Nonlinear Optimization Methods of Parameters of Pulse Testing Model

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Abstract

Pulse Testing is one of interference testing. Conventional method of well testing parameters explanation includes the tangent method, the secant method, the plate fitting method and the parameter matching method. All the above methods employ chart interpretation using some special points of production data. So there exist some disadvantages, such as low utilization of data, slow explaination speed, and poor fitting accuracy. This paper establishes pulse testing well storage and skin models. Here, on the basis of combination of nonlinear regression technique, the bounded trust region optimization method is adopted to obtain the storage coefficient, flow coefficient, skin, wellbore storage coefficient of pulse testing models. The maximum relative error of fitting values is 6.49% and the average error is 2.98% in fitting production data of pulse testing of a reservoir with this method. This method is an automatic fitting technique having super-linear fitting speed without any effect of human factor.

Keywords

Pulse Testing; Data Utilization; Well Storage; Skin model; Nonlinear Regression; Bounded Trust Region.

1. Introduction

Pulse testing is the main method of measuring well connectivity. Storage coefficient and flow coefficient are important parameters to determine well connectivity. Classical methods are tangent method and secant method, which use some special points of production data to make a line to obtain pressure response amplitude, and then obtaining formation parameters according to the dimensionless time delay and dimensionless pressure response amplitude [1-11]. The parameter matching method is a special case of the chart fitting method [12]. Classical analysis methods only use special points to explain, which leads to low utilization of data, which involve artificial participation and are time-consumed. Secondly, traditional method can only explain limited parameters which requires most parameters of reservoir for obtaining flow coefficient and storage coefficient and so on. However the initial exploration and development data can’t provide all these reservoir geological parameters, which extend interpretation cycle of well testing. Based on the above analysis, this paper establishes pulse testing well storage and skin model, on the basis of combination of nonlinear regression techniques introducing bounded trust region method to solve multi-parameter problem of pulse testing. Case analysis shows that the method can explain more parameters than traditional methods with fast calculation speed and high precision.

2. Mathematical description of pulse testing well storage and skin models

Basic assumptions: the reservoir is homogeneous, isotropy and infinite; the fluid in the reservoir is single-phase and slightly compressible; ignore gravity and capillary force; the initial pressure is equal everywhere; the thickness is uniform everywhere; the inner boundary is a point source. According to the above assumptions, the dimensionless control equation is as follows:

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \tag{1}$$

The outer boundary condition:

$$p_D(\infty, t) = 0 \tag{2}$$

A. Response well is is shut in and denoted by subscript 1. The wellbore flow rate change is equal to the flow rate change of sandstone surface:

$$C_{D1}\left[\frac{dP_{wD1}}{dt_D}\right] - \lim_{rD1\to 0}\left[r_{D1}\frac{\partial P_D}{\partial r_{D1}}\right] = 0 \tag{3}$$

$$P_{wD1} = P_D(1, t_D) - S_1\left[r_{D1}\frac{\partial P_D}{\partial r_{D1}}\right]_{rD1\to 0} \tag{4}$$

B. Flow rate of active well on the ground keeps constant and is denoted by subscript 2. Sum of wellbore flow rate change and flow rate change in the sandstone surface is equal to flow rate change on the ground.

$$C_{D2}\left[\frac{dP_{wD2}}{dt_D}\right] - \lim_{rD2\to 0}\left[r_{D2}\frac{\partial P_D}{\partial r_{D2}}\right] = 1 \tag{5}$$

$$P_{wD2} = P_D(1, t_D) - S_2\left[r_{D2}\frac{\partial P_D}{\partial r_{D2}}\right]_{rD2\to 0} \tag{6}$$

Here,

$$p_D = \frac{kh(p_i - p)}{141.2q\mu B} \qquad t_D = \frac{0.0002637kt}{\phi\mu c_t r_w^2} \qquad r_D = \frac{r}{r_w} \qquad C_D = \frac{0.8935}{\phi c_t h r_w^2}C$$

Solve Eq.1 to Eq.6 using Laplace transform and principle of superposition to get the Laplace space solution of any time and any space at any point:

$$\overline{P}_D = \frac{1}{\Delta}\left\{-C_{D1}lK_0(r_D\sqrt{l})K_0(r_{D1}\sqrt{l}) + \left[1 + C_{D1}l\left(S_1 + K_0(\sqrt{l})\right)\right]K_0(r_{D2}\sqrt{l})\right\} \tag{7}$$

r_{D1} is distance of any point in the formation to the response well; r_{D2} is distance of any point in the formation to the active well; l is time variable;

$$\Delta = l\left\{\left[1 + C_{D1}l(S_1 + K_0(\sqrt{l}))\right]\left[1 + C_{D2}l(S_2 + K_0(r_D\sqrt{l}))\right] - C_{D1}C_{D2}\left[lK_0(r_D\sqrt{l})\right]^2\right\}$$

The Laplace space pressure of the responding well can be obtained through solving the Eq.7:

$$\overline{P}_{WD1} = \frac{K_0(r_D\sqrt{l})}{\Delta} \tag{8}$$

The bottom-hole pressure of responding well can be obtained by Stehfest numerical inversion on Eq.8. And the numerical solution of the N-th pulse can be obtained by superposition principle:

$$\Pr esp(t_D) = P_{wD1}(t_D) + \sum_{i=0}^{N-1}(-1)^i P_{wD1}(t_D - t_{iD}) \tag{9}$$

3. Nonlinear optimization techniques

Eq.9 shows strong nonlinear characteristics , which has 6 unknown parameters(C_1,C_2,S_1,S_2,T,S). For this problem, on the basis of combination of the non-linear least square this paper introduces bound-ed trust region to solve the multiple parameters of Eq.9. The bottom-hole pressure of responsonding well is $Pw(\vec{a},t_j)$; the measured bottom-hole pressure is P_j; M is the minimum function,; $\vec{a}$ is th-e vector representation of 6 unknown parameters. The below equation can be got:

$$M = \min \sum_{j=1}^{N} (Pw(\vec{a},t_j) - P_j)^2 c \tag{10}$$

Trust region algorithm bases on quadratic approximations of the objective function in a certain neighborhood of the current iterative point $\vec{a}_k$ (usually in the trust region),which need repeated iteration and correction of the trust region radius and then obtain the new iterative point $\vec{a}_{k+1}$.

Utilizing the quadratic approximations on Eq.10 the trust region subproblem is as follows:

$$\min m_k(\Delta \vec{a}) = M(\vec{a}_k) + \nabla M(\vec{a}_k)^T \Delta \vec{a} + \frac{1}{2} \Delta \vec{a}^T B_K \Delta \vec{a} \tag{11}$$

Eq.11 should satisfy the condition:

$$\left\| \Delta \vec{a} \right\| < \eta_k \text{ , } l < \vec{a}_k + \Delta \vec{a} < u \quad \text{(l, u as the upper and lower bounds)} \tag{12}$$

Here,

$$A_k = \nabla M(\vec{a}_k) = \left[\frac{\partial M_i(\vec{a}_k)}{\partial \vec{a}_i} \right]_{\vec{a}=\vec{a}_k} = \begin{bmatrix} \frac{\partial M_1}{\partial a_1} & \frac{\partial M_1}{\partial a_2} & \cdots & \frac{\partial M_1}{\partial a_n} \\ \frac{\partial M_2}{\partial a_1} & \frac{\partial M_2}{\partial a_2} & \cdots & \frac{\partial M_2}{\partial a_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial M_m}{\partial a_1} & \frac{\partial M_m}{\partial a_2} & \cdots & \frac{\partial M_m}{\partial a_n} \end{bmatrix}$$

This paper adopts DFP selection method.

$$B_{k+1} = B_k + \Delta B_k \tag{13}$$

$$\Delta B_k = \frac{s_k s_k^T}{s_k^T y_k} - \frac{(B_k y_k)(B_k y_k^T)}{y_k^T B_k y_k} \tag{14}$$

$s_k = \vec{a}_{k+1} - \vec{a}_k \quad y_k = \nabla M(\vec{a}_{k+1}) - \nabla M(\vec{a}_k) \quad B_0 = I$ (I as a unit matrix)

η_k is trust region radius; $\|.\|$ is a norm. We usually employ the second order norm on any preliminary step,which we can define the predict declining quantity as follows:

$$\Pr ed = m(0) - m(\Delta \vec{a}_k)$$

The actual reduction is :

$$Ared = M(\vec{a}_k) - M(\vec{a}_k + \Delta \vec{a}_k)$$

The definition of the ratio is :

$$\rho_k = \frac{Ared(\Delta \vec{a}_k)}{\Pr ed(\Delta \vec{a}_k)} \tag{15}$$

Use Eq.15 to measure the approximation degree of quadratic approximation function and objective function. Set 0<μ<η<1, the optimal solution of the trust region subproblem (11)-(12) is $\Delta \vec{a}^*$. If $\rho \le \mu$ and $\vec{a}_{k+1} \ne \vec{a}_k$, set $\vec{a}_{k+1} = \vec{a}_k + \Delta \vec{a}^*$ and return the problem (11)-(12) to iterate until get :

$$\left\| \nabla M(\vec{a}_k) \right\| \le \varepsilon \tag{16}$$

The modification principle of the trust region radius is as follows:

$$\eta_{k+1} = \begin{cases} 0.5\eta_k, \rho \le \mu \\ \eta_k, \mu < \rho < \eta \\ 2\eta_k, \rho \ge \eta \end{cases} \tag{17}$$

ε is the permissible error range. We can calculate 6 unknown parameters of pulse testing by the application of the above method and programming.

4. Example analysis

The basic parameters of an oil well are: q=8612.727, r=347.44, r_w=0.5, Δt_g=8.5, Δt_k=8.5, Δt_c=17, B=1.117 (Note: The units are English units) . This paper adopts two methods which we usually use (L-M method and G-N method) . However, the two methods to solve the problem of multiple-parameter recognition is misconvergent. The calculation results are shown in Table 1. So the trust region method is proposed in this paper. The maximum error of fitting result is 6.49% and the average error is 2.98%. It can be seen that the fitting result is reliable and successful. They are shown in Fig.1.

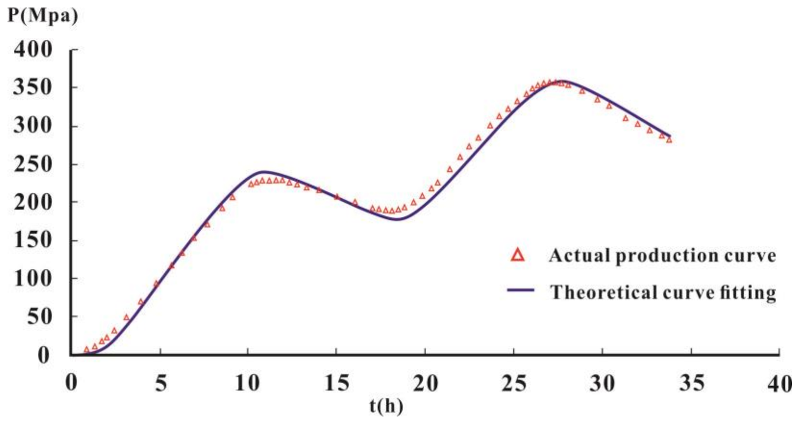

Fig.1 Data fitting results

Table 1. Comparison table of trust region to obtain parameters

	T 10^3md*ft/cp	S 10^{-5}ft/Psi	C_1 RB/psi	C_2 RB/psi	S_1	S_2	Maximum error	Average error
Trust region	1.69	6.98	0.001	0.339	0.336	-6.758	6.49%	2.98%
G-N	misconvergence, no numerical solution							
L-M	misconvergence, no numerical solution							

5. Conclusion

1. Pulse well testing storage and skin model is established in this paper. Nonlinear regression technique with bounded trust region optimization method is employed to work out the parameters of the pulse testing model. The maximum error of the fitting result is 6.49% and the average error is 2.98%.

2. The frequently-used G-N and L-M can't get the final result in solving the problem of multiple parameters identification, but the trust region optimization can do it. Practice indicates that the trust region method is more effective than conventional method such as G-N and L-M in solving the problem that the number of the testing parameter is greater than four.

3. The explanation cycle of well testing parameter is shorter with the method employed in this paper than the conventional tangent and secant method without manual intervention,which avoids the defect of time consuming process of previous chart explanation and has good economic value and practicality.

4. Application of the method can be extended to oil and gas well productivity prediction, production decline law etc.

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References

[1] Johnson C R, Greenkorn R A, Woods E G. Pulse-testing: a new method for describing reservoir flow properties between wells[J]. JPT, 1966,18(12):1599-1604.
[2] Kamal M, Brigham W E. Pulse-testing response for un-equal pulse and shut-in periods [J]. SPE, 1975, 15(5) :399-410.
[3] Dong YM, Ren L C , Han D K. Single Pulse Method of Multiwell Testing with Elimination of Unknown Pressure Trends[R] . SPE 17819 ,1988 :1～24.
[4] Coskuner G, Ramler L B , Brown M W, etc. Design Implementation and Analysis of Multilayer Pressure Transient Tests in White Rose Field [R] .SPE , 63080 , 2000 :475～489.
[5] Li Xiaoping , Zhang Dawei. Mathematical Physics Method and Mathematical Model of Well Test [M].Beijing , Petroleum Industry Press , 1993: 130-132.
[6] Liu Zhenyu , Zeng Zhaoying , Zai Yunfang, et al. Study on Connectivity of Low-permeability Reservoirs with Pulse-testing Method [J]. Acta Petrolem Sinica , 2003, 24(1): 73-77.
[7] Kamal M, Brigham W E. Design and Analysis of Pulse Tests With Unequal Pulse and Shut-in Period[J]. JPT, 1976, 205-212.
[8] Wang Lei , Wang Xiaodong, Luo Erhui. Theoretical Type Curves of Pulse Test with Un-Equal Duration in Low-permeability Reservoirs and Their Application[J].Well Testing, 2009, 18(5): 13-16.
[9] Zhang Tongyi, Li Quanxiu and Xie Ying. Theory Study of Scant Method for Unequal Time Pulse Testing[J]. Well testing, 1997, 6(1): 12-15.
[10] Liao Xinwei, et al. Theory and Practice of Pulse Well Testing in Non-shut-in Oil wells[J]. Petroleum Exploration and Development, 1999, 26(1), 20-52.
[11] Zeng Zhaoying, Zhang Dawei, An Yanming. Curve and Model of Interwell Pulse Testing in Two Layer Reservoir[J].Journal of Daqing Pertoleum Institute, 2003, 27(1): 19-22.

Appendix

Table 2. Symbol description

p_D — dimensionless pressure	B_k— the Hessian matrix of M
t_D — dimensionless time	B — formation volume factor
r_{D1} — the dimensionless distance from the responding well	K — the formation permeability
r_{D2} — the dimensionless distance from the active well	h — the formation effective thickness
r_D — the dimensionless distance between the responding well and active well	N — the number of impulse
l — Laplace time , the image function of the time	P — the reservoir pressure

by Laplace transform	
C_1 — well bore storage factor of the responding well	Pi — the initial reservoir pressure
C_2 — well bore storage factor of the active well	μ — the oil viscosity
S_1 — skin factor of the responding well	t — time , h
S_2 — skin factor of the active well	r — the distance between the responding well and the active well
p_{wd1} — dimensionless bottom hole pressure of the responding well	C_t — the total compressibility
p_{resp} — dimensionless bottom hole pressure of the Nth pulse well	Φ — porosity
M — the objective function	q — production
$\vec{a}$ — the variable of the objective function	T — flow coefficient
$\Delta\vec{a}$ — the variable step of the objective function	S — storage coefficient
$m_k(\Delta\vec{a})$ — the quadratic approximation of the objective function	G-N — the Gauss-Newton algorithm
$\nabla M(\vec{a})$ — the Jacobi matrix of M	L-M — the Levenberg-Marquarat algorithm
K_0 — the second species zeroth-order Bessel function	r_w — wellbore radius
η — the radius of the trusty region, dimesionless	Δt_g — shut-in impulse time
YC — the prediction decline rate	Δt_k — open impulse time
ZS — the actual decline rate	Δt_c — pulse period
ρ — the ratio between YC and ZS	ε — the iterative error

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