Assessment of Residential Building Energy-Saving with Intuitionistic Fuzzy Information

Dexue Deng*, Xinrui Xu, Linggang Ran

School of Civil Engineering, Chongqing University of Arts and Sciences, Yongchuan, 402160, Chongqing, China

*Corresponding author, E-mail: 617230150@qq.com

Abstract

With the acceleration of the process of our country's urbanization, the investment of urban infrastructure construction continues to increase and the construction industry has become a pillar industry of the national economy. However, the proportion of building energy consumption in the total energy is increasing at the same time. In order to achieve the sustainable development of the society, the energy-saving target is proposed in the12th Five-Year Plan of China, which sets for the requirements for building energy-saving in Shaanxi Province. Evaluation of building energy-saving is an effective method to promote the healthy development of the work of building energy efficiency, but there is not a complete, reasonable and practicable the evaluation system. In this paper, we investigate the multiple attribute decision making problems for evaluating the residential building energy-saving with intuitionistic fuzzy variables. We utilize the intuitionistic fuzzy correlate geometric mean (IFCGM) operator to aggregate the intuitionistic fuzzy numbers corresponding to each alternative and get the overall value of the alternatives, then rank the and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example is given.

Keywords

Multiple attribute decision making (MADM); intuitionistic fuzzy information; intuitionistic fuzzy correlate geometric mean (IFCGM) operator; interval-valued residential building energy-saving

1. Introduction

In urban areas, energy-saving design is more and more attentioned, but relatively remote and backward rural areas, building energy-saving design is almost a distant subject, residential construction is still in the traditional way. City project involves the number of immigrants mountains, large new residential area, in order to reduce energy consumption, energy-saving design of immigrant housing is very important. Through field visits to investigate City mountain immigration residential status and energy consumption profiles, and sub-immigrant village house for the object winter indoor thermal environment tests, while sub-immigrant village, street acupuncture immigrant village Tobu ditch resettlement village50villagers for the object winter indoor thermal environment subjective survey results show that high immigration residential energy consumption, winter indoor thermal comfort is poor. In order to implement the policy of building energy efficiency improves energy efficiency of homes, giving residents a comfortable and healthy living environment while minimizing energy consumption.

In this paper, we investigate the multiple attribute decision making problems[1-12] for evaluating the residential building energy-saving with intuitionistic fuzzy variables. We utilize the intuitionistic

fuzzy correlate geometric mean (IFCGM) operator to aggregate the intuitionistic fuzzy numbers corresponding to each alternative and get the overall value of the alternatives, then rank the and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example is given.

2. Preliminaries

In the following, we introduce some basic concepts related to IFS.

Definition 1. An IFS A in X is given by

$$A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle\middle|x\in X\right\} \tag{1}$$

where $\mu_A: X\to[0,1]$ and $\nu_A: X\to[0,1]$, with the condition $0\le\mu_A(x)+\nu_A(x)\le 1$, $\forall\, x\in X$ .The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non- membership degree of the element x to the set A [13].

Definition 2. Let $\tilde{a}=(\mu,\nu)$ be an intuitionistic fuzzy number, a score function S of an intuitionistic fuzzy value can be represented as follows [14]:

$$S(\tilde{a})=\mu-\nu,\quad S(\tilde{a})\in[-1,1]. \tag{2}$$

Definition 3. Let $\tilde{a}=(\mu,\nu)$ be an intuitionistic fuzzy number, a accuracy function H of an intuitionistic fuzzy value can be represented as follows [15]:

$$H(\tilde{a})=\mu+\nu,\quad H(\tilde{a})\in[0,1]. \tag{3}$$

to evaluate the degree of accuracy of the intuitionistic fuzzy value $\tilde{a}=(\mu,\nu)$, where $H(\tilde{a})\in[0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the intuitionistic fuzzy value $\tilde{a}$.

For a collection of IFVs $\tilde{a}_j=(\mu_j,\nu_j)(j=1,2,\cdots,n)$, then

(1) The intuitionistic fuzzy weighted geometric (IFWG) operator [16]:

$$\mathrm{IFWG}_{\omega}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\overset{n}{\underset{j=1}{\otimes}}(\tilde{a}_j)^{\omega_j}=\left(\prod_{j=1}^{n}\mu_j{}^{\omega_j},1-\prod_{j=1}^{n}(1-\nu_j)^{\omega_j}\right) \tag{4}$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $\tilde{a}_j\,(j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$.

(2) The intuitionistic fuzzy ordered weighted geometric (IFOWG) operator [16]:

$$\mathrm{IFOWG}_{w}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\overset{n}{\underset{j=1}{\otimes}}(\tilde{a}_{\sigma(j)})^{w_j}=\left(\prod_{j=1}^{n}\mu_{\sigma(j)}{}^{w_j},1-\prod_{j=1}^{n}(1-\nu_{\sigma(j)})^{w_j}\right) \tag{5}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $\tilde{\alpha}_{\sigma(j-1)}\ge\tilde{\alpha}_{\sigma(j)}$ for all $j=2,\cdots,n$ and $w=(w_1,w_2,\cdots,w_n)^T$ is the aggregation-associated vector such that $w_j\in[0,1]$, $\sum_{j=1}^{n}w_j=1$.

Definition 4. Let $\tilde{a}_j=(\mu_j,\nu_j)(j=1,2,\cdots,n)$ be a collection of intuitionistic fuzzy values on Q, and μ be a fuzzy measure on Q, the intuitionistic fuzzy correlate geometric mean(IFCGM) operator of $\tilde{a}_j$ with respect to μ is defined by

$$
\begin{aligned}
&IFCGM_{\mu}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\\
&=\left(\tilde{a}_{\sigma(1)}\right)^{\left(\mu\left(A_{(1)}\right)-\mu\left(A_{(2)}\right)\right)}\otimes\left(\tilde{a}_{\sigma(2)}\right)^{\left(\mu\left(A_{(2)}\right)-\mu\left(A_{(3)}\right)\right)}\otimes\cdots\otimes\left(\tilde{a}_{\sigma(n)}\right)^{\left(\mu\left(A_{(n)}\right)-\mu\left(A_{(n+1)}\right)\right)}\\
&=\overset{n}{\underset{i=1}{\otimes}}\left(\tilde{a}_{\sigma(i)}\right)^{\left(\mu\left(A_{(i)}\right)-\mu\left(A_{(i+1)}\right)\right)} \qquad (6)\\
&=\left(\prod_{i=1}^{n}\mu_{\sigma(i)}^{\left(\mu\left(A_{(i)}\right)-\mu\left(A_{(i+1)}\right)\right)},1-\prod_{i=1}^{n}\left(1-\nu_{\sigma(i)}\right)^{\left(\mu\left(A_{(i)}\right)-\mu\left(A_{(i+1)}\right)\right)}\right)
\end{aligned}
$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $(1,2,\cdots,n)$, such that $\tilde{a}_{\sigma(j-1)}\le\tilde{a}_{\sigma(j)}$ for all $j=2,\cdots,n$, $A_{(i)}=\left((i),\cdots,(n)\right)$, $A_{(n+1)}=\phi$[17].

3. An approach to multiple attribute decision making for evaluating the residential building energy-saving with intuitionistic fuzzy Information

Energy is base of human life. At present，humans the main energy is oil and gas, as the two kinds of resources by continual exploration so all countries in the world are aware of this serious problem. Energy and national have more relation in the country's rise and fall. In a sense, the progress and development of human society depends on the emergence of high-quality energy and the use of advanced energy technologies. China's land area is 960 square kilometers, across several climatic zones, which for the most part in north China town all need heating in winter. At the same time, along with the global climate change in recent years, our country throughout most parts of the summer air temperature is unusually hot, becoming increasingly prevalent in refrigeration and air conditioning, rapid increase of energy consumption in summer. But, our country old existing residential buildings do not have any energy saving measures, its heat preservation and heat insulation performance is poor, equipment system efficiency low, lead to severe waste of heating and cooling energy consumption. According to the measure, if you don't take effective measures, at the current trends, by 2020 China's building energy consumption will be more than three times now; building energy consumption will reach 1.09 billion tons of standard coal. Building energy conservation is to carry out the basic state policy of energy saving, environmental protection, also is the current global development trend.

The following assumptions or notations are used to represent the MADM problems for evaluating the residential building energy-saving with intuitionistic fuzzy variables. Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives. Let $G=\{G_1,G_2,\cdots,G_n\}$ be a set of attributes. The information about attribute weights is completely known. Let $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ be the weight vector of attributes, where $\omega_j\ge 0$, $j=1,2,\cdots,n$. Suppose that $\tilde{R}=\left(\tilde{r}_{ij}\right)_{m\times n}=\left(\mu_{ij},\nu_{ij}\right)_{m\times n}$ is the intuitionistic fuzzy decision matrix, where μ_{ij} indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, ν_{ij} indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $\mu_{ij}\subset[0,1]$, $\nu_{ij}\subset[0,1]$, $\mu_{ij}+\nu_{ij}\le 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

In the following, we apply the IFCGM operator to multiple attribute decision making (MADM) problems for evaluating the socialist new countryside democratic construction with intuitionistic fuzzy numbers.

Step 1. Determine the fuzzy measure of attribute of $G_j\,(j=1,2,\cdots,n)$ and attribute sets of G. There are a few methods for the determination of the fuzzy measure. For example, linear methods, quadratic methods, heuristic-based methods and genetic algorithms and so on are available in the literature.

Step 2.Utilize the IFCGM operator

$$\begin{aligned}\tilde{r}_i &= \mathrm{IFCGM}_{\mu}\left(\tilde{r}_{i1},\tilde{r}_{i2},\cdots,\tilde{r}_{in}\right)\\ &= \left(\tilde{r}_{\sigma(i1)}\right)^{\left(\mu\left(G_{(1)}\right)-\mu\left(G_{(2)}\right)\right)}\otimes\left(\tilde{r}_{\sigma(i2)}\right)^{\left(\mu\left(G_{(2)}\right)-\mu\left(G_{(3)}\right)\right)}\otimes\cdots\otimes\left(\tilde{r}_{\sigma(in)}\right)^{\left(\mu\left(G_{(n)}\right)-\mu\left(G_{(n+1)}\right)\right)}\\ &= \overset{n}{\underset{j=1}{\otimes}}\left(\tilde{r}_{\sigma(ij)}\right)^{\left(\mu\left(G_{(j)}\right)-\mu\left(G_{(j+1)}\right)\right)}\\ &= \left(\prod_{i=1}^{n}\mu_{\sigma(ij)}^{\left(\mu\left(G_{(i)}\right)-\mu\left(G_{(i+1)}\right)\right)},1-\prod_{i=1}^{n}\left(1-\nu_{\sigma(ij)}\right)^{\left(\mu\left(G_{(i)}\right)-\mu\left(G_{(i+1)}\right)\right)}\right)\end{aligned} \tag{5}$$

to derive the overall preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i.

Step 3. Calculate the scores $S\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$ of the collective overall intuitionistic fuzzy preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s).

Step 4. Rank all the enterprises $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$.

4. Numerical example

Since China's reform and opening up， along with the speeding up of urbanization in our country and the continuous improvement of people's living standard， the proportion of total energy consumption of building energy consumption is becoming more and more big，building energy consumption has gradually grown to and industrial， transportation， energy consumption and energy consumption becomes our country energy consumption of the big three "large energy consumption". How to effectively do a good job of the building energy conservation has subjected to more and more widely attention of the professionals. Slurry sampling study by the ministry of construction，China's building energy consumption accounts forabout33%of the national total energy consumption， and a rising trend， the main reason is that the building volume is growing. Through the nearly30years of research and practice in the field of energy， is generally believed that the construction of energy-efficient buildings is one of the largest and most efficient potential method， especially the large market construction. Shopping malls though accounts for only a small part of public buildings，but it is high in energy consumption per unit area. Therefore， how to reduce the large stores of energy consumption， with the progress of China's public building energy efficiency technology development has the vital significance. In this section, we shall present a numerical example to illustrate the method proposed. There is a panel with five possible residential building energy-saving projects $A_i\left(i=1,2,3,4\right)$ to select. The company selects four attribute to evaluate the five possible residential building energy-saving projects: ①G_1 is the logistics; ②G_2 is the strategy; ③G_3 is the economy; ④G_4 is the technology. The five possible residential building energy-saving projects $A_i\left(i=1,2,3,4\right)$ are to be evaluated using the intuitionistic fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix}(0.7,0.1) & (0.6,0.3) & (0.5,0.3) & (0.5,0.4)\\ (0.6,0.4) & (0.5,0.5) & (0.7,0.2) & (0.7,0.3)\\ (0.8,0.2) & (0.7,0.3) & (0.4,0.4) & (0.6,0.3)\\ (0.5,0.4) & (0.6,0.4) & (0.8,0.2) & (0.8,0.1)\\ (0.6,0.4) & (0.3,0.4) & (0.6,0.4) & (0.6,0.4)\end{bmatrix}$$

Then, we utilize the intuitionistic fuzzy approach developed to get the most desirable residential building energy-saving projects.

Step 1. Suppose the fuzzy measure of attribute of $G_j\ (j=1,2,3,4)$ and attribute sets of G as follows:

$$\mu(G_1)=0.30, \mu(G_2)=0.20, \mu(G_3)=0.35, \mu(G_4)=0.25$$
$$\mu(G_1,G_2)=0.50, \mu(G_1,G_3)=0.65, \mu(G_1,G_4)=0.70$$
$$\mu(G_2,G_3)=0.65, \mu(G_2,G_4)=0.60, \mu(G_3,G_4)=0.50$$
$$\mu(G_1,G_2,G_3)=0.80, \mu(G_1,G_2,G_4)=0.85$$
$$\mu(G_1,G_3,G_4)=0.85, \mu(G_2,G_3,G_4)=0.78$$
$$\mu(G_1,G_2,G_3,G_4)=1.00$$

Step 2. Utilize the decision information given in matrix $\tilde{R}$, and the IFCGM operator, we obtain the overall preference values $\tilde{r}_i$ of the residential building energy-saving projects $A_i\ (i=1,2,\cdots,5)$.

$$\tilde{r}_1=(0.45,0.23), \tilde{r}_2=(0.55,0.38), \tilde{r}_3=(0.61,0.34)$$
$$\tilde{r}_4=(0.47,0.35), \tilde{r}_5=(0.46,0.22)$$

Step 3. Calculate the scores $S(\tilde{r}_i)\ (i=1,2,3,4,5)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i=1,2,3,4,5)$

$$S(\tilde{r}_1)=0.22, S(\tilde{r}_2)=0.17, S(\tilde{r}_3)=0.27$$
$$S(\tilde{r}_4)=0.12, S(\tilde{r}_5)=0.24$$

Step 4. Rank all the residential building energy-saving projects $A_i\ (i=1,2,3,4,5)$ in accordance with the scores $S(\tilde{r}_i)\ (i=1,2,3,4,5)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i\ (i=1,2,3,4,5)$: $A_3 \succ A_5 \succ A_1 \succ A_2 \succ A_4$, and thus the most desirable residential building energy-saving project is A_3.

5. Conclusion

The energy consumption of heating systems is quite serious, as compared to other countries with high energy consumption, pollution, especially cold regions building and heating system energy consumption has become a major burden on energy. Building and heating system to solve the energy problem is an important strategy to achieve long-term stable development. For the establishment of energy-saving society, China has issued a series of energy-related policies, but the lack of a comprehensive building energy efficiency and energy-saving heating systems and levels of performance evaluation system and effective assessment tools, and complete establish energy index evaluation system is the basis of energy conservation. In this paper, we investigate the multiple attribute decision making problems for evaluating the residential building energy-saving with intuitionistic fuzzy variables. We utilize the intuitionistic fuzzy correlate geometric mean (IFCGM) operator to aggregate the intuitionistic fuzzy numbers corresponding to each alternative and get the overall value of the alternatives, then rank the and select the most desirable one(s) according to the score function and accuracy function. Finally, an illustrative example is given. In the future, we shall extend the proposed modles to other domains[18-29].

Acknowledgements

The work was supported by the Humanities and Social Sciences Foundation of Ministry of Education of the People's Republic of China (14XJCZH002).

References

[1] Haiyan Geng, Changhong Wu, "An Approach to Evaluating the Physical Education Teaching Quality in Colleges and Universities with Intuitionistic Fuzzy Information", JCIT: Journal of Convergence Information Technology, Vol. 7, No. 14, pp. 368 ~ 374, 2012.

[2] Chunying Long, Cheng Zhang, Jiaquan Zhang, Yuanqiu Liu, "Assessment on Vegetation Restoration by Soil Spray Seeding Technique on Road Verge Slopes of Expressway with Triangular Fuzzy Information", JCIT: Journal of Convergence Information Technology, Vol. 7, No. 14, pp. 324 ~ 330, 2012.

[3] Yong Xu, "An Approach to Evaluating the Software Systems Risk Based on the Intelligence Computation with Linguistic Information", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 14, pp. 480 ~ 486, 2012.

[4] Rongguang Qi, Shiying Liu, "An Approach to Evaluating the Project Risk Management with 2-tuple Linguistic Information", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 14, pp. 512 ~ 518, 2012.

[5] Hong Fan, "Research on the Physical Education Teaching Effectiveness of the Higher Colleges and Universities with Intuitionistic Fuzzy Information", JCIT: Journal of Convergence Information Technology, Vol. 7, No. 15, pp. 463 ~ 469, 2012.

[6] Zefeng Zheng, Ji Zhang, "Model for Evaluating the Industrial Design with Interval Grey Linguistic Variables", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 15, pp. 136 ~ 142, 2012.

[7] Shaoying Wang, Xianzhang Meng, "Research on Evaluation of Teaching Quality of University with Triangular Fuzzy Information", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 15, pp. 201 ~ 207, 2012.

[8] Xiaoyan Huang, Shujiang Xi, "Study on the Software Quality Evaluation with Hesitant Fuzzy Information", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 17, pp. 317 ~ 322, 2012.

[9] Li-Ping Xu, "An Approach to HAZOP Safety Assessment Technology of Chemical Production Process with Linguistic Information", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 17, pp. 358 ~ 364, 2012

[10] Chun Jiang, Xiang Luo, "An Approach to Evaluating the Exchange Rate Risk in the International Trade with Linguistic Information", JDCTA: International Journal of Digital Content Technology and its Applications, Vol. 6, No. 17, pp. 365 ~ 371, 2012.

[11] Zeshui Xu, "Intuitionistic fuzzy aggregation operators", IEEE Transations on Fuzzy Systems, vol.15, no.6, pp.1179-1187, 2007.

[12] Dengfeng Li, "Multiattribute decision making models and methods using intuitionistic fuzzy sets", Journal of Computer and System Sciences, vol.70, no.3, pp.73-85, 2005.

[13] K. Atanassov, "More on intuitionistic fuzzy sets, Fuzzy Sets and Systems", vol.33, no.5, pp.37-46, 1989.

[14] S. M. Chen and J. M. Tan, "Handling multicriteria fuzzy decision-making problems based on vague set theory", Fuzzy Sets and Systems vol. 67, no.4, pp.163-172, 1994.

[15] D. H. Hong and C. H. Choi, "Multicriteria fuzzy problems based on vague set theory", Fuzzy Sets and Systems, vol.114, no.3, pp.103-113, 2000.

[16] Zeshui Xu and R. R. Yager, "Some geometric aggregation operators based on intuitionistic fuzzy sets", International Journal of General System, vol.35, no.6, pp.417-433, 2006.

[17] Zeshui Xu, "Choquet integrals of weighted intuitionistic fuzzy information", Information Sciences, vol.180, no.5, pp.726-736, 2010.

[18] D. H. Hong and C. H. Choi, "Multicriteria fuzzy problems based on vague set theory", Fuzzy Sets and Systems, vol.114, no.3, pp.103-113, 2000.

[19] Z. J. Wang, K. W. Li, and J. H. Xu, "A mathematical programming approach to multi-attribute decision making with interval-valued intuitionistic fuzzy assessment information," Expert Systems with Applications, vol. 38, no. 10, pp. 12462-12469, Sep, 2011.
[20] Z. S. Xu, "Models for multiple attribute decision making with intuitionistic fuzzy information," International Journal of Uncertainty Fuzziness and Knowledge-Based Systems, vol. 15, no. 3, pp. 285-297, Jun, 2007.
[21] Z. S. Xu, "Multi-person multi-attribute decision making models under intuitionistic fuzzy environment," Fuzzy Optimization and Decision Making, vol. 6, no. 3, pp. 221-236, Sep, 2007.
[22] Z. S. Xu, "Some similarity measures of intuitionistic fuzzy sets and their applications to multiple attribute decision making," Fuzzy Optimization and Decision Making, vol. 6, no. 2, pp. 109-121, Jun, 2007.
[23] F. Herrera and L. Nartinez, "A 2-tuple fuzzy linguistic representation model for computing with words", IEEE Transactions on Fuzzy Systems, vol.8, no.6, pp.746-752, 2000.
[24] F. Herrera and L. Nartinez, "A 2-tuple fuzzy linguistic representation model for computing with words", IEEE Transactions on Fuzzy Systems, vol.8, no.6, pp.746-752, 2000.
[25] F. Chiclana, F. Herrera and E.Herrera-Viedma, "The ordered weighted geometric operator: properties and application", Proceedings of the 8th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Madrid, Spain, pp. 985–991, 2000.
[26] G. W. Wei, R. Lin, X.F. Zhao, H.J. Wang, Models for multiple attribute group decision making with 2-tuple linguistic assessment information, International Journal of Computational Intelligence Systems, vol.3, no.3, pp.315-324, 2010.
[27] G. W. Wei, Extension of TOPSIS method for 2-tuple linguistic multiple attribute group decision making with incomplete weight information, Knowledge and Information Systems, Vol. 25, pp.623-634, 2010.
[28] F. Herrera and L. Martinez, "A model based on linguistic 2-tuples for dealing with multigranularity hierarchical linguistic contexts in multiexpert decision-making", IEEE Transactions on Systems, Man and Cybernetics-Part B: Cybernetics, vol.31, no. 2, pp.227-234, 2001.
[29] Yanping Jiang, Ziping Fan. "Property analysis of the aggregation operators for 2-tuple linguistic information", Control and Decision, vol.18, no.6, pp. 754-757, 2003.