Non-linear Model Parameter Estimation based on Quantum Genetic Algorithm

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Abstract

Describes the quantum genetic algorithm (QGA) overview of the development, from individual behavior and the evolution of both groups, describes the basic principles of quantum genetic algorithm, the solution step and so on; and it is applied to estimate the parameters of the nonlinear model, Finally, an example analyzes the feasibility and advantages of quantum genetic algorithm to nonlinear parameter estimation applied.

Keywords

Quantum Genetic Algorithm; Non-linear Model; Parameter Estimation;Swarm Intelligence; Evolutionary Computing.

1. Introduction

The results of high-precision observation have the characteristics of complexity, constraint, non-linearity, multi-pole value and difficult modeling et al. in data processing in practice of surveying and mapping works. Intelligent algorithm as an advanced data processing tool has been incorporated into the field of surveying and mapping, has now become an important research direction. These algorithms are generally developed by simulating some natural phenomena and can effectively solve some problem that traditional methods cannot solve. However, any single algorithm will have some shortcomings. And they have to be constantly tested and improved in practical applications. The most obvious drawback of the genetic algorithm is the convergence problem. It includes problems such as slow convergence and premature convergence (easy to fall into local optimum) [1,2] (Li Shiyong, 1998, 2006). To make continuous improvement of the algorithm, we cannot only to adjust and find the optimal parameters, but also consider the advantages of absorbing other types of algorithms. Under the inspiration of this "combinatorial optimization", the fusion algorithm of various distinct algorithms is expected to be a new way to study the future intelligent algorithm.

Quantum Genetic Algorithm (QGA) is a new multi-algorithm fusion technology, which is a combination of quantum theory and the genetic algorithm. The quantum genetic algorithm was first proposed by Narayanan[3] (1996) et al. Quantum genetic algorithms inspired by quantum computing is proposed, and the concept of quantum multi-universe is included in genetic algorithms. Quantum bits and quantum revolving gates, which is preferable to traditional genetic algorithm, are introduced into the genetic algorithm[4](Han, 2000).

In recent years, the development of quantum information technology has shown superior computing power. This super power is mainly dependent on the quantum mechanism. Combining quantum algorithm with intelligent optimization, a new efficient optimization algorithm is generated. The new algorithm not only preserves the advantages of intelligent algorithms, but also absorbs the benefits of quantum theory, so the new algorithm becomes more effective[5] (Laboudi Z, 2012). In this paper, the quantum genetic algorithm is applicable to the field of surveying and mapping. The basic principle and the solving steps of the algorithm are presented. The feasibility of the algorithm is applicable to the nonlinear model of data processing. It can provide a new analytical approach to the nonlinear problem.

2. Methodology

The idea of Quantum computation is introduced into the genetic algorithm, and the quantum genetic algorithm is formed. The most important content of the algorithm is the construction of the coding and evolutionary strategies of the population. Using some concepts and theories of quantum computing(such as quantum bits, quantum superposition, and other tectonic chromosomes) is the essence of population coding. The information of multiple states can be simultaneously characterized by a quantum chromosome in this way. Strong parallelism is implied, and population diversity and avoidance of selection pressures can be maintained. Build on the information of the current optimal individuals, evolutionary search is done through quantum gate functions and quantum gate updates. Strong parallelism is implied, and population diversity and avoidance of selection pressures can be maintained. Based on the information of the current optimal people. Evolutionary search is done through quantum gate functions and quantum gate updates.

Individuals in the quantum genetic algorithm are chromosomes containing multiple qubits, which are characterized by superposition and entanglement. A quantum chromosome can present a plurality of discrete states. Quantum chromosomes are updated by means of quantum revolving gates or quantum nematic and other variation mechanisms, and rich species diversity is obtained. With constant iterations, the superposition of each qubit will collapse to a definite state, which will stabilize and tend to converge. Through such a way, quantum genetic algorithm continues to explore, evolve, and finally achieve the goal of optimization. The chromosome is present as a quantum state vector in quantum coding. A chromosome is given in a superposition of multiple states. In this approach, the diversity of the population has increased. So that the algorithm can obtain the optimal solution under the smaller population size. The introduction of quantum gate algorithm has the capability to develop and explore the ability to ensure the convergence of the algorithm.

2.1 The Theoretical Basis of Quantum Genetic Algorithm

2.1.1 Quantum chromosome

The quantum genetic algorithm has a population that can be iteratively evolved and uses chromosomes as information carriers. Each chromosome in the population is the basis for the algorithm. So the structure of the chromosome is also the application of such algorithms to the primary problem. Quantum chromosomes directly contain not the solution of the problem, but the qubit in quantum computation. Quantum chromosomes are constructed in a quantum-based coding manner. The information of a certain state is reflected in the form of probability.

The qubit has a superposition. So the new individual is produced by the probability of the qubit. The state is no longer part fixed on every bit of the information. A chromosome does not correspond to only a specified state, but instead becomes a kind of information carrying different superposition states. Because of this nature, the evolutionary algorithm based on quantum chromosome coding has better population diversity than traditional evolutionary algorithms. After several iterations, the probability amplitude for a certain qubit is close to 0 or 1. The diversity of this uncertainty will gradually disappear, eventually collapsed to a definite state, so that the algorithm eventually converge. This indicates that quantum chromosomes have both the ability to study and develop at the same time. Therefore, compared with the genetic algorithm, the quantum genetic algorithm can get the optimal solution under the smaller population size.

2.1.2 Quantum revolving door

The population update can be obtained by a quantum gate transformation matrix. As the actuator of the evolutionary operation, the quantum gate can be chosen according to the specific problem. The choice of quantum gate can affect the convergence performance of the algorithm. At present there are various kinds of quantum doors. According to calculation characteristics of quantum genetic algorithm, it is more appropriate to select quantum revolving door.

In the quantum theory, the transfer between the various quantum states is mainly achieved through the quantum gate. Probability amplitude angle of the qubit is rotated by the quantum gate, and the change of the quantum state can also be realized. Thus, in quantum genetic algorithms, the variation of quantum chromosomes can be achieved using quantum revolving gates. Since the information of the optimal individual is considered as the angle of rotation, thus the population tends to optimize the optimal solution under the guidance of the optimal individual information. So the convergence of the algorithm is speeded up.

2.1.3 Quantum non-portal variation

Chromosome variation can be achieved using quantum non-gate. First of all, quantum chromosomes need to be mutated were randomly selected from the population. Then, the mutation is made on a number of quantum bits of these quantum chromosomes. The variant operation of the quantum-gate implementation is essentially interchangeable by the two probable plans of the qubit. Since the state of the quantum bit state superposition is changed, the tendency to collapse in state 1 tends to collapse to state 0 or vice versa. So it played a role in the variation. Obviously, the mutation operation has the exact same effect on all superposition states of the chromosome.

From another point of view, this variation is equally a rotation of the magnitude of the qubit. If the amplitude of a quantum bit is q, the amplitude is ($\pi/2$-q) after variation. That is, the amplitude is rotated by $\pi/2$. This rotation does not compare with the current best chromosome. But rotated all in the positive direction. This contributes to the increase in the diversity of the population. And the probability of premature convergence is reduced.

2.2 The basic flow of quantum genetic algorithm

The basic flow of quantum genetic algorithm is as follows:

A) Given algorithm parameters, including population size, maximum number of iterations, crossover probability, mutation probability;

B) Population initialization;

C) Assess the fitness of each identified solution;

D) Store the optimal solution, record the optimal individual and the corresponding fitness;

E) To determine whether the calculation process can be completed: if the end of the conditions to meet; otherwise continue to calculate;

F) Perform a measure of each individual in the population to obtain the corresponding definite solution;

G) To assess the accuracy of the solution;

H) The use of quantum revolving door to adjust the individual to get a new population;

I) Record the optimal individual and the corresponding fitness;

J) Increase the number of iterations in step a) and return to step e).

3. Example Analysis

3.1 Example

This example is taken from Example 2-1-1 of reference [6] (Wang Xinzhou, 2002). In this case, the nonlinear model is known as: $L_i = x_1 e^{ix_2}$. The true value of the parameter is $X = (5.420136187 \ -0.25436189)'$. This example requires parameter estimation. The corresponding five independent observations with the accuracy are listed in Table 1. The results of the quantum genetic algorithm are compared with those of other algorithms (Table 2).

Table 1. The true value of L_i and the corresponding observations

Numble	True value	Observations
1	4.202 834	4.20
2	3.258 924	3.25
3	2.527 006	2.52
4	1.959 469	1.95
5	1.519 394	1.51

In Table 2, for true errors (the penultimate2), linearize least squares estimation has the largest error (0.0263). The results of the following two algorithms are closer (0.0028 and 0.0017). The consequences of quantum genetic algorithm (0.0017) were better than those of other algorithms. The main reason is that the generalized least squares estimation is linearly approximated in the calculation. In this process, a part of the model error due to the linear approximation is generated, and the error in this part is larger than the error in the observation. Avoiding the model error caused by linear approximation,nonlinear least squares and intelligent optimization algorithms do not, need to linearize the nonlinear observation equations. So the results of the latter two methods are much better than the least squares estimation.

Table 2. Comparison of calculation results of various algorithms

Algorithm name	$\hat{x}_1$	$\hat{x}_2$	$\|\Delta X\|$
True value	5.420 136	-0.254 362	----
Least squares method	5.394 142	-0.250 246	0.026 3
Genetic algorithm	5.420 016	-0.257 130	0.002 8
Quantum genetic algorithm	5.423 806	-0.255 871	0.001 7

The results of quantum genetic algorithm are better than those of other algorithms. The reasons are as follows: for genetic algorithms, individual variation is achieved by mutation operator. In other words, different individuals produce by changing one or several genes. The gene of the genetic algorithm expresses a precise information. In the quantum genetic algorithm, the genes expressed by the quantum bits contain all the possible information because of the superposition of the quantum information. In the case of constant population size, the number of candidate pairs of quantum genetic algorithm is twice as high as that of the genetic algorithm. The diversity of the solution space is increased and the probability of success is improved. In addition, in the quantum genetic algorithm, the quantum revolving door is utilized to change the phase of the quantum bit. The purpose is to update the probability of quantum beat amplitude and achieve the effect of genetic variation.

3.2 Analysis of Group Intelligence Search Strategy Based on Quantum Genetic Algorithm

3.2.1 Analysis of Information Interaction between Individual Bank and Individual

Individual information is expressed based on the quantum state vector in the quantum genetic algorithm. Since each quantum chromosome can represent the superposition of multiple quantum states, the evolutionary algorithm using quantum bit coding has better population diversity than traditional evolutionary algorithms. Even if a smaller quantum group is chosen, the performance of the algorithm is not covered.

For the quantum genetic algorithm, although the probability of observation in the linear superposition state of quantum chromosomes can also be changed by using traditional crossover and mutation operator, performance of quantum genetic algorithm will be declined significantly. The reason is perhaps that the more abundant species diversity brought by the superposition of the quantum chromosome itself. And the probability of using crossover and mutation will be more higher.

In quantum genetic algorithms, evolutionary operations are done by quantum gates. In contrast, in quantum theory, the transfer between states is achieved by the quantum gate transformation matrix[7] (Yang Shuying, 2012). The rotation angle of the current optimal solution is invoked as the expression of quantum chromosome variation in quantum revolving door operation. The direction and size of the rotation angle are determined by observing the state of the optimal individual and the corresponding unusual bits of the current individual and comparing their fitness values. Since the rotation angle is related to the probability of each bit of the quantum chromosome, the mutation of each bit of the quantum chromosome is realized to produce a new individual. Search process of quantum revolving door driven is the process of optimal discrete information driven to the optimal solution approximation. It leads the direction of the evolution of the population, allowing the population to evolve towards the optimal region with maximum probability, which makes the convergence rate of the algorithm more faster. In order to avoid premature and local extremes, on this basis, we can further use the quantum nematic to achieve chromosome mutation operation, which can maintain the diversity of species and avoid the choice of pressure.

3.2.2 Group evolution analysis

In order to achieve group updates,individuals across and mutate to generate new individuals in evolutionary algorithms. And select the outstanding individuals to form the next generation of the population. By selecting the operator based on the fitness value. Various evolutionary operators have completed the diversity expansion of the population, stipulated the evolution direction, and realized the biological evolution mechanism of "survival of the fittest".

Cross and selection operators of traditional genetic operators are not used in quantum genetic algorithms. Group updates through quantum revolving doors and quantum kernels. The preferred operation is just not performed. Each quantum chromosome is rotated toward the corresponding amplitude of the optimal solution by the introduction of the optimal individual information. So that the population can be upgraded to make the overall trend of the population become better[7] (Yang Shuying, 2012). Quantum genetic algorithm has high search efficiency, wide adaptability and rapid convergence.

Each qubit is given in itself and the current optimal solution. This mechanism may appear to be due to the tendency of each individual to optimize the individual, resulting in a decrease in population diversity and a high population density. This easily leads to precocious, into the local optimal. However, due to the uncertainty caused by the superposition of quantum chromosomes, a very rich population diversity is produced, which reduces the risk of early maturation.

3.2.3 The realization of population diversity

Compared with the traditional genetic algorithm, the quantum genetic algorithm increases the diversity of the population, and summarizes the following characteristics:

A) In the process of iteration, the effective size of the population will continue to decrease, but diversity will increase. This is because the superposition of quantum bits indicates that the individual quantum superposition states tend to be the same and the effective size of the population decreases. Nevertheless, by the addition of quantum gates, the difference (diversity) between individuals increases.

B) Even if the same degree of fitness, individual similarity may not be the same, or even a great difference. In this way, the opportunities for better people will increase significantly.

C) Excellent genes can be preserved to a great extent. The operation object is the superposition of each individual. In this process, the possibility of advantageous genes being eliminated is minimized and well inherited.

4. Conclusion

The principle of quantum genetic algorithm and the concrete implementation steps is introduced in this paper. Convergence time, population diversity increase, quantum gate rotation angle and search step size control was discussed. Aiming at the problem of nonlinearity, multi-extremum and so on in the measurement data processing, quantum genetic algorithm can be employed as an effective solution. The results of this paper show that the quantum genetic algorithm is a global and efficient inversion algorithm, especially for non-linear, multi-parameter, multi-polar problem. And it has wide application prospect.

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