Global Exponential Stabilization for Brushless DC Motors

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Abstract

In this paper, global exponential stabilization for chaotic brushless DC motor(BLDCM) is described. Based on Lyapunov theorem and stability theory, we design two controller to realize the global stabilization of BLDCM with exponential convergence rate. Numerical simulations suggest that the motor with the two controllers can converge efficiently.

Keywords

Brushless DC Motor; Chaos; Exponential Stability; Exponential Convergence Rate.

1. Introduction

Due to the superior features on reliability, efficiency and lifetime, the brushless DC motor (BLDCM) has been used extensively in modern motion control applications, such as aerospace[1,2], robotic manipulators [3,4], electric vehicles[5] electric flights [6], automation manufacturing, etc. In the last two decades, many researchers observed that the angle speed of the BLDCM may exhibit aperiodic, random, sudden, or intermittent morbid oscillations with certain parameters [7,9]. Such chaotic behavior can destroy the stable operation of the motor seriously or even induce industrial drive system collapse. Therefore, the stability of BLDCM, especially restraining chaos, is an essential problem for industrial engineering.

In recent years, much effort has been made to control chaos in BLDCM by scientists from various background, and many powerful methods were proposed to stabilize BLDCM. Ren and Chen utilized a piecewise quadratic state feedback method to restrain chaos in the BLDCM[10]. Ge et al. studied chaos anti-control and synchronization in BLDCM by addition of an external nonlinear term [11], backstepping technique[12], and adaptive control[13]. Dadras et al. stabilized a chaotic uncertain BLDCM by both feedback linearization and sliding mode control methods[14]. Uyaroglu and Cevher studied sliding-mode control for single time-scale BLDCM[15]. Wei et al. proposed an adaptive synchronization control method the chaotic brushless DC motors based on the LaSalle invariance principle [16]. All of the above methods is local, uniform or asymptotic, which is weaker than exponential stability. Therefore, how to stabilize BLDCM more efficiently becomes a significant and challenge problem. Some scientists have devoted themselves to find simpler controllers with global exponential stability.

In this paper, we will propose two controllers which can achieve global exponential stabilization for the BLDCM. With the new controllers, numerical simulations suggest that the BLDCM can become stable very efficiently.

The rest of this paper is organized as follows: Section 2 presents the mathematical model of the BLDCM; Section 3 proposes two controllers and discusses the global stability of controlled BLDCM with Lyapunov stability theory; Section 4 verifies the effectivity of controllers by numerical simulations; and Section 5 draws conclusions.

2. Stabilization for chaotic BLDCM

In this section, we first recall the mathematical model of the chaotic BLDCM, then we define the concept of global exponential stabilization, at last we present two different way to stabilize this system by constructing Lyapunov functions.

2.1 Description of the chaotic BLDCM

The mathematical dq- model of BLDCM is given by [12, 17, 18],

$$\begin{cases} \dot{i}_d = u_d - \delta i_d + i_q\omega \\ \dot{i}_q = u_q - i_q - i_d\omega + \gamma\omega \\ \dot{w} = \sigma(i_q - \omega) + \eta i_d i_q - T_L \end{cases} \quad (1)$$

where ω, i_q and i_d denote the angle speed, quadrature-axis and direct-axis currents of the motor, respectively. σ, δ, γ and η are system parameters, which determine the type of the dynamical regime of the motor. Considering the case that the motor is running freely under no loading conditions, namely, $u_q = 0$, $u_d = 0$ and $T_L = 0$. For convenience, let $i_d = x_1$, $i_q = x_2$ and $\omega = x_3$, then system (1) becomes:

$$\begin{cases} \dot{x}_1 = -\delta x_1 + x_2x_3 \\ \dot{x}_2 = -x_2 - x_1x_3 + \gamma x_3 \\ \dot{x}_3 = \sigma(x_2 - x_3) + \eta x_1x_2 \end{cases} \quad (2)$$

System (2) exhibits a chaotic attractor for a wide range of parameters. For example, when we take the following two groups of parameters and initial values:

$\sigma = 4$, $\gamma = 55$, $\delta = 0.875$, and $x(0) = [30, 20, -10]^T$

the phase portraits of the chaotic attractor are shown in Fig. 1. In the following paper, we are going to stabilize the initial values of (2) to the zero equilibrium.

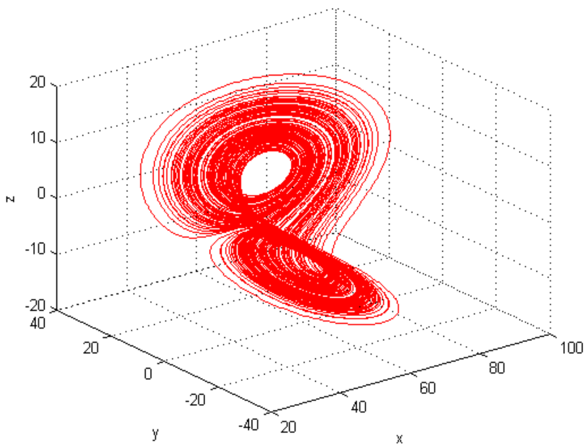

Fig. 1 A phase portrait of the chaotic attractor of (2)

2.2 Global Exponential Stability

Consider the following general nonlinear dynamical system

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(t, \mathbf{x}) \quad (3)$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T \in R^n$, $\mathbf{f} = (f_1, f_2, \cdots, f_n)^T$ is a continuous function from $I \times R^n$ to R^n ensuring existence and uniqueness of solutions of (3), and the origin is an equilibrium, i.e., $\mathbf{f}(t, \mathbf{0}) \equiv 0$. Here, I

indicates the time interval $[t_0,+\infty)$. Then, exponential stability, rate of convergence and a criteria of global exponential stability can be described as follows[19,20]

Definition 1 $\mathbf{x}=\mathbf{0}$ is called a globally exponentially stable equilibrium of (3) if there exist $c,\alpha>0$ such that solution $\mathbf{x}(t)$ satisfies

$$\|\mathbf{x}(t)\|<c\mathrm{e}^{-\alpha(t-t_0)}\|\mathbf{x}(t_0)\|$$

For all $\mathbf{x}(t_0)\in R^n$, and the constant α is called the rate of convergence.

Lemma 1 If there exists a radially unbounded positive-definite Lyapunov function $V(t,x)\in C^1[I\times R^n,R_+]$ satisfies:

1) $\mu\|\mathbf{x}\|^\lambda\le V(t,\mathbf{x})$;

2) $\frac{\mathrm{dV}}{\mathrm{dt}}\le-\beta V(t,\mathbf{x})$,

where $\mu,\lambda,\beta>0$, then the zero solution of (3) is exponentially stable and an estimation is as follows.

$$\|\mathbf{x}(t)\|<(\frac{V(x_0)}{\mu})^{\frac{1}{\lambda}}\mathrm{e}^{-\frac{\beta}{\lambda}(t-t_0)}$$

Especially, when $\lambda=2,\mu=1$, we have

$$\|\mathbf{x}(t)\|<V(x_0)^{\frac{1}{2}}\mathrm{e}^{-\frac{\beta}{2}(t-t_0)}$$

According to this lemma, when $V(t,\mathbf{x})=\|\mathbf{x}(t)\|^2$, $\|\mathbf{x}(t)\|<\mathrm{e}^{-\beta(t-t_0)/2}\|\mathbf{x}(t_0)\|$, which suggests that the rate of convergence $\alpha=\beta/2$.

2.3 The design of two controllers

In order to stabilize the chaotic BLDCM via the quadrature-axis current, we add a single controller u to the second equation of system (2) as Wei et al. did in [17]. And then the controlled system can be expressed as

$$\begin{cases}\dot{x}_1=-\delta x_1+x_2x_3\\ \dot{x}_2=-x_2-x_1x_3+\gamma x_3+u\\ \dot{x}_3=\sigma(x_2-x_3)+\eta x_1x_2\end{cases}\qquad(4)$$

According to Lemma 2 the key to global exponential stabilization of (4) is to find an appropriate controller u and an applicable Lyapunov function, which is often a hard job.

In the following paper, we try to find control law u such that the global exponential stabilization can still be guaranteed. Besides, we also estimate the exponential convergence rate of

Theorem 1 In system (4), take the following simple feedback control law:

$$u=-(\gamma+\sigma+\eta x_1)x_3\qquad(5)$$

then the zero solution of controlled system (4) is globally exponentially stable, where the guaranteed exponential convergence rate is given by: $\alpha=\min\{\sigma,1,\delta\}$.

Proof : Let us construct the following radially unbounded Lyapunov function:

$$V(\mathbf{x}(t))=x_1^2+x_2^2+x_3^2\qquad(6)$$

We take $\sigma=4$, $\gamma=55$, $\delta=0.875$, and $x(0)=[30,20,-10]^T$. it is easy to see that condition 1) of **Lemma 1** is satisfied, i.e., $\|\mathbf{x}\|\to+\infty$ implies $V(x(t))\to+\infty$.

Now, we prove that the second condition of **Lemma 1** is satisfied. Take the derivative of $V(x(t))$ along system (4), it is found to be

$$\begin{aligned}\dot{V}(x(t)) &= 2(x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3) \\ &= 2(x_1(-\delta x_1 + x_2x_3) + x_2(-x_2 - x_1x_3 + \gamma x_3 - (\gamma + \sigma + \eta x_1)x_3) + x_3(\sigma(x_2 - x_3))) \\ &= 2(-\delta x_1^2 - x_2^2 - \sigma x_3^2) \\ &\le -2\min(\sigma,1,\delta)(x_1^2 + x_2^2 + x_3^2) \\ &= -2\min(\sigma,1,\delta)V(x(t))\end{aligned}$$

So, by **Lemma 1**, we have

$$V(x(t)) \le V(x(t_0))\mathrm{e}^{-2\min(\sigma,1,\delta)(t-t_0)}$$

Thus, the zero equilibrium is globally exponentially stable. Since $V(x(t)) = \|\mathbf{x}(t)\|^2$, we have

$$\|\mathbf{x}(t)\|^2 \le \|\mathbf{x}(t_0)\|^2 \mathrm{e}^{-2\min(\sigma,1,\delta)(t-t_0)}$$

According to **Definition 1,** the exponential convergence rate of $\|\mathbf{x}(t)\|$ is $\alpha = \min\{\sigma,1,\delta\}$.

Now, we will discuss adding a single controller u to the third equation of system, then the controlled system can be expressed as:

$$\begin{cases}\dot{x}_1 = -\delta x_1 + x_2x_3 \\ \dot{x}_2 = -x_2 - x_1x_3 + \gamma x_3 \\ \dot{x}_3 = \sigma(x_2 - x_3) + u\end{cases} \tag{7}$$

For this controlled system, we can also find two simple control law u with just one state variable such that the global exponential stabilization can still be guaranteed. First, let the angle speed x_2 (i.e., w) be our state variable in the controller, then we have the following theorem.

Theorem 2 If the controller u is designed as

$$u = -(\gamma + \sigma + \eta x_1)x_2 \tag{8}$$

Then, the controlled system (9) is globally exponentially stable at the zero equilibrium, where the guaranteed exponential convergence rate is given by: $\alpha = \min\{\sigma,1,\delta\}$

Proof : We still take (7) to be our Lyapunov function which satisfies condition 1) of Lemma 2, then the derivative of Lyapunov function along system (9) is as follows：

$$\begin{aligned}\dot{V}(x(t)) &= 2(x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3) \\ &= 2(x_1(-\delta x_1 + x_2x_3) + x_2(-x_2 - x_1x_3 + \gamma x_3) + x_3(\sigma(x_2 - x_3) - (\sigma + \gamma + \eta x_1)x_2)) \\ &= 2(-\delta x_1^2 - x_2^2 - \sigma x_3^2) \\ &\le -2(\min(\sigma,1,\delta))(x_1^2 + x_2^2 + x_3^2) \\ &= -2(\min(\sigma,1,\delta))V(x(t))\end{aligned}$$

Therefore,

$$V(x(t)) \le V((x(t_0))\mathrm{e}^{-2\min\{\sigma,1,\delta\}(t-t_0)},$$

Thus, the zero equilibrium is globally exponentially stable. Since $V(x(t)) = \|\mathbf{x}\|^2$, we have

$$\|\mathbf{x}(t)\|^2 \le \|\mathbf{x}(t_0)\|^2 \mathrm{e}^{-2\min\{\sigma,1,\delta\}}$$

According to **Definition 1,** the exponential convergence rate of $\|\mathbf{x}(t)\|$ is $\alpha = \min\{\sigma,1,\delta\}$

3. Numerical Simulations

In Section 2, we have proposed two controllers theoretically. Now, let us verify their effectivity with numerical simulations. The numerical results are obtained with a fourth-fifth order Runge-Kutta method in MATLAB software.

First we stabilize the system via the quadrature-axis current. we choose $\sigma = 4$, $\gamma = 55$, $\delta = 0.875$, and $x(0) = [30, 20, -10]^T$, We add controller $u = -(\gamma + \sigma + \eta x_1)x_3$ to the second equation of (2), i.e., the controlled system(4). According to Theorem 1, the zero equilibrium is globally exponentially stable, and the converge rate $\alpha = \min\{\sigma, 1, \delta\} = \sigma = 0.875$. The simulation results are shown in Fig. 2. It is clear to see that all the stable variables and their norm $\|\mathbf{x}\|$ converge to zero exponentially.

Second, we stabilize the system via the angle speed with controlled system(7). The controller u takes $-(\gamma + \sigma + \eta x_1)x_2$. According Theorem 2, the zero equilibrium is globally exponentially stable and the converge rate is also $\alpha = \min\{\sigma, 1, \delta\} = \sigma = 0.875$, as shown in Fig.3.

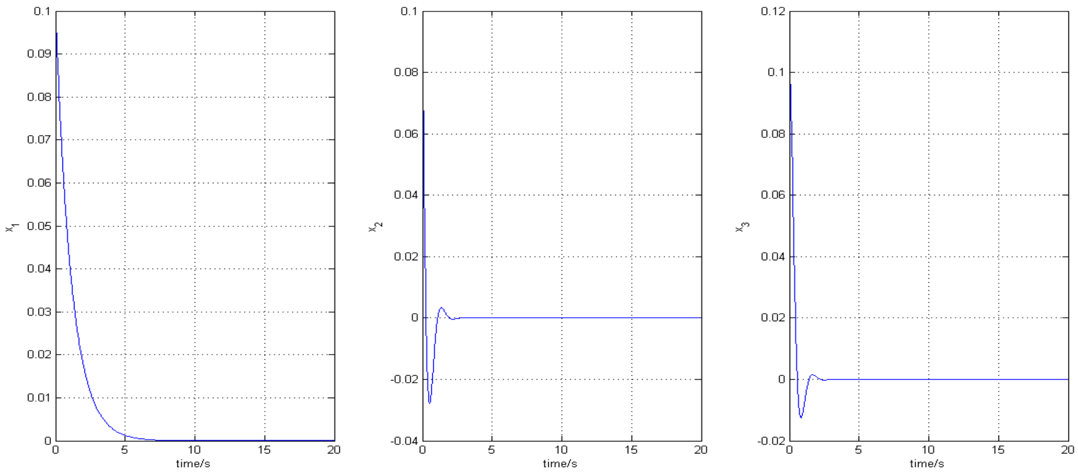

Fig. 2 Simulation results of the tests for the controller(5)

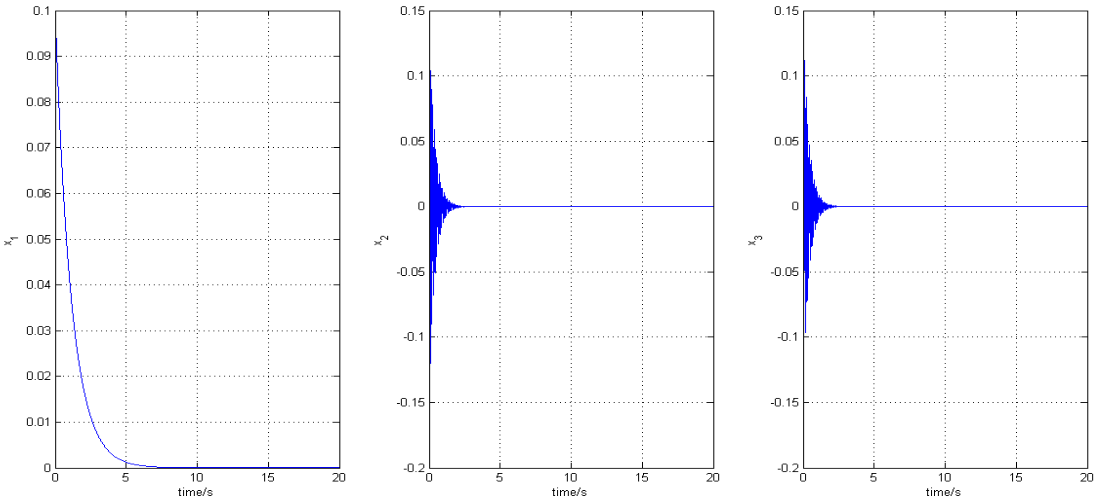

Fig. 3 Simulation results of the tests for the controller(5)

4. Conclusions

We have studied the global exponential stabilization for chaotic brushless DC motors by Lyapunov stablility theory. We propose two controllers to realize the global stabilization of BLDCM with exponential convergence rate. The theory and numerical simulation results show the effectiveness of the proposed methods at the same. So, the control strategy is feasible.

Acknowledgments

This work is supported in part by National Natural Science Foundation of China (61104150), Science Fund for Distinguished Young Scholars of Chongqing (cstc2013jcyjjq40001).

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