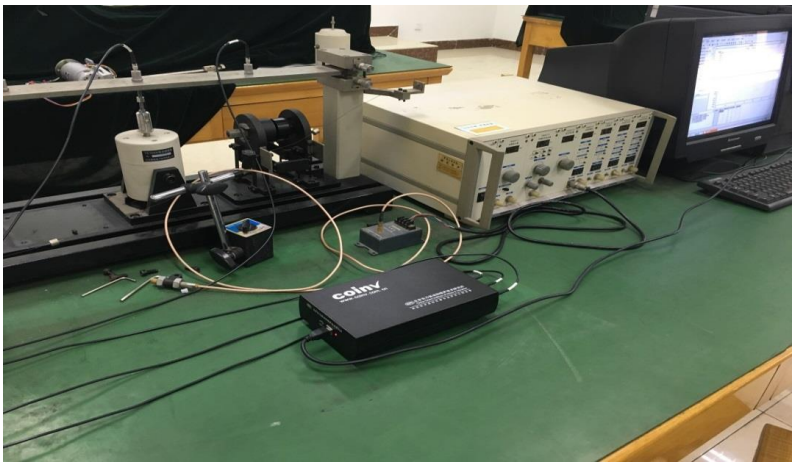

Fig.4 Simply supported beam testing system

5.3 Experimental Methods

The test system used the INV3018 acquisition instrument of the Oriental vibration and noise Technology Research Institute, and the frequency range was from 10 Hz to 500 Hz by using the sine sweep frequency method. Signal generated by the signal generator, through the power amplifier to amplify the signal, and then the vibration exciter was actted on the simple beam The acceleration sensor was mounted on the corresponding position of the simply supported beam, and the vibration signal was collected and input to the computer by the acquisition instrument, then started analysis. Vibration test schematic diagram, as shown in figure 5.

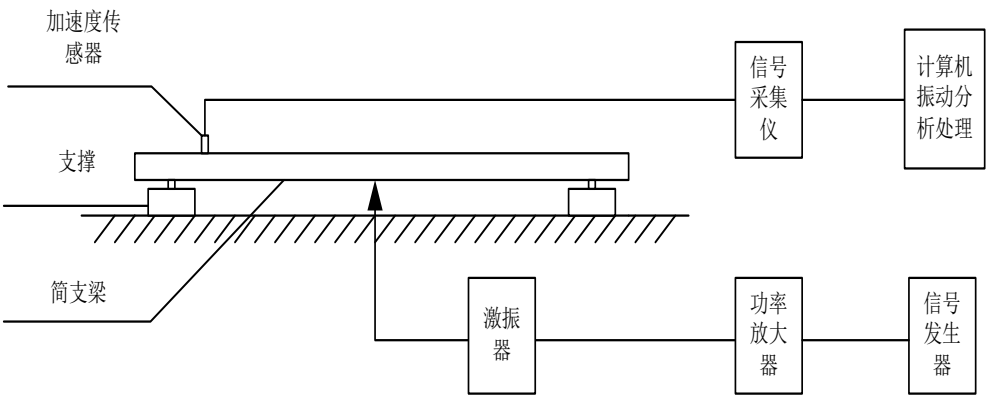

Fig.5 Vibration test principle diagram

5.4 Data Analysis

The analytical value of the theoretical analysis about the simply supported beam continuum system was compared with that of the simple supported beam discrete system, as shown in Tab 1.

Tab.1 Theoretical value and measured value of natural frequency

Frequency	First order	Second order	Third order
Continuous system/Hz	44.7	178.8	402.3
Discrete system/Hz	45	181.3	410
Error/%	0.7	1.4	4.4

The beam was discretized into three degrees of freedom vibration system, used the lumped mass method to solve the first three natural frequencies of the beam, and the continuum system comparison results showed that the first two natural frequencies were basically the same, but with the increase of the order, the error increases. In order to reduce the error, the mass distribution of the beam is more refined, and the order of the equation (15) is increased, and then the characteristic equation was solved by Matlab. The smaller order, the higher accuracy, which provides a simple and convenient calculation method for the complex problems in engineering, and saves a lot of work time.

The collected vibration signal was introduced into the DASP visual modal analysis software, and got the full name waveform diagram as shown in Figure 6. Through the analysis of the software FFT transform, got the natural frequency, as shown in figure 7.

The first three order natural frequencies 46Hz, 183Hz and 395Hz were obtained by experimental measurement. The theoretical value and measured value as shown in Tab 2and Tab 3.

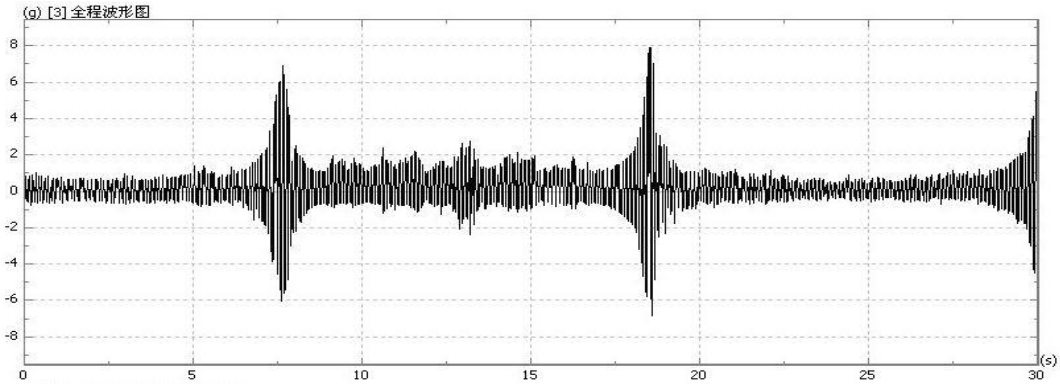

Fig.6 Full waveform graph

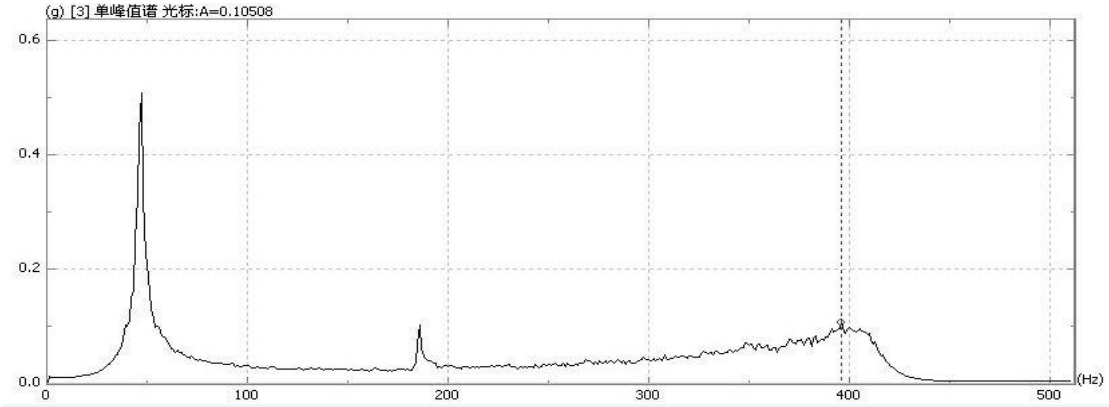

Fig.7 Spectrum analysis

Tab.2 Theoretical value and measured value of the natural frequency of continuous

Frequency	First order	Second order	Third order
Continuous system/Hz	44.7	178.8	402.3
Measured value/Hz	46	183	395
Error/%	2.9	2.3	1.8

Tab.3 Theoretical value and measured value of the natural frequency of the discrete system

Frequency	First order	Second order	Third order
Continuous system/Hz	45	181.3	410
Discrete system/Hz	46	183	395
Error/%	2	0.9	3.7

It is known that the natural frequency was obtained by the experimental measurement is almost the same as the theoretical analysis, and the frequency error range was 1.5% ~ 4%. In the experiment, the factors which influenced the experiment result are quite many, it is impossible to reach the theoretical condition, analyzed the error, the acceleration sensor was fixed on the simply supported beam, and the total weight of the simply supported beam is increased.

$$f' = \sqrt{m/(m+m_t)} \bullet f \tag{16}$$

In addition, f' is coupled with the natural frequency of the sensor, m_t is the quality of the acceleration sensor. Visible, plus the sensor will make the measurement value small.

The density and elastic modulus of simply supported beam and the actual value of the material parameters have the error, which affects the accuracy of the theoretical calculation. The simple supported beam experimental platform is not stable, easy to produce the interference signal, and at the same time, the excitation force is not stable, and the response signal is affected. The vibration isolation measure of the experiment failed to completely eliminate the negative impact of the vibration source. Various factors affect the damping, which was not considered in the theoretical calculation, but in the actual measurement, there was the influence of the damping, which resulted in the error of the experiment. The temperature of the system and the level of the operator can also affect the results. The first two natural modes obtained by hammering method are shown in Figure 8 and 9.

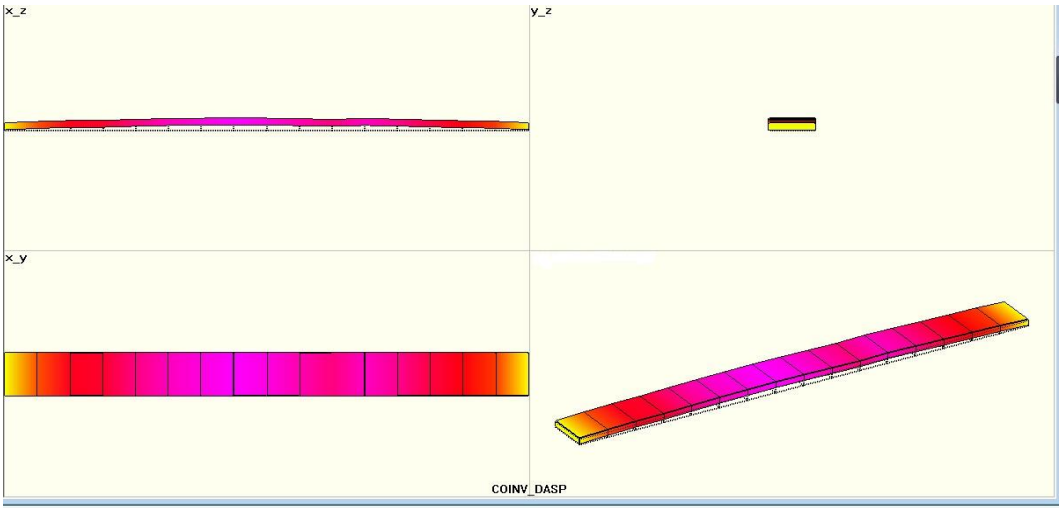

Fig.8 First order mode

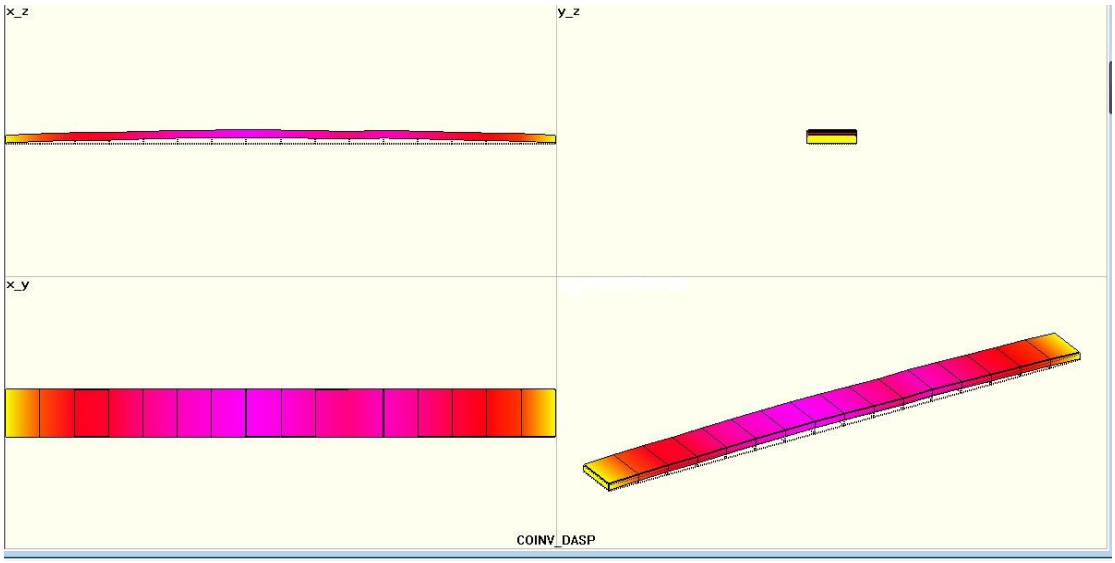

Fig.9 Second order mode

6. Conclusion

It is not easy to direct analyze and measured the small span beam bridge, we can simplify its model, simplified into uniform beam structure. By means of theoretical analysis and experimental measurement of three degree of freedom vibration system, the first three order natural frequencies of simply supported beam are obtained. Compared them we found than the continuum system and the discrete system error was small, provided a simple and convenient method for complex engineering problems. However, experimental measurements due to the presence of factors can not be eliminated, there was a deviation between the theoretical value, but the error in the acceptable range, can also be considered consistent with the experimental data the theory of value. By measurement and analysis of equal cross section beam modal parameters, to deepen the understanding of the beam vibration characteristics, vibration analysis in engineering vibration problems of beams and structure like beam, and thus avoid the resonance phenomenon, provided a scientific method, lay the foundation for the further study of complex structure vibration. For some structures which need to be subjected to mechanical shock and vibration, the method of modal analysis is carried out by experimental modeling, which has become an important tool in the system analysis and design of engineering structures. For complex problems, the use of high performance instruments is more conducive to improve work efficiency.

References

[1] Y.M. Zhang: Mechanical Vibration (Tsinghua University Press, 2007), (In Chinese)
[2] B. Qian, J.Y. Gao: Numerical calculation of natural frequency of transverse vibration of beam, Journal of Shihezi University, Vol. 26 (2008) No.1 ,p. 87-90. (In Chinese)
[3] L.L. Dai: Laboratory measurement and theoretical analysis of natural frequencies and natural modes of simply supported beam, Journal of Kunming College, Vol. 30 (2008) No.4 ,p. 86-89. (In Chinese)
[4] W.M. Li, W. He, L. Yang: Theoretical analysis and experimental study on vibration characteristics of simply supported beam, Journal of Guizhou Educational College, 2007 No.4 ,p, 44-49. (In Chinese)
[5] H.W. Liu: Mechanics of Materials (Higher Education Press, 2004), (In Chinese)
[6] Y.Y. Huang, Y.G. Zhao: Natural frequency analysis of simply supported beam, Journal of Qinghai University, Vol. 28 (2010) No.6, p. 20-23. (In Chinese)
[7] W. Li, J.J. Chen: Determination and analysis of modal parameters of simply supported beam [J]. Journal of Beijing Union University: Vol. 25 (2011) No. 2 , p. 20-23. (In Chinese)
[8] X.J. Liu, Q.F. Jia: Engineering Vibration theory and testing technology (Higher Education Press, 2004), (In Chinese)
[9] R.E. Coleman, R.J. Allenmang: Experimental Structural Dynamics (Tsinghua University Press, 2012).