Effective Subcontractor Selection Model in the Construction Engineering with the Interval Grey Uncertain Linguistic Variables

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Abstract

In this paper, we investigate the multiple attributes group decision making (MAGDM) problems in which attribute values take the form of interval grey uncertain linguistic variables. Firstly, we develop the interval grey uncertain linguistic variables weighted geometric (IGULWG) operator, the interval grey uncertain linguistic variables ordered weighted geometric (IGULOWG) operator and the induced interval grey uncertain linguistic variables ordered weighted geometric (I-IGULOWG) operator and study some desirable properties of the I-IGULOWG operator. Furthermore, we apply the I-IGULOWG operator and the IGULWG operator to the group decision making with interval grey uncertain linguistic information. Finally, an illustrative example for subcontractor selection in the construction engineering with interval grey uncertain linguistic information is given to verify the developed approach and to demonstrate its practicality and effectiveness.

Keywords

Multiple attribute decision making (MADM); interval grey uncertain linguistic variables; interval grey uncertain linguistic variables ordered weighted geometric (IGULOWG) operator; subcontractor selection; construction engineering

1. Introduction

Multiple attribute decision making (MADM) has been one of the fastest growing areas during the last decades depending on the changings in the business sector. Decision maker(s) need a decision aid to decide between the alternatives and mainly excel less preferrable alternatives fast. With the help of computers the decision making methods have found great acceptance in all areas of the decision making processes. Since MADM has found acceptance in areas of operation research and management science, the discipline has created several methodologies. Especially in the last years, where computer usage has increased significantly, the application of MADM methods has considerably become easier for the users the decision makers as the application of most of the methods are corresponded with complex mathematics[1-9].

The general concepts of domination structures and non-dominated solutions play an important role in describing the decision problems and the decision maker's revealed preferences described above [1]. So far, various approaches have been developed as the decision aid (see, for example [2]). That is, for many such problems, the decision maker wants to solve a MADM problem. In MADM problems, there does not necessarily exist the solution that optimizes all objectives functions, and then the concept which is called Pareto optimal solution (or efficient solution) is introduced. Usually, there exist a number of Pareto optimal solutions, which are considered as candidates of final decision making solution. It is an issue how decision makers decide one from the set of Pareto optimal solutions as the final solution (see, for more details [3]). A MADM problem can be concisely expressed in matrix format as

$$\boldsymbol{A} = \begin{matrix} & \begin{matrix} G_1 & G_2 & & G_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \\ A_m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{matrix}$$

$$W = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix}$$

where $A_1, A_2, \cdots, A_m$ are possible alternatives among which decision makers have to choose, $G_1, G_2, \cdots, G_n$ are attribute with which alternative performance are measured, a_{ij} is the rating of alternative A_i with respect to attribute G_j, w_j is the weight of attribute G_j.

In this paper, we investigate the multiple attributes group decision making (MAGDM) problems in which attribute values take the form of interval grey uncertain linguistic variables. Firstly, we develop the interval grey uncertain linguistic variables weighted geometric (IGULWG) operator, the interval grey uncertain linguistic variables ordered weighted geometric (IGULOWG) operator and the induced interval grey uncertain linguistic variables ordered weighted geometric (I-IGULOWG) operator and study some desirable properties of the I-IGULOWG operator. Furthermore, we apply the I-IGULOWG operator and the IGULWG operator to the group decision making with interval grey uncertain linguistic information. Finally, an illustrative example for subcontractor selection in the construction engineering with interval grey uncertain linguistic information is given to verify the developed approach and to demonstrate its practicality and effectiveness.

2. Preliminaries

In this section, we briefly review some basic concepts to be used throughout the paper.

Definition 1[10-11]. Let $\tilde{A}(x)$ be the fuzzy subset in the space $X = \{x\}$, if the membership degree $\mu_A(x)$ of x to $\tilde{A}(x)$ is the grey in the interval $[0,1]$, and its grey is $\nu_A(x)$, then $\tilde{A}(x)$ is called the grey fuzzy set in space X (GF set, for short), denoted by $\underset{\otimes}{\tilde{A}}(x)$, as follows:

$$\underset{\otimes}{\tilde{A}}(x) = \left\{ \left(x, \mu_A(x), \nu_A(x) \right) \middle| x \in X \right\} \tag{1}$$

The set pair mode is $\underset{\otimes}{\tilde{A}}(x) = \left(\tilde{A}(x), \underset{\otimes}{A}(x) \right)$, where $\tilde{A}(x) = \left\{ \left(x, \mu_A(x) \right) \middle| x \in X \right\}$ is called the fuzzy part of $\underset{\otimes}{\tilde{A}}(x)$, and $\underset{\otimes}{A}(x) = \left\{ \left(x, \nu_A(x) \right) \middle| x \in X \right\}$ is called the grey part of $\underset{\otimes}{\tilde{A}}(x)$. So the grey fuzzy set is regarded as the generalization of the fuzzy set and the grey set.

Definition 2 [10]. Let $\underset{\otimes}{\tilde{A}}(x) = \left(\tilde{A}(x), \underset{\otimes}{A}(x) \right)$ be the grey fuzzy number, if its fuzzy part is a linguistic variable $s_\alpha \in S$, where S is a finite and totally ordered discrete term set, and its grey part $\underset{\otimes}{A}(x)$ is a closed interval $\left[g_A^L, g_A^U \right]$, then $\underset{\otimes}{\tilde{A}}(x)$ is called the interval grey linguistic variable.

Supposed that $\underset{\otimes}{\tilde{A}}(x) = \left(s_\alpha, \left[g_A^L, g_A^U \right] \right)$, $\underset{\otimes}{\tilde{B}}(x) = \left(s_\beta, \left[g_B^L, g_B^U \right] \right)$ be two interval grey linguistic variables.

The continuous ordered weighted averaging (C-OWA, for short) operator which is developed by Yager [12] can be usefully applied to aggregate the grey part, the greyness of the grey part would be transformed into a real number, and then the fuzzy part integrates with the grey part. That is to say, the size of the interval grey linguistic variables can get through comparing the size of $s_\alpha \times f_\rho\left(\left[\left(1-g_A^L\right),\left(1-g_A^U\right)\right]\right)$ and $s_\beta \times f_\rho\left(\left[\left(1-g_B^L\right),\left(1-g_B^U\right)\right]\right)$. Assume the ordering value $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)=\alpha \times f_\rho\left(\left[\left(1-g_A^L\right),\left(1-g_A^U\right)\right]\right)$, $Q\left(\underset{\otimes}{\tilde{B}}(x)\right)=\beta \times f_\rho\left[\left(1-g_B^L\right),\left(1-g_B^U\right)\right]$, then $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)=\alpha-\alpha\times\int_0^1 \frac{d\rho(y)}{dy}\left(g_A^U+y\left(g_A^L-g_A^U\right)\right)dy$, $Q\left(\underset{\otimes}{\tilde{B}}(x)\right)=\beta-\beta\times\int_0^1 \frac{d\rho(y)}{dy}\left(g_B^U+y\left(g_B^L-g_B^U\right)\right)dy$, which can be obtained based on the continuous ordered weighted averaging (C-OWA) operator, such as $f_p([a,b])=\int_0^1 \frac{d\rho(y)}{dy}\left(b-y(b-a)\right)dy$.

The operation rules of ranking are defined as follows:

(1) If $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)>Q\left(\underset{\otimes}{\tilde{B}}(x)\right)$, then we have $\underset{\otimes}{\tilde{A}}(x)>\underset{\otimes}{\tilde{B}}(x)$;

(2) If $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)<Q\left(\underset{\otimes}{\tilde{B}}(x)\right)$, then we have $\underset{\otimes}{\tilde{A}}(x)<\underset{\otimes}{\tilde{B}}(x)$;

(3) If $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)=Q\left(\underset{\otimes}{\tilde{B}}(x)\right)$ and $s_\alpha \ge s_\beta$, then we have $\underset{\otimes}{\tilde{A}}(x)\ge\underset{\otimes}{\tilde{B}}(x)$;

(4) If $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)=Q\left(\underset{\otimes}{\tilde{B}}(x)\right)$ and $s_\alpha < s_\beta$, then we have $\underset{\otimes}{\tilde{A}}(x)<\underset{\otimes}{\tilde{B}}(x)$.

The function ρ is denoted as basic unit-interval monotonic (BUM) functions. If $(\delta \ge 0)$ $\rho(y)=y^\delta$, then we have $Q\left(\underset{\otimes}{\tilde{A}}(x)\right)=\alpha-\alpha\times\frac{\delta g_A^L+g_A^U}{\delta+1}$ and $Q\left(\underset{\otimes}{\tilde{B}}(x)\right)=\beta-\beta\times\frac{\delta g_B^L+g_B^U}{\delta+1}$.

The operation rules of the interval grey linguistic variables are defined as follows:

(1) $\underset{\otimes}{\tilde{A}}(x)\times\underset{\otimes}{\tilde{B}}(x)=\left(s_{\alpha\times\beta},\left[\left(1-\left(1-g_A^L\right)\times\left(1-g_B^L\right)\right),\left(1-\left(1-g_A^U\right)\times\left(1-g_B^U\right)\right)\right]\right)$;

(2) $\left(\underset{\otimes}{\tilde{A}}(x)\right)^k=\left(s_{\alpha^k},\left[g_A^L,g_A^U\right]\right)$.

3. Some geometric aggregation operators with the interval grey uncertain linguistic variables

Supposed that $\underset{\otimes}{\tilde{A}}(x)=\left(\tilde{s}_1,\left[g_A^L,g_A^U\right]\right)$, $\underset{\otimes}{\tilde{B}}(x)=\left(\tilde{s}_2,\left[g_B^L,g_B^U\right]\right)$ be two interval grey uncertain linguistic variables, where $\tilde{s}_1=\left[s_{\alpha_1},s_{\beta_1}\right]$ and $\tilde{s}_2=\left[s_{\alpha_2},s_{\beta_2}\right]$. The continuous ordered weighted averaging (C-OWA, for short) operator which is developed by Yager [12] can be usefully applied to aggregate the grey part, the greyness of the grey part would be transformed into a real number, and then the fuzzy part integrates with the grey part. That is to say, the size of the interval grey linguistic variables can get through comparing the size of $\left[s_{\alpha_1},s_{\beta_1}\right]\times f_\rho\left(\left[\left(1-g_A^L\right),\left(1-g_A^U\right)\right]\right)$ and $\left[s_{\alpha_2},s_{\beta_2}\right]\times f_\rho\left(\left[\left(1-g_B^L\right),\left(1-g_B^U\right)\right]\right)$. Assume the ordering value $\tilde{Q}\left(\underset{\otimes}{\tilde{A}}(x)\right)=\left[s_{\alpha_1},s_{\beta_1}\right]\times f_\rho\left(\left[\left(1-g_A^L\right),\left(1-g_A^U\right)\right]\right)$ and $\tilde{Q}\left(\underset{\otimes}{\tilde{B}}(x)\right)=\left[s_{\alpha_2},s_{\beta_2}\right]\times f_\rho\left(\left[\left(1-g_B^L\right),\left(1-g_B^U\right)\right]\right)$, which can be obtained based on the continuous ordered weighted averaging (C-OWA) operator, such as $f_p([a,b])=\int_0^1 \frac{d\rho(y)}{dy}\left(b-y(b-a)\right)dy$.

Definition 3. An IGULWG operator of dimension n is a function IGULWG: $\Omega^n \to \Omega$, which has associated a set of weights or weighting vector $w=(\omega_1,\omega_2,\cdots,\omega_n)$ with $\omega_j \in [0,1]$, $\sum_{j=1}^{n}\omega_j = 1$, and is defined to aggregate a list of values $\left\{\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right\}$, where $\underset{\otimes}{\tilde{A}}(x_j)=\left(\left[\tilde{s}_{\alpha_j},\tilde{s}_{\beta_j}\right],\left[g_j^L,g_j^U\right]\right)$. According to the following expression:

$$\begin{aligned}
&IGULWG\left(\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right)\\
&=\bigotimes_{j=1}^{n}\left(\underset{\otimes}{\tilde{A}}(x_j)\right)^{\omega_j}\\
&=\left(\left[s_{\prod_{j=1}^{n}(\alpha_j)^{\omega_j}},s_{\prod_{j=1}^{n}(\beta_j)^{\omega_j}}\right],\left[\left(1-\prod_{j=1}^{n}\left(1-g_j^L\right)\right),\left(1-\prod_{j=1}^{n}\left(1-g_j^U\right)\right)\right]\right)
\end{aligned} \tag{2}$$

where there is a permutation such that $\underset{\otimes}{\tilde{A}}(x_{\tau(1)}) \ge \underset{\otimes}{\tilde{A}}(x_{\tau(2)}) \ge \cdots \ge \underset{\otimes}{\tilde{A}}(x_{\tau(n)})$, i.e., $\underset{\otimes}{\tilde{A}}(x_{\tau(j)})$ the j th largest value in the set $\left\{\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right\}$.

Definition 4. An IGULOWG operator of dimension n is a function IGULOWG: $\Omega^n \to \Omega$, which has associated a set of weights or weighting vector $w=(\omega_1,\omega_2,\cdots,\omega_n)$ with $\omega_j \in [0,1]$, $\sum_{j=1}^{n}\omega_j = 1$, and is defined to aggregate a list of values $\left\{\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right\}$, where $\underset{\otimes}{\tilde{A}}(x_j)=\left(\left[\tilde{s}_{\alpha_j},\tilde{s}_{\beta_j}\right],\left[g_j^L,g_j^U\right]\right)$. According to the following expression:

$$\begin{aligned}
&IGULOWG\left(\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right)\\
&=\bigotimes_{j=1}^{n}\left(\underset{\otimes}{\tilde{A}}(x_{\tau(j)})\right)^{\omega_j}\\
&=\left(\left[s_{\prod_{j=1}^{n}(\alpha_{\tau(j)})^{\omega_j}},s_{\prod_{j=1}^{n}(\beta_{\tau(j)})^{\omega_j}}\right],\left[\left(1-\prod_{j=1}^{n}\left(1-g_{\tau(j)}^L\right)\right),\left(1-\prod_{j=1}^{n}\left(1-g_{\tau(j)}^U\right)\right)\right]\right)
\end{aligned} \tag{3}$$

where there is a permutation such that $\underset{\otimes}{\tilde{A}}(x_{\tau(1)}) \ge \underset{\otimes}{\tilde{A}}(x_{\tau(2)}) \ge \cdots \ge \underset{\otimes}{\tilde{A}}(x_{\tau(n)})$, i.e., $\underset{\otimes}{\tilde{A}}(x_{\tau(j)})$ the j th largest value in the set $\left\{\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right\}$.

Inspired by ideal of the induced OWG operator[13], we developed the I-IGULOWG operator with interval grey uncertain linguistic variables information based on the IGULOWG operator, which is defined as follows:

Definition 5. An I-IGLUOWG operator is defined as follows:

$$\begin{aligned}
&I\text{-}IGULOWG\left(\underset{\otimes}{\tilde{A}}(x_1),\underset{\otimes}{\tilde{A}}(x_2),\cdots,\underset{\otimes}{\tilde{A}}(x_n)\right)\\
&=\bigotimes_{j=1}^{n}\left(\underset{\otimes}{\tilde{A}}(x_{\sigma(j)})\right)^{\omega_j}\\
&=\left(\left[s_{\prod_{j=1}^{n}(\alpha_{\sigma(j)})^{\omega_j}},s_{\prod_{j=1}^{n}(\beta_{\sigma(j)})^{\omega_j}}\right],\left[\left(1-\prod_{j=1}^{n}\left(1-g_{\sigma(j)}^L\right)\right),\left(1-\prod_{j=1}^{n}\left(1-g_{\sigma(j)}^U\right)\right)\right]\right)
\end{aligned} \tag{4}$$

where $w=(\omega_1,\omega_2,\cdots,\omega_n)$ is a weighting vector, such that $\omega_j\in[0,1],\sum_{j=1}^{n}\omega_j=1$, $\underset{\otimes}{\tilde{A}}\left(x_{\sigma(j)}\right)$ is the $\underset{\otimes}{\tilde{A}}(x_j)$ value of the pair $\left\langle u_j,\underset{\otimes}{\tilde{A}}(x_j)\right\rangle$ having the j th largest u_j, and u_j in $\left\langle u_j,\underset{\otimes}{\tilde{A}}(x_j)\right\rangle$ is referred to as the order inducing variable and $\underset{\otimes}{\tilde{A}}(x_j)$ is an interval grey uncertain linguistic value while u_j can be drawn from ordinal Ω.

Similar to the IOWG, the I-IGLUOWG operator has the following properties.

Theorem 1 (Commutativity).

$$\begin{aligned}&I\text{-}IGLUOWG\left(\left\langle u_1,\underset{\otimes}{\tilde{A}}(x_1)\right\rangle,\left\langle u_2,\underset{\otimes}{\tilde{A}}(x_2)\right\rangle,\cdots,\left\langle u_n,\underset{\otimes}{\tilde{A}}(x_n)\right\rangle\right)\\&=I\text{-}IGLUOWG\left(\left\langle u_1',\underset{\otimes}{\tilde{A}'}(x_1)\right\rangle,\left\langle u_2',\underset{\otimes}{\tilde{A}'}(x_2)\right\rangle,\cdots,\left\langle u_n',\underset{\otimes}{\tilde{A}'}(x_n)\right\rangle\right)\end{aligned}$$

where $\left(\left\langle u_1',\underset{\otimes}{\tilde{A}'}(x_1)\right\rangle,\left\langle u_2',\underset{\otimes}{\tilde{A}'}(x_2)\right\rangle,\cdots,\left\langle u_n',\underset{\otimes}{\tilde{A}'}(x_n)\right\rangle\right)$ is any permutation of $\left(\left\langle u_1,\underset{\otimes}{\tilde{A}}(x_1)\right\rangle,\left\langle u_2,\underset{\otimes}{\tilde{A}}(x_2)\right\rangle,\cdots,\left\langle u_n,\underset{\otimes}{\tilde{A}}(x_n)\right\rangle\right)$.

Theorem 2 (Idempotency). If $\underset{\otimes}{\tilde{A}}(x_j)=\left(\left[s_{\alpha_j},s_{\beta_j}\right],\left[g_j^L,g_j^U\right]\right)=\underset{\otimes}{\tilde{A}}(x)=\left(\left[s_\alpha,s_\beta\right],\left[g^L,g^U\right]\right)$ $(j=1,2,\cdots,n)$ for all j, then

$$I\text{-}IGLUOWGA\left(\left\langle u_1,\underset{\otimes}{\tilde{A}}(x_1)\right\rangle,\left\langle u_2,\underset{\otimes}{\tilde{A}}(x_2)\right\rangle,\cdots,\left\langle u_n,\underset{\otimes}{\tilde{A}}(x_n)\right\rangle\right)=\underset{\otimes}{\tilde{A}}(x)=\left(\left[s_\alpha,s_\beta\right],\left[g^L,g^U\right]\right)$$

Theorem 3 (Monotonicity). If $\underset{\otimes}{\tilde{A}}(x_j)\le\underset{\otimes}{\tilde{A}'}(x_j)(j=1,2,\cdots,n)$, then

$$\begin{aligned}&I\text{-}IGLUOWGA\left(\left\langle u_1,\underset{\otimes}{\tilde{A}}(x_1)\right\rangle,\left\langle u_2,\underset{\otimes}{\tilde{A}}(x_2)\right\rangle,\cdots,\left\langle u_n,\underset{\otimes}{\tilde{A}}(x_n)\right\rangle\right)\\&\le I\text{-}IGLUOWGA\left(\left\langle u_1,\underset{\otimes}{\tilde{A}'}(x_1)\right\rangle,\left\langle u_2,\underset{\otimes}{\tilde{A}'}(x_2)\right\rangle,\cdots,\left\langle u_n,\underset{\otimes}{\tilde{A}'}(x_n)\right\rangle\right)\end{aligned}$$

Theorem 4 (Bounded).

$$\underset{j}{Min}\,\underset{\otimes}{\tilde{A}}(x_j)\le I\text{-}IGLUOWGA\left(\left\langle u_1,\underset{\otimes}{\tilde{A}}(x_1)\right\rangle,\left\langle u_2,\underset{\otimes}{\tilde{A}}(x_2)\right\rangle,\cdots,\left\langle u_n,\underset{\otimes}{\tilde{A}}(x_n)\right\rangle\right)\le\underset{j}{Max}\,\underset{\otimes}{\tilde{A}}(x_j)$$

4. Effective Subcontractor Selection Model in the Construction Engineering with the Interval Grey Uncertain Linguistic Variables

In this section, we shall develop an approach to multiple attribute group decision making for subcontractor selection in the construction engineering with interval grey uncertain linguistic variables information as follows. Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, and $X=\{x_1,x_2,\cdots,x_n\}$ be the set of attributes, $w=(\omega_1,\omega_2,\cdots,\omega_n)$ is the weighting vector of the attribute x_j $(j=1,2,\cdots,n)$, where $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$. Assume $D=\{D_1,D_2,\cdots,D_t\}$ be the set of decision makers, $v=(v_1,v_2,\cdots,v_t)$ be the weighting vector of decision makers, with $v_k\in[0,1]$, $\sum_{k=1}^{t}v_k=1$. Suppose that $\tilde{R}_k=\left(\underset{\otimes i}{\tilde{A}^{(k)}}(x_j)\right)_{m\times n}$ is the interval grey uncertain linguistic decision matrix and

$U=\left(u_j^{(k)}\right)_{t\times n}$ is the inducing values given by the experts, where $\tilde{A}_{\otimes i}^{(k)}\left(x_j\right)$ indicates the attribute value that the alternative A_i satisfies the attribute x_j given by the decision maker D_k and $u_j^{(k)}$ indicates the inducing values that the decision maker D_k satisfies the alternative A_i, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$, $k=1,2,\cdots,t$.

In the following, we apply the IGULWG and I-IGULOWG operator for subcontractor selection in the construction engineering based on interval grey uncertain linguistic information. The method involves the following steps:

Step 1. Utilize the decision information given in matrix $\tilde{R}_k$, and the I-IGULOWG operator which has associated weighting vector $w=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$.

$$\tilde{r}_i^{(k)}=I\text{-}IGULOWG\left(\left\langle u_1,\tilde{A}_{\otimes i}^{(k)}\left(x_1\right)\right\rangle,\left\langle u_2,\tilde{A}_{\otimes i}^{(k)}\left(x_2\right)\right\rangle,\cdots,\left\langle u_n,\tilde{A}_{\otimes i}^{(k)}\left(x_n\right)\right\rangle\right)=\overset{n}{\underset{j=1}{\otimes}}\left(\tilde{A}_{\otimes i}^{(k)}\left(x_{\sigma(j)}\right)\right)^{\omega_j}$$

$$i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ k=1,2,\cdots,t \tag{5}$$

to aggregate all the decision matrices $\tilde{R}_k\left(k=1,2,\cdots,t\right)$ into a collective decision matrix $\tilde{R}=\left(\tilde{r}_i^{(k)}\right)_{m\times t}$, where $w=\left(\omega_1,\omega_2,\cdots,\omega_n\right)$ is the weighting vector of the attribute $x_j\left(j=1,2,\cdots,n\right)$.

Step 2. Utilize the decision information given in matrix $\tilde{R}$ and the IGULWG operator

$$\tilde{z}_i=IGULWG\left(\tilde{r}_i^{(1)},\tilde{r}_i^{(2)},\cdots,\tilde{r}_i^{(t)}\right)=\overset{t}{\underset{k=1}{\otimes}}\left(\tilde{r}_i^{(k)}\right)^{v_k}\quad i=1,2,\cdots,m \tag{6}$$

to derive the collective overall preference values $\tilde{z}_i\ \left(i=1,2,\cdots,m\right)$ of the alternative A_i, where $v=\left(v_1,v_2,\cdots,v_t\right)$ be the weighting vector of decision makers.

Step 3. Let the basic unit-interval monotonic (BUM) function is $\rho(y)=y^{\delta}$. We get the size of the collective overall preference values $\tilde{z}_i\ \left(i=1,2,\cdots,m\right)$ by using:

$$\begin{aligned}\tilde{Q}\left(\tilde{z}_i\right)&=\left[s_{\alpha_i},s_{\beta_i}\right]\times f_{\rho}\left(\left[\left(1-g_i^L\right),\left(1-g_i^U\right)\right]\right)\\&=\left[s_{\alpha_i},s_{\beta_i}\right]\times\int_0^1\frac{d\rho\left(y\right)}{dy}\left(b-y\left(b-a\right)\right)dy\end{aligned} \tag{7}$$

Step 4. To rank these collective overall preference values $\tilde{Q}\left(\tilde{z}_i\right)\ \left(i=1,2,\cdots,m\right)$, we first compare each $\tilde{Q}\left(\tilde{z}_i\right)$ with all $\tilde{Q}\left(\tilde{z}_i\right)\ \left(i=1,2,\cdots,m\right)$ by using (8). For simplicity, we let $p_{ij}=p\left(\tilde{Q}\left(\tilde{z}_i\right)\geq\tilde{Q}\left(\tilde{z}_j\right)\right)$, then we develop a complementary matrix as $P=\left(p_{ij}\right)_{n\times n}$, where $p_{ij}\geq0$, $p_{ij}+p_{ji}=1$, $p_{ii}=\frac{1}{2}$, $i,j=1,2,\cdots,n$. Summing all elements in each line of matrix P, we have $p_i=\sum_{j=1}^{n}p_{ij}$, $i=1,2,\cdots,n$ [14].

$$p\left(\tilde{Q}\left(\tilde{z}_i\right)\geq\tilde{Q}\left(\tilde{z}_j\right)\right)=\frac{Max\left\{0,len\left(\tilde{s}_i\right)+len\left(\tilde{s}_j\right)\right\}-Max\left(\beta_j-\alpha_i,0\right)}{len\left(\tilde{s}_i\right)+len\left(\tilde{s}_j\right)} \tag{8}$$

Then we rank $\tilde{Q}\left(\tilde{z}_i\right)\ \left(i=1,2,\cdots,m\right)$ in descending order in accordance with the values of $p_i\left(i=1,2,\cdots,m\right)$.

Step 5. The ranking of the alternatives can be gained and the best one can be find out.

5. Numerical example

A practical use of the proposed approach involves the subcontractor selection in the construction engineering. There are four subcontractors in the construction engineering $\{A_1, A_2, A_3, A_4\}$, the attributes is shown as: the ability of innovative resources input (x_1), the ability of innovation management (x_2), the ability of innovation tendency (x_3) and the ability of research and development (x_4). Based on the four attributes, the three experts $\{D_1, D_2, D_3\}$ evaluated the four subcontractors in the construction engineering. Supposed that $w=(0.24,0.26,0.23,0.27)$ is the weighting vector of the attributes, $v=(0.4,0.32,0.28)^T$ is the weighting vector of decision makers, and the inducing values given by the experts take the form of real number. The decision matrices as listed in the following matrices $\tilde{R}_k = \left(\tilde{A}^{(k)}_{\otimes i}(x_j)\right)_{4\times 4}$ $(k=1,2,3)$ as follows:

$$\tilde{R}_1 = \begin{bmatrix} ([s_{3.2}, s_4],[0.2,0.5]) & ([s_{1.9}, s_2],[0.4,0.43]) & ([s_{4.3}, s_{4.6}],[0.35,0.4]) & ([s_{2.4}, s_3],[0.2,0.35]) \\ ([s_{3.6}, s_{3.9}],[0.4,0.4]) & ([s_{4.1}, s_{4.3}],[0.4,0.5]) & ([s_{2.1}, s_3],[0.1,0.2]) & ([s_{3.4}, s_4],[0.5,0.5]) \\ ([s_{2.5}, s_{2.8}],[0.35,0.4]) & ([s_{3.2}, s_{3.7}],[0.33,0.4]) & ([s_{3.5}, s_{3.8}],[0.23,0.3]) & ([s_{2.9}, s_{3.5}],[0.3,0.45]) \\ ([s_{2.6}, s_3],[0.5,0.6]) & ([s_{1.6}, s_2],[0.2,0.2]) & ([s_{3.5}, s_3],[0.2,0.4]) & ([s_{2.3}, s_3],[0.3,0.4]) \end{bmatrix}$$

$$\tilde{R}_2 = \begin{bmatrix} ([s_{3.4}, s_4],[0.2,0.4]) & ([s_{2.6}, s_3],[0.2,0.35]) & ([s_{2.7}, s_3],[0.3,0.4]) & ([s_{3.5}, s_4],[0.4,0.5]) \\ ([s_{3.6}, s_{4.1}],[0.4,0.5]) & ([s_{2.3}, s_3],[0.3,0.4]) & ([s_{3.6}, s_4],[0.2,0.4]) & ([s_{2.5}, s_3],[0.2,0.3]) \\ ([s_{3.2}, s_{3.8}],[0.2,0.4]) & ([s_{3.6}, s_4],[0.2,0.4]) & ([s_{1.6}, s_2],[0.3,0.4]) & ([s_{2.6}, s_3],[0.4,0.5]) \\ ([s_{3.7}, s_4],[0.3,0.4]) & ([s_{3.2}, s_4],[0.4,0.5]) & ([s_{1.8}, s_2],[0.3,0.4]) & ([s_{3.6}, s_4],[0.2,0.4]) \end{bmatrix}$$

$$\tilde{R}_3 = \begin{bmatrix} ([s_{3.5}, s_4],[0.2,0.4]) & ([s_{2.4}, s_3],[0.3,0.4]) & ([s_{3.2}, s_4],[0.4,0.5]) & ([s_{3.2}, s_{4.2}],[0.2,0.3]) \\ ([s_{3.7}, s_4],[0.3,0.3]) & ([s_{3.4}, s_{3.9}],[0.3,0.5]) & ([s_{2.8}, s_{3.2}],[0.1,0.2]) & ([s_{2.4}, s_3],[0.3,0.4]) \\ ([s_{3.2}, s_{3.6}],[0.3,0.4]) & ([s_4, s_{4.3}],[0.4,0.6]) & ([s_{2.9}, s_{3.3}],[0.2,0.4]) & ([s_{3.6}, s_4],[0.2,0.4]) \\ ([s_{2.8}, s_3],[0.2,0.3]) & ([s_{2.3}, s_{3.2}],[0.1,0.3]) & ([s_{3.4}, s_4],[0.3,0.4]) & ([s_{3.1}, s_{3.7}],[0.4,0.5]) \end{bmatrix}$$

Tables 6. Inducing variables U.

Experts	Attribute (x_1)	Attribute (x_2)	Attribute (x_3)	Attribute (x_4)
D_1	17	15	22	12
D_2	15	22	25	13
D_3	16	21	25	28

Step1. Utilize the decision information given in decision matrix $\tilde{R}_k$ $(k=1,2,3)$ and inducing variables U, and the I-IGULOWG operator

$$\tilde{r}_i^{(k)} = I\text{-}IGULOWG\left(\left\langle u_1, \tilde{A}^{(k)}_{\otimes i}(x_1)\right\rangle, \left\langle u_2, \tilde{A}^{(k)}_{\otimes i}(x_2)\right\rangle, \left\langle u_3, \tilde{A}^{(k)}_{\otimes i}(x_3)\right\rangle, \left\langle u_4, \tilde{A}^{(k)}_{\otimes i}(x_4)\right\rangle\right)$$

$$= \bigotimes_{j=1}^{4}\left(\tilde{A}^{(k)}_{\otimes i}\left(x_{\sigma(j)}\right)\right)^{\omega_j}$$

To derive the individual overall preference value $r_i^{(k)}$ of subcontractors in the construction engineering A_i:

$$r_1^{(1)} = I\text{-}IGLOWGA\left(\left\langle u_1, \underset{\otimes i}{\tilde{A}^{(k)}}(x_1)\right\rangle, \left\langle u_2, \underset{\otimes i}{\tilde{A}^{(k)}}(x_2)\right\rangle, \left\langle u_3, \underset{\otimes i}{\tilde{A}^{(k)}}(x_3)\right\rangle, \left\langle u_4, \underset{\otimes i}{\tilde{A}^{(k)}}(x_4)\right\rangle\right)$$
$$= \left([s_{4.3}, s_{4.6}], [0.35, 0.4]\right)^{0.24} \otimes \left([s_{3.2}, s_4], [0.2, 0.5]\right)^{0.26} \otimes$$
$$\left([s_{1.9}, s_2], [0.4, 0.43]\right)^{0.23} \otimes \left([s_{2.4}, s_3], [0.2, 0.35]\right)^{0.27}$$
$$= \left([s_{2.819}, s_{3.263}], [0.750, 0.889]\right)$$

Similarly, we have

$$r_1^{(1)} = \left([s_{2.819}, s_{3.263}], [0.750, 0.889]\right),\ r_2^{(1)} = \left([s_{3.209}, s_{3.771}], [0.838, 0.880]\right),$$
$$r_3^{(1)} = \left([s_{2.986}, s_{3.412}], [0.765, 0.861]\right),\ r_4^{(1)} = \left([s_{2.416}, s_{2.733}], [0.776, 0.885]\right).$$
$$r_1^{(2)} = \left([s_{3.024}, s_{3.464}], [0.731, 0.883]\right),\ r_2^{(2)} = \left([s_{2.904}, s_{3.454}], [0.731, 0.874]\right),$$
$$r_3^{(2)} = \left([s_{2.642}, s_{3.097}], [0.731, 0.892]\right),\ r_4^{(2)} = \left([s_{2.975}, s_{3.387}], [0.764, 0.892]\right).$$
$$r_1^{(3)} = \left([s_{3.068}, s_{3.788}], [0.731, 0.874]\right),\ r_2^{(3)} = \left([s_{3.042}, s_{3.502}], [0.691, 0.832]\right),$$
$$r_3^{(3)} = \left([s_{3.378}, s_{3.760}], [0.731, 0.914]\right),\ r_4^{(3)} = \left([s_{2.884}, s_{3.451}], [0.698, 0.853]\right).$$

Step 2. Utilize the weight vector of decision makers, $v = (0.4, 0.32, 0.28)^T$ and the IGULWG operator $\tilde{z}_i = IGULWG\left(\tilde{r}_i^{(1)}, \tilde{r}_i^{(2)}, \tilde{r}_i^{(3)}\right) = \overset{3}{\underset{k=1}{\otimes}} v_k \tilde{r}_i^{(k)}$ $(i = 1, 2, 3, 4)$ to aggregate the individual overall preference values $\tilde{r}_i^{(k)}$ $(k = 1, 2, 3)$ and thus get the collective overall preference value $\tilde{r}_i$ of subcontractors in the construction engineering A_i:

$$\tilde{z}_1 = IGULWG\left(\tilde{r}_1^{(1)}, \tilde{r}_1^{(2)}, \tilde{r}_1^{(3)}\right) = \overset{3}{\underset{k=1}{\otimes}} \left(\tilde{r}_1^{(k)}\right)^{v_k}$$
$$= \left([s_{2.819}, s_{3.263}], [0.750, 0.889]\right)^{0.4} \otimes \left([s_{3.024}, s_{3.464}], [0.731, 0.883]\right)^{0.32} \otimes$$
$$\left([s_{3.068}, s_{3.788}], [0.731, 0.874]\right)^{0.28}$$
$$= \left([s_{2.952}, s_{3.468}], [0.982, 0.998]\right)$$

Similarly, we have

$$\tilde{z}_1 = \left([s_{2.952}, s_{3.468}], [0.982, 0.998]\right),\ \tilde{z}_2 = \left([s_{3.062}, s_{3.591}], [0.987, 0.997]\right),$$
$$\tilde{z}_3 = \left([s_{2.972}, s_{3.399}], [0.983, 0.999]\right),\ \tilde{z}_4 = \left([s_{2.714}, s_{3.125}], [0.984, 0.998]\right).$$

Step 3. Let the basic unit-interval monotonic (BUM) function is $\rho(y) = y^2$.We can get the size of the collective overall preference values $\tilde{Q}(\tilde{z}_i)$ $(i = 1, 2, 3, 4)$.

$$\tilde{Q}(\tilde{z}_1) = [s_{0.037}, s_{0.044}],\ \tilde{Q}(\tilde{z}_2) = [s_{0.030}, s_{0.035}],$$
$$\tilde{Q}(\tilde{z}_3) = [s_{0.035}, s_{0.040}],\ \tilde{Q}(\tilde{z}_4) = [s_{0.030}, s_{0.035}].$$

Step 4. To rank these collective overall preference values $\tilde{Q}(\tilde{z}_i)$ $(i = 1, 2, 3, 4)$ by using (7), and then construct a complementary matrix

$$P=\begin{bmatrix}0.5 & 1 & 0.757 & 1\\ 0 & 0.5 & 0.033 & 0.484\\ 0.243 & 0.967 & 0.5 & 0.978\\ 0 & 0.516 & 0.022 & 0.5\end{bmatrix}$$

Summing all elements in each line of matrix P , we have

$$p_1=0.355,\ p_2=0.168,\ p_3=0.307,\ p_4=0.170.$$

Then we rank $\tilde{Q}(\tilde{z}_i)$ $(i=1,2,3,4)$ in descending order in accordance with the values of $p_i\ (i=1,2,3,4)$: $\tilde{Q}(\tilde{z}_1)\succ\tilde{Q}(\tilde{z}_3)\succ\tilde{Q}(\tilde{z}_4)\succ\tilde{Q}(\tilde{z}_2)$, so we get the $\tilde{z}_1\succ\tilde{z}_3\succ\tilde{z}_4\succ\tilde{z}_2$.

Step 5. Rank all the subcontractors in the construction engineering $A_i\ (i=1,2,3,4)$ in accordance with $\tilde{z}_i\ (i=1,2,3,4)$: $A_1\succ A_3\succ A_4\succ A_2$. Thus the best subcontractor in the construction engineering is A_1.

6. Conclusion

In this paper, we investigate the multiple attributes group decision making (MAGDM) problems in which attribute values take the form of interval grey uncertain linguistic variables. Firstly, we develop the interval grey uncertain linguistic variables weighted geometric (IGULWG) operator, the interval grey uncertain linguistic variables ordered weighted geometric (IGULOWG) operator and the induced interval grey uncertain linguistic variables ordered weighted geometric (I-IGULOWG) operator and study some desirable properties of the I-IGULOWG operator. Furthermore, we apply the I-IGULOWG operator and the IGULWG operator to the group decision making with interval grey uncertain linguistic information. Finally, an illustrative example for subcontractor selection in the construction engineering with interval grey uncertain linguistic information is given to verify the developed approach and to demonstrate its practicality and effectiveness.

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