Study on the Constant Water Temperature of Bathtub based on 0-1 Planning of Lumped Parameter Method

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Abstract

According to model of temperature distribution of zero dimensional unsteady, the natural convection heat transfer between water is accelerated by human action. The water temperature is almost the same, so the space is reduced to a point. Draw the simulation temperature distribution of people taking a bath. Bubble bath agent on the surface of the bathtub, the heat transfer coefficient of the water in the bathtub is decreased, and the thermal resistance is increased. The limit method is used to calculate the contact area. $S'_{max} = 2S_1$ .Assuming that the bubble is made of a homogeneous l layer, Total thermal conductivity is $k = l\lambda_p/d$ (λ_p is equivalent coefficient of thermal conductivity of air holes; d is pore diameter) 0□1 planning model was established, and the effect of foam on heat transfer was determined by identifying the relationship between the size of 1. $\eta = Q'/Q_3$ (Q' is heat dissipating capacity with air bubbles; Q_3 is heat dissipating capacity without bubble) When $l = 20$, $\eta = 0.43 < 1$.It can be known that there are bubbles, slow down the heat transfer of water and can play the role of insulation.

Keywords

Lumped Parameter Method, 0-1 Planning, Model of Temperature Distribution of Zero Dimensional Unsteady

1. Introduction

Taking a shower is a part of people’s daily life, but ordinary bathtub does not have secondary heating system and circular jet. With time going by, water gradually becomes cold. We must continuously put heating water into bathtub in order to adapt to temperature the body need. We will study the changes in water temperature during movement and the change in water temperature when the water surface is surrounded by bubbles, and how to control water flow and choose the shape of bathtub is the problem we need to urgently resolve.

2. Assumptions

1) Neglecting the spatial distribution of temperature, and simplified the space to be a point.
2) The bubbles cover all the water homogeneous.

3) Bubble's size is homogeneous.
4) Multilayer bubbles superposition homogeneously to form a bubble layer.

3. The temperature distribution model of zero dimensional unsteady (Lumped parameter method)

3.1 Analysis model:

People taking a bath, due to the movement of people accelerating the heat convection between water everywhere is uniform, ignore spatial temperature distribution, the space is simplified to a point, that is, the temperature field is only a function of time, the simplified zero dimensional unsteady problem. And because the lumped parameter method is easy to deal with the related problems of irregular geometric shapes. Therefore, this method is used to solve the effect of the action of the human in the bathtub.

3.2 Model building:

The zero dimensional problem, because the space is reduced to a point, so the internal heat source heat transfer interface.

According to the method, the heat conduction differential equation of the lumped parameter method can be listed:

$$\frac{dt}{d\tau} = \frac{\dot{\Phi}}{\rho c} = -\frac{Ah(t-t_\infty)}{\rho c V}$$

$\dot{\Phi}$ Is heat flux[W];

A is sectional area for vertical heat conduction $[m^2]$;

λ is volume of water[$W/(m\cdot k)$];

Defined excess temperature: $\theta = t - t_\infty$

Heat conduction differential equation with lumped parameter method: $\frac{d\theta}{d\tau} = -\frac{Ah\theta}{\rho c V}$

h Is heat transfer coefficient for convective heat transfer surface[$W/(m^2\cdot K)$].

Initial condition:: $\tau = 0$ $\theta_0 = t_0 - t_\infty$ (t_∞ is water temperature for steady state)

Separation of variables integral solution:

Temperature distribution has nothing to do with the spatial coordinate, and over time temperature changes exponentially with time.

Temperature distribution based on lumped parameter method:

$$\frac{\theta}{\theta_0} = \frac{t-t_\infty}{t_0-t_\infty} = e^{-\frac{hA}{\rho V c}\tau} \tag{1}$$

Define time constant: $\tau_c = \rho c V / hA$

$$\frac{\theta}{\theta_0} = \frac{t-t_\infty}{t_0-t_\infty} = e^{-\frac{\tau}{\tau_c}} \tag{2}$$

The smaller the time constant, the excess temperature of theta changes with time faster, faster response, thermocouple temperature,see Fig.1.

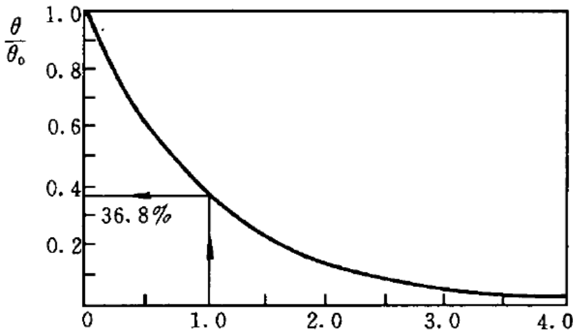

Fig.1 Graph of relation with time

$\tau = \tau_c$ $\frac{\theta}{\theta_0} = 36.8\%$ $\tau = 4\tau_c$ $\frac{\theta}{\theta_0} = 1.83\%$

Heat conduction body has reached thermal equilibrium.

Defined Fourier number:

$$F_{ov} = \frac{\tau}{\rho c \left(V/A \right)^2} = \frac{a\tau}{\left(V/A \right)^2} \tag{3}$$

$$\frac{\theta}{\theta_0} = e^{\left(-Bi_V Fo_V\right)}$$

Non dimensional time: $Fo = \tau / \left(l^2 / a \right)$

l^2/a the time required for the diffusion of thermal disturbance to the area .

Fu Liye number indicates the depth of the unsteady heat conduction process. The larger the F_0 is, the more deeply the thermal disturbance can be spread to the interior of the object. Thus, the temperature of the object is close to the temperature of the surrounding medium.

Transient heat flux based on lumped parameter method:

$$\Phi = hA\theta = hA\theta_0 e^{\left(-\tau/\tau_c\right)} \tag{4}$$

Total heat quantity in 0~τ time based on the method of total parameter:

$$Q_\tau = \int_0^\tau \Phi(\tau) d\tau = \rho V c \theta_0 \left(1 - e^{-\frac{hA}{\rho V c}} \right) [J] \tag{5}$$

The applicable conditions of the lumped parameter method: Biota number

$$Bi = \frac{h\left(V/A \right)}{\lambda} < 0.1M \tag{6}$$

The bathtub shape we designed is a cylinder: $V/A = R/2$ $M = 1/2$.

To meet the above conditions, the difference in excess temperature of each point in the object is less than 5%.

4. 0~1 Planning

When people take a bubble bath, the water will produce bubbles. Bubbles will produce two effects ,see Fig.2.

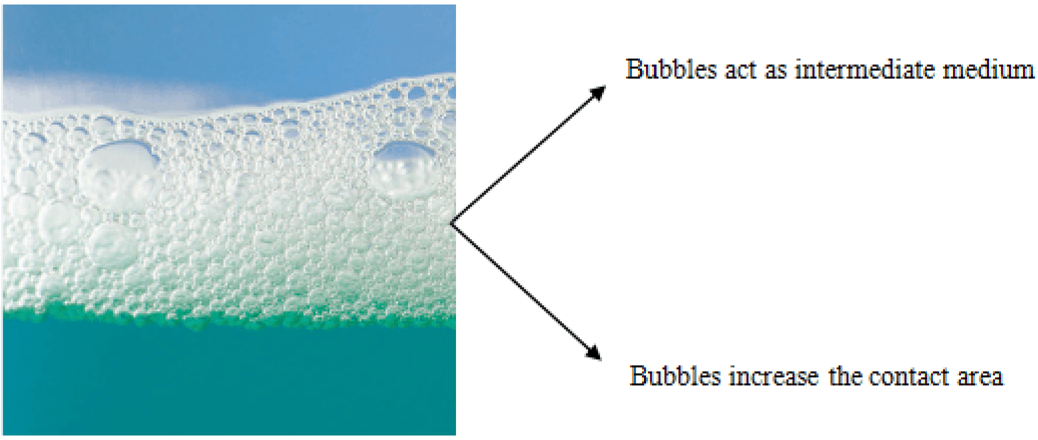

Fig.2 Bubble effect diagram

4.1 Contact area of the outermost bubbles

In the model a solution that the bath on the surface of the transverse section for a circular, the same as the horizontal section of the bubble ,see Fig.3.

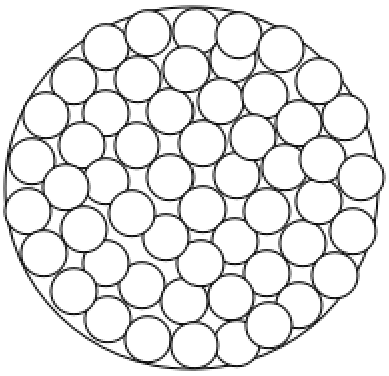

Fig.3 Section of foam layer

Bubbles are uniformly distributed, when the bubble radius r is small enough (m is the number of bubbles):

$$\pi \cdot R^2 = m\pi \cdot r^2 \tag{7}$$

In order to calculate conveniently, assuming that bubbles are arranged in a hemispherical order, the surface area is the largest:

$$S' = \frac{1}{2} \cdot 4\pi \cdot r^2 \cdot m \tag{8}$$

4.2 Bubbles will be separated from the water, bubbles act as an intermediate medium

1)Coefficient of thermal conductivity and thermal resistance

Thermal conductivity coefficient λ is said physical quantity of heat conducting material degree of difficulty, it is material of a important thermophysical parameters unit $W/(m \cdot K)$. because the bubbles are formed by the bubble wall and the hole, so that the heat transfer mechanism is very complex, molecule conduction, elastic wave, quantum theory of radiation and convection, phase transition and.

By Fu Liye's law, the heat conductivity coefficient is defined as:

$$\lambda = \frac{q}{-gradt}.$$

Thermal resistance and thermal conductivity of the corresponding I have an important thermal parameters, using R , and the coefficient of thermal conductivity λ inverse relationship, the expression for the $R=\delta/\lambda$, unit $m^2 \cdot K/W$. Which is the thickness of δ , the unit is m .

- **Total heat transfer coefficient** k

According to the heat transfer theory of porous body, the equivalent coefficient of thermal conductivity can be expressed as:

$$\lambda_p = \lambda_a + \lambda_c + \lambda_y \quad (9)$$

λ_a is thermal conductivity of air molecules in holes;

λ_c is thermal conductivity of air molecular convection in a cavity;

λ_y is Radiation thermal conductivity of hole wall.

If the bubble layer is a vertical bubble layer, and the hole width or diameter is d , then the expression of λ_c and λ_y is:

$$\lambda_c = 1.09 d^{\frac{3}{2}} (t_2 - t_1)^{\frac{1}{2}} \quad (10)$$

$$\lambda_y = 4.19 \times 10^{-2} d \left(T_2^4 - T_1^4\right) / (t_2 - t_1) \quad (11)$$

From the above we can know that under certain conditions, the convection heat transfer coefficient λ_c and the radiation heat transfer coefficient λ_y are respectively with the $3/2$ of the d and one time equation . When the thickness of the air layer is very small, the heat transfer and radiation heat transfer value is small, can be ignored, then the bubble layer of heat conduction is only the heat transfer.
The foam layer is a layer of layers of foam accumulation, assuming that there is a l layer of foam, the total thermal conductivity of k :

$$k = l \frac{1}{d/\lambda_p} \quad (12)$$

In this case, the unit of time for heat loss:

$$Q' = kS'(t_w - t_f) \quad (13)$$

4.3 0~1 planning model establishment

In order to compare whether there is no bubble on the heat dissipation effect, when there is a bubble in the amount of heat Q' and the ratio of Q_3 heat dissipation η .

$$\eta = Q' / Q_3 \quad (14)$$

When $\eta > 1$,bubble can accelerate the amount of heat;

When $\eta = 1$,there is no effect of air bubble on heat dissipation;

When $\eta < 1$,bubble can slow down the amount of heat.

- **Model Solution**

Through the calculation of the $\eta = 0.033 < 1$, we can know that there are bubbles, slow down the amount of heat, the bubble can play a role in insulation.

5. Sensitivity Analysis

- **Continuous heating performance analysis**

In the continuous heating process, the temperature of the bath water temperature heating system is a constant, the heating process is a stable heat transfer process, heat transfer tube heat can be considered to be absorbed by the water. Heat emanating from the pipeline of unit time assumption to a certain value, the change emanating from the heat pipe heat can be mainly used in two ways, one part of heat water absorbed by, to raise the water temperature; another part of heat through the surface of the water will heat is transferred to the air convection thermonuclear radiation radiation heat transfer simultaneously. In order to calculate the simple approximation, it is considered that the heat conservation equation is:

$$Q_1 \Delta\tau = c\rho V(t_{j+1} - t_i) + h_w A_0 (t_i - t_w)\Delta\tau \tag{15}$$

Formula for the heating process of the energy equation, t_i for the i time of the water average temperature, $\Delta\tau$ for the time, the meaning of the other parameters in the formula. For the initial time, the initial temperature of water is $t = t_0$, which constitutes the dynamic simulation control equation and the solution conditions of the continuous heating process. In the case of t_i and Q_1, the temperature of t_{i+1} can be obtained, see Fig.4.

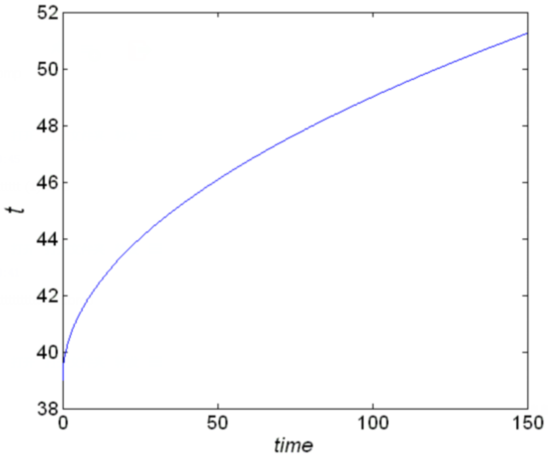

Fig.4 The Temperature Curve of Continuity Heating

From the graph we can see, for the beginning of heating the water in the pool, due to generally lower the temperature of the water, and small temperature difference of the air, maintenance the heat dissipating capacity of the structure is relatively small (negligible) for heat pipe heat is mainly used to improve the water temperature, heating speed is faster. When the water temperature reaches a steady state, namely bath water heating system to thermal equilibrium and heat loss due to the constant heat flow of the whole system, then the heat pipe heat loss and surface air natural convection heat transfer caused by the approximation considered equal.

Reduce the waste of energy, save energy. Bath water temperature heating system of another characteristic is the heating time, achieve different heating rate Q_1 heating pool a predetermined temperature at different times, so again heated time to design temperature are important parameters to evaluate the performance.

6. Weaknesses and Strengths

If the water temperature is almost uniform in the bath, the model is simplified, and it is more efficient to deal with it. And the combination of surface area and heat transfer coefficient are combined together to avoid the restriction of single consideration.

We ignore the spatial distribution of temperature, the space is simplified to a point, to a certain extent, simplify the model. But this can only get the water temperature with the time of the simulation curve, can no longer space analysis of the changes in water temperature. And the size of the bubble may have an impact on the results.

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