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# Optimization Design of the Pitch Gear Reducer for A Port Cranes

Yun Xu <sup>1, 2, a</sup>, Yuankai Meng <sup>1, 2, b</sup> and Yinghua Liao <sup>1, 2, c</sup>

<sup>1</sup>School of Mechanical Engineering, Sichuan University of Science & Engineering, Zigong 643000, China;

<sup>2</sup>Sichuan Provincial key Lab of Process Equipment and Control, Zigong 643000, China.

<sup>a</sup>1214004131@qq.com, <sup>b</sup>Bryant413@163.com, <sup>c</sup>liaoyinghua118@163.com

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## Abstract

The traditional reducer design is based on the experience of the designer, so design is usually not optimal. The port crane pitch gear reducer is a series three stage helical gear reducer, which is more prominent in the design of the three stage helical gear reducer. A mathematical model of optimization design is established for three grade helical cylindrical gear reducer to ensure the bearing capacity and performance specifications of gear, and achieve three volume and weight reducer. And the minimum value of the gear center distances is regarded as the optimization objective values of the model. Furthermore, the MATLAB optimization toolbox is used to simplify the complexity of programming and improve efficiency and quality of the design.

## Keywords

Three-grade helical cylindrical gear reducer; Optimization design; Single objective optimization; Objective function.

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## 1. Introduction

Gear reducer which can reduce speed and increase torque is an independent closed mechanical transmission device between prime mover and working machine. In recent years, the optimization design on the gear reducers have been widely discussed in many literatures [1-3], especially those about how to select the optimization design variables, objective function and the constraint function of the optimization model. However, the systematic and comprehensive study on the optimization design of the three-stage cylindrical gear reducer is relatively rarely, and there are many problems which are worth studying and exploring. The advanced MATLAB optimization toolbox is regarded as a simple and effective means for the optimization design by which a mathematical model for the three-stage helical gear reducer for the port crane can be constructed to accomplish its optimization design [4-6], and the method mentioned in this paper also can provide some basis for the optimization design for the other cylindrical gear reducer.

## 2. Optimization model of a three stage helical cylinder gear reducer

### 2.1 Design Representation

Figure 1 shows the three-stage helical cylindrical gear reducer for a port crane. The input power in high-speed shaft  $p=375\text{KW}$ , and the input speed of the motor drive for high-speed shaft  $n_1=2000\text{r/min}$ . The reducer works 8 hours a day, 200 days per year. The operation life is at least 20 years. The gear ratio is  $i=102$ , and the tooth width coefficient is  $\phi_d=b/d=0.8$ , the gear transmission efficiency of each stage is  $\eta=0.97$ . The material of the large gears is 45 steel (quenched and tempered), whose tooth

surface hardness is HB216 and the material of the small gears is 40Cr (quenched and tempered), whose tooth surface hardness is HB250.

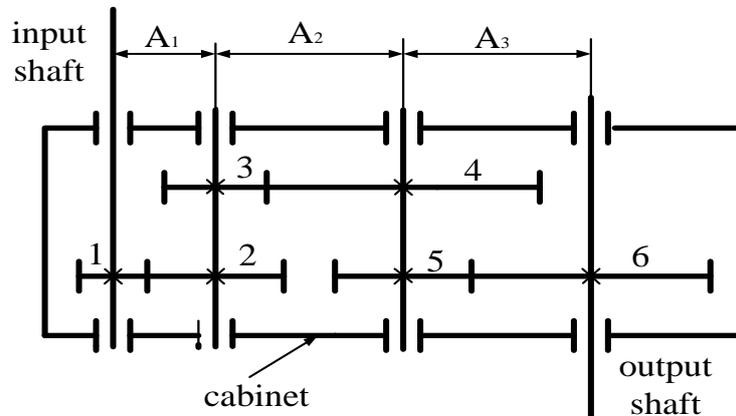


Fig. 1 Three-grade helical cylindrical gear reducer for a port crane

### 2.2 Objective function

The decelerator optimization of the three-grade helical cylindrical gear reducer involves many parameters such as gear strength, performance specifications and other conditions of load capacity, so the decelerator optimization design is a process of multi-objective optimization. But in practice, the most important parameters are often used as the optimization target, the other parameters as constraints single objective optimization is converted to improve the accuracy and efficiency of optimization. The port cranes lightweight design’s goal is to minimize the total center distance and to make the reducer smaller and lighter. Therefore, total minimum Center distance as the optimization goal, build three-grade helical cylindrical gear reducer lightweight single-objective optimization model for gearbox[7-11].

As Fig.1 is shown the total center distance is of the three-stage helical gear reducer.

$$A = A_1 + A_2 + A_3 = \frac{m_{n12}z_1(1+i_{12})}{2 \cos \beta_{12}} + \frac{m_{n34}z_3(1+i_{34})}{2 \cos \beta_{34}} + \frac{m_{n56}z_5[1+i/(i_{12} \cdot i_{34})]}{2 \cos \beta_{56}} \quad (1)$$

### 2.3 Design variables

There are eleven independent design variables in equation (1) below:

$$x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}]^T = [z_1, i_{12}, z_3, i_{34}, z_5, m_{n12}, m_{n34}, m_{n56}, \beta_{12}, \beta_{34}, \beta_{56}]^T \quad (2)$$

$F(x) = A$ ,

the target function can be expressed as:

$$f(x) = \frac{x_6 \cdot x_1(1+x_2)}{2 \cos x_9} + \frac{x_7 \cdot x_3(1+x_4)}{2 \cos x_{10}} + \frac{x_8 \cdot x_5[1+i/(x_2 \cdot x_4)]}{2 \cos x_{11}} \quad (3)$$

The objective value of the function  $f(x)$  is the minimum value, namely  $\min f(x)$ .

### 2.4 Constraint condition

#### 2.4.1 Tooth number of a helical gear

For helical gears without undercut, the minimum number of teeth is usually equal to 15, and constraint condition is:

$$\begin{cases} g_1(x) = 15 - \frac{x_1}{\cos^3 x_9} \leq 0 \\ g_2(x) = 15 - \frac{x_1 \cdot x_2}{\cos^3 x_9} \leq 0 \\ g_3(x) = 15 - \frac{x_3}{\cos^3 x_{10}} \leq 0 \\ g_4(x) = 15 - \frac{x_3 \cdot x_4}{\cos^3 x_{10}} \leq 0 \\ g_5(x) = 15 - \frac{x_5}{\cos^3 x_9} \leq 0 \\ g_6(x) = 15 - \frac{x_5 \cdot \frac{102}{x_2 \cdot x_4}}{\cos^3 x_9} \leq 0 \end{cases} \quad (4)$$

2.4.2 Modulus of a helical gear in Normal plane

The modulus of a power transmission gear is usually not less than 2mm, and then the constraint condition is:

$$\begin{cases} g_7(x) = 2 - x_6 \leq 0 \\ g_8(x) = 2 - x_7 \leq 0 \\ g_9(x) = 2 - x_8 \leq 0 \end{cases} \quad (5)$$

2.4.3 Helical angle of a helical gear

The helix angle of helical gear is  $8^\circ \sim 20^\circ$ , and then the constraint condition is:

$$\begin{cases} g_{10}(x) = 8^\circ \times \frac{\pi}{180^\circ} - x_9 \leq 0 \\ g_{11}(x) = x_9 - 20^\circ \times \frac{\pi}{180^\circ} \leq 0 \\ g_{12}(x) = 8^\circ \times \frac{\pi}{180^\circ} - x_{10} \leq 0 \\ g_{13}(x) = x_{10} - 20^\circ \times \frac{\pi}{180^\circ} \leq 0 \\ g_{14}(x) = 8^\circ \times \frac{\pi}{180^\circ} - x_{11} \leq 0 \\ g_{15}(x) = x_{11} - 20^\circ \times \frac{\pi}{180^\circ} \leq 0 \end{cases} \quad (6)$$

2.4.4 Reduction ratio between two adjacent levels

In order to adopt the oil bath lubrication, the immersion depth of the gears in one level is approximately equal to that in the other level for a multistage gear reducer. For a three-stage pitch gear reducer of port crane, value ranges  $i_{12}=(1.18\sim 1.62) i_{34}$ ,  $i_{34}=(1.18\sim 1.62) i_{56}$ , and obtain the constraint condition :

$$\begin{cases} g_{16}(x) = 1.18 \cdot x_4 - x_2 \leq 0 \\ g_{17}(x) = x_2 - 1.62 \cdot x_4 \leq 0 \\ g_{18}(x) = \frac{120.36}{x_2 \cdot x_4} - x_4 \leq 0 \\ g_{19}(x) = x_4 - \frac{165.24}{x_2 \cdot x_4} \leq 0 \end{cases} \quad (7)$$

2.4.5 Conditions of no interference between high speed gear and low speed shaft

$$\begin{cases} g_{20}(x) = \frac{x_6 \cdot x_1 \cdot x_2}{x_9} + 20 - \frac{x_7(x_3 + x_3 \cdot x_4)}{x_{10}} \leq 0 \\ g_{21}(x) = \frac{x_7 \cdot x_3 \cdot x_4}{x_{10}} + 20 - \frac{x_8(x_5 + \frac{102 \cdot x_5}{x_2 \cdot x_4})}{x_{11}} \leq 0 \end{cases} \quad (8)$$

#### 2.4.6 Bending fatigue strength of the tooth root at the high speed level

From the formula 10-17 in the reference [1], high-speed pinion and large gear root bending fatigue strength constraints are:

$$\begin{cases} g_{22}(x) = \frac{17563319.69 \cdot \cos^2 x_9}{x_6^3 \cdot x_1^2} - 210 \leq 0 \\ g_{23}(x) = \frac{16350511.57 \cdot \cos^2 x_9}{x_6^3 \cdot x_1^2} - 161 \leq 0 \end{cases} \quad (9)$$

#### 2.4.7 Tooth surface contact fatigue strength of the gears at the high speed level

The contact fatigue strength constraint of high-speed pinion and large gear tooth surface can be obtained from equations (10) – (22) in reference [1]:

$$g_{24}(x) = 355.6 \sqrt{\frac{6714862.5 \cdot (1 + x_2)}{x_6^3 \cdot x_1^3 \cdot x_2}} \leq 0 \quad (10)$$

#### 2.4.8 Bending fatigue strength of the tooth root at intermediate level

$$\begin{cases} g_{25}(x) = \frac{77514000.09 \cdot \cos^2 x_{10}}{x_7^3 \cdot x_3^2} - 199 \leq 0 \\ g_{26}(x) = \frac{71226151.13 \cdot \cos^2 x_{10}}{x_7^3 \cdot x_3^2} - 153 \leq 0 \end{cases} \quad (11)$$

#### 2.4.9 Tooth surface contact fatigue strength of the gears at the intermediate level

$$g_{27}(x) = 355.6 \sqrt{\frac{28845037.5 \cdot (1 + x_4)}{x_7^3 \cdot x_3^3 \cdot x_4}} \leq 0 \quad (12)$$

#### 2.4.10 Bending fatigue strength of the tooth root at the low speed level

$$\begin{cases} g_{28}(x) = \frac{352199295.7 \cdot \cos^2 x_{11}}{x_8^3 \cdot x_5^2} - 186 \leq 0 \\ g_{29}(x) = \frac{323629282.9 \cdot \cos^2 x_{11}}{x_8^3 \cdot x_5^2} - 143 \leq 0 \end{cases} \quad (13)$$

#### 2.4.11 Tooth surface contact fatigue strength of the gears at the low speed level

$$g_{30}(x) = 355.6 \sqrt{\frac{1.3 \times 10^3 \cdot x_2 \cdot x_4 \cdot (1 + \frac{102}{x_2 \cdot x_4})}{x_8^3 \cdot x_5^3}} \leq 0 \quad (14)$$

Above all, it can be simplified by an eleven dimensional optimization model with 30 inequality constraint conditions for the optimization design of three grade helical gear reducer whose optimization objective is the minimum value of center distance.

### 3. Numerical analysis

The optimization function is solved by the fmincon toolbox under the MATLAB environment. First of all, the objective function of the M file gear\_obj.m is constructed. Secondly M file gear\_nonlcon.m is programmed. Finally, the optimization procedure M-file optimization is called. And the initial values of design variables  $x_0 = [21; 4.43; 19; 4.68; 19; 12; 20; 32; 0.2715; 0.2681; 0.2712]^T$  is obtained by seeking the tables and calculation. After the optimization function is executed, the optimized design parameter Vector  $x$  will be obtained, as follow.

$x = [16.6288, 6.0000, 16.4665, 5.0000, 16.3482, 9.4685, 15.1685, 24.9851, 0.1396, 0.1396, 0.1396]^T$   
The target function value fval is equal to 2.2205e+03 for above  $x$ . After rounded, the design parameter  $x$  will be obtained, as follow.

$$x = [17, 6, 17, 5, 17, 6, 17, 9.5, 15, 25, 8^\circ, 8^\circ, 8^\circ]^T$$

Take  $i_{56}=102/(6*5)=3.4$ ,  $z_2=102$ ,  $z_4=85$ ,  $z_6=58$ . Compared the center distance before and after the optimization, the total center distance a before the optimization is 3690mm, and yet that a after the optimization is 2370mm, the reduce ratio of the center distance is equal to  $((3690-2370)) / 2370=35.7\%$ . The total center distance reducer is dramatically reduced on the premise that each constraint conditions are met.

#### 4. Conclusion

The optimization model of the pitch gear reducer for port crane is solved by the MATLAB toolbox, the design efficiency is improved, and the volume of the reducer and manufacturing costs are all reduced on the premise that the carrying capacity of reducer under three grade helical cylindrical gear reducer is met. The optimization model for multi-level helical gear optimal design, which is mentioned in this paper, can be customized as a MATLAB optimization tool. This method can greatly improve the modeling efficiency of multistage helical gear transmission, and it has not only good theoretical significance and engineering application value, but also can provide an important basis for the optimization design of a similar transmission mechanism with the gear transmission.

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