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Research on Approximation High-order Spectrum Model of Non-Gaussian Wind Pressure on Large-span Roofs

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Abstract

Describing the probability characteristics of non Gaussian wind pressure through frequency domain information remains a challenge. This article construct the high-order moment spectrum model of non Gaussian wind pressure through the polynomial model based on the time history information of non Gaussian wind pressure. Firstly, the approximation 2nd order moment spectrum model is constructed by polynomial form, and the truncation degree is determined based on the variance convergence criterion. Then, the approximate third-order moment spectrum is determinted by the approximation 2nd order moment spectrum model based on the definition of third-order moment spectrum. The results indicate that the proposed method can be effectively applied to the construction of high-order moment spectrum models.

Keywords

Wind Pressure; Non-Gaussian; High-order Spectrum.

1. Introduction

In the field of civil engineering, the perspective that wind pressure on large-span roof structures exhibits non-Gaussian characteristics has been accepted[1]. Generally, time-domain analysis and frequency-domain analysis are two effective methods for studying non-Gaussian wind pressures. In time-domain analysis, wind pressures are often described as stochastic processes that vary over time, in which numerous simulation methods for non-Gaussian wind pressures proposed[2][3]. For frequency-domain analysis, the dynamic long information of non-Gaussian wind pressures is mapped to the frequency domain through Fourier transforms. Compared to time-domain analysis, frequency-domain analysis can more comprehensively describe the probabilistic characteristics and energy characteristics of non-Gaussian wind pressures. Therefore, the frequency-domain analysis is consider in this paper.

For the frequency-domain analysis of non-Gaussian wind pressure, the power spectral density model is most widely used in existing studies. Howevere, the power spectral density is the 2nd spectrum in the requency-domain analysis, which can provisded an complete stochastic description for Gaussian processes. It is worth pointing out that the higher-order spectrum is an extension of the 2nd order moment spectrum, which can provide a complete stochastic description for non-Gaussian processes[4]. At present, the higher-order spectra of non-Gaussian stochastic processes have been applied in field as diverse as underwater acoustics[5], electroencephalogram data analysis[6]. In all of the above researches, the focus has been entirely on characterization of higher-order spectra, and the approximation method of higher-order moment spectrum attract less attention. Therefore, the

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approximation high-order spectrum model of non-Gaussian wind pressure on large-span roofs is proposed in this paper.

2. The Approximation High-order Spectrum Model of Non-Gaussian Wind Pressure

Since non-Gaussian wind pressures have attracted a lot of attention from researchers, many universities have carried out in-depth studies on wind pressures on large-span roofs and shared aerodynamic databases. In this paper, the non-Gaussian wind pressure coefficients on a low-rise building, which are obtained from the 21st century Coe program "Wind effects on building and urban environment"[7], are investigated. Meanwhile, the flat roof type as a classic roof form will be studied in this paper. The test model is a flat roof, which has a full-scale size of 160 × 160 × 40 m, and the model scale is 1:100. The different points of wind pressure coefficients are shown in Fig. 1. As shown in Fig. 1, the wind direction of 0º is considered. The sampling frequency was 500 Hz and the sampling period was 18 seconds. The time duration is 0.2 s, and the time period is 600 s. According to the time history of different points, the skewness and kurtosis are calculated and plotted in Fig. 2. As shown in these figures, the wind pressures at different points show as non-Gaussian characteristic bsecasuse the skewness and kurtosis of Gaussian stochastic process are equal 0 and 3, respectively. Therefore, the approximation high-order spectrum model of non-Gaussian wind pressure is proposed. Meanwhile, we focus on analysis of the 2nd order moment spectrum and 3rd order moment spectrum, because 3rd order moment spectrum is the simplest to obtain for all high-order moment spectra and the analysis procedures for 3rd order moment spectrum and other high-order moment spectra are very similar.

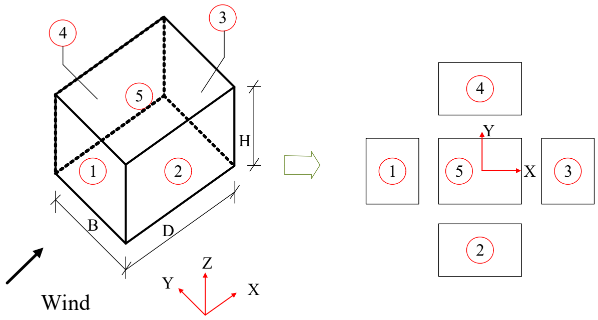
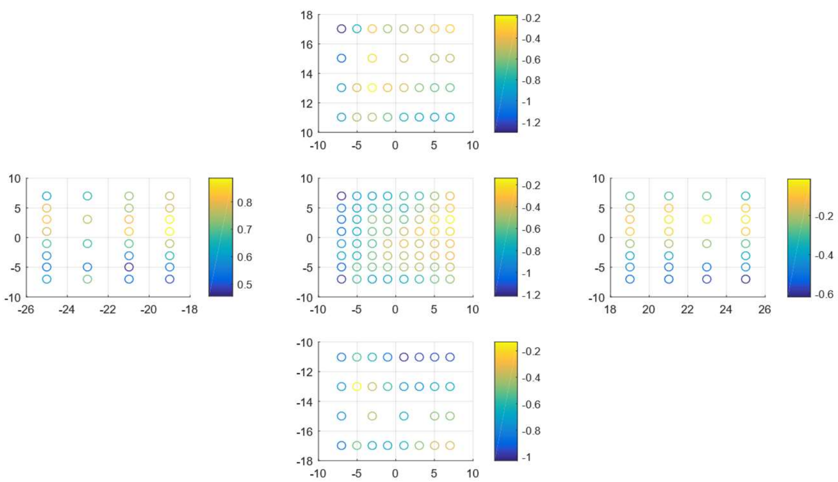
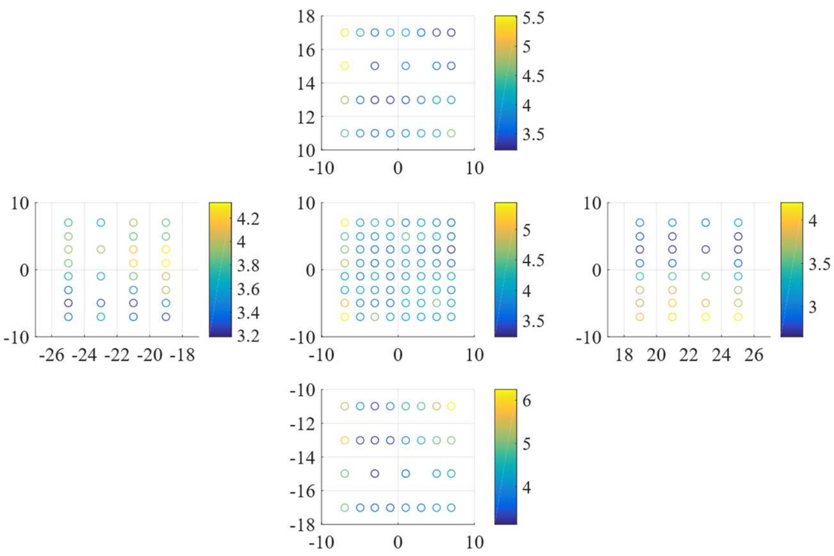


Fig. 1 The test model of flat roof.



(a) The skewness of different points



(b) The kurtosis of different points

Fig. 2 The test model of flat roof.

2.1 The Calculation Method of Approximation 2nd Order Moment Spectrum Model

Due to the 2nd order moment spectrum is the most popular in the refrequency domain analysis, many approximation models are proposed, in which the exponential form and polynomial form of the approximate model are the most classical. Compared with the exponential form, the polynomial form is more flexible. Then, the polynomial form is employed to reconstructed the 2nd order moment spectrum of the non-Gaussian wind pressure based on the estimated 2nd order moment spectrum from time-history samples.

According to the aerodynamic databases, the time-history of point is plotted in Fig. 3. Based on the time history, the estimated 2nd order moment spectrum $S_{X,X}(\omega)$ is plotted in Fig. 4, where the estimated $S_{X,X}(\omega)$ shows the obvious oscillation. Furtheremore, the polynomial form of approximation 2nd order moment spectrum $S_{X,X,2}(\omega)$can be shown as:

$$S_{X,X,2}(\omega)=\sum_{k_2=1}^{l_2} A_{k_2}\omega^{A_{k_2}-1} \Big/ \sum_{k_3=1}^{l_3} A_{k_3}\omega^{A_{k_3}-1}, \tag{1}$$

in which the A_{k2} and A_{k3} are the parameters, l_2 and l_3 ($l_2 \geq l_3$) are the number of truncation, respectively. As shown in Eq. (1), the critical procedure of approximation 2nd order moment spectrum is determing the value of l_2 and l_3. The variance convergence criterion is employed to determint l_2 and l_3 in the following contents.

According to Eq. (1), the variance can be calculated as:

$$m_{a,2,l_2,l_3}=\int_{-\infty}^{\infty}\left(\sum_{k_2=1}^{l_2} A_{k_2}\omega^{k_2-1} \Big/ \sum_{k_3=1}^{l_3} A_{k_3}\omega^{k_3-1}\right)\mathrm{d}\omega, \tag{2}$$

The step is described in Algorithm 1.

Algorithm 1.

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Step 1:	**Define** l_2=1, l_3=1
Step 2:	calculate $m_{a,2,l2,l3}$ through Eq.(2)
Step 3:	**if** $(m_{a,2,l2,l3}- m_{e,2,l2,l3})/ m_{e,2,l2,l3} > \varepsilon$ then
Step 4:	$l_2= l_2 + 1$, $l_3= l_3 + 1$
Step 5:	repeat step 2~3
Step 6:	**end if**
Step 7:	**Get** the truncated order of the l_2 and l_3

Note: $m_{e,2,l2,l3}$ is the variance from samples, ε is the relative error of $m_{a,2,l2,l3}$ and $m_{e,2,l2,l3}$. In thsi paper, ε denotes 0.01.

According to the Algorithm 1, the values of l_2 and l_3 equal 2, respectively. The approximation $S_{X,X,2}(\omega)$ is shown as:

$$S_{X,X,2}(\omega)=\frac{-0.0922\omega^{-0.0922}+0.6221\omega^{0.6221}}{2.3345\omega^{2.3345}+1.4331\omega^{1.4331}}, \tag{3}$$

The corresponding approximation $S_{X,X}(\omega)$ is also ploted in Fig. 4. As shown the figure, the approximation $S_{X,X}(\omega)$ agree very well with those by sample in the entire frequency domain, showing that the polynomial form is accurate enough.

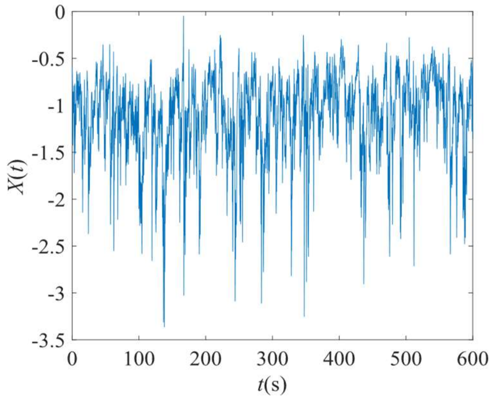


Fig. 3 The time-history of non-Gaussian wind pressure.

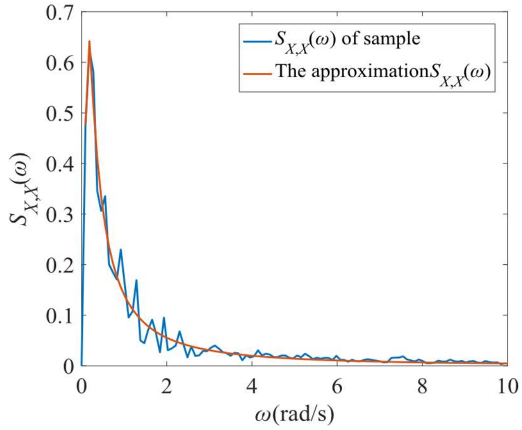


Fig. 4 The comparison of 2nd order moment spectrum.

2.2 The Calculation Method of Approximation 3rd Order Moment Spectrum Model

According to the definition of 3rd order moment spectrum model, $S_{X,X,X}(\omega_1, \omega_2)$ is calculated as:

$$S_{X,X,X}\left(\omega_1,\omega_2\right)=X\left(\omega_1\right)X\left(\omega_2\right)X^*\left(\omega_1+\omega_2\right), \tag{4}$$

Setting ω_2=0, there exists:

$$S_{X,X,X}\left(\omega_1,0\right)=X\left(\omega_1\right)X\left(0\right)X^*\left(\omega_1\right), \tag{5}$$

According to Eq. (5), 3rd order moment spectrum contain the inforemation of 2nd order moment spectrum. Therefore, the 3rd order moment spectrum can be expressed by the approximation 2nd order moment spectrum model:

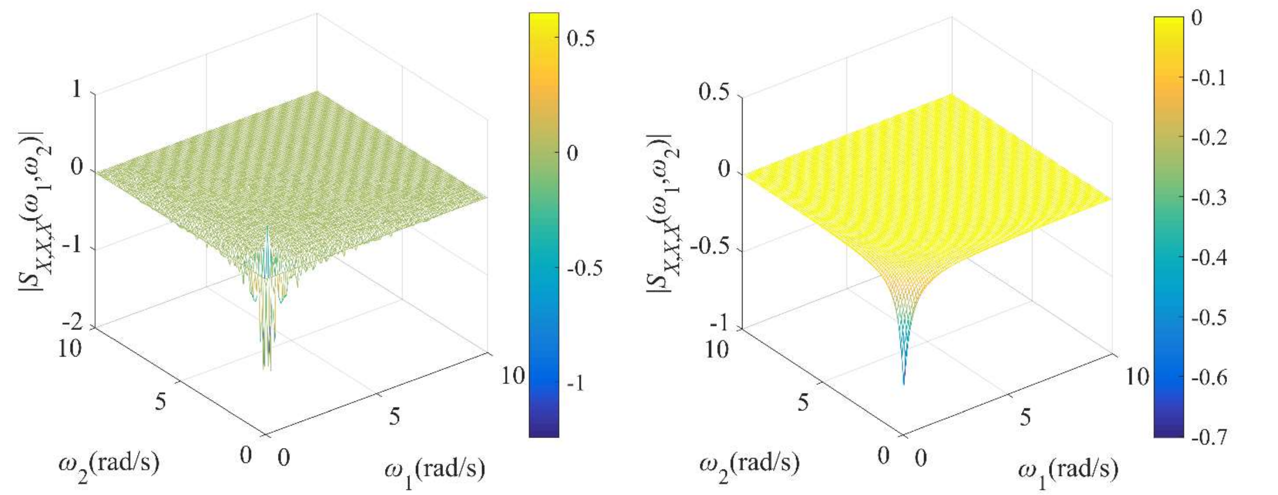
$$S_{X,X,X,3}\left(\omega_1,\omega_2\right)=A_{k_4}\left[S_{X,X,2}\left(\omega_1\right)+S_{X,X,2}\left(\omega_2\right)\right]+A_{k_5}S_{X,X,2}\left(\omega_1\right)S_{X,X,2}\left(\omega_2\right), \tag{6}$$

where A_{k4} and A_{k5} are the parameters.

According to the time-history of point, the estimation $S_{X,X,X}(\omega_1, \omega_2)$ is shown as Fig. 5(a). According to Eq. (6), the 3rd order statistical moment can be calculated as:

$$m_{a,3,\mathrm{k}_4,\mathrm{k}_5}=\int_{-\infty}^{\infty}\left\{A_{k_4}\left[S_{X,X,2}\left(\omega_1\right)+S_{X,X,2}\left(\omega_2\right)\right]+A_{k_5}S_{X,X,2}\left(\omega_1\right)S_{X,X,2}\left(\omega_2\right)\right\}\mathrm{d}\omega_1\mathrm{d}\omega_2, \tag{7}$$

Subsquently, the value of A_{k4} and A_{k5} is calculated through the data fitting methods based on the estimation $S_{X,X,X}(\omega_1, \omega_2)$. Then, A_{k4} and A_{k5} is equal to 0.00059 and -1.7065, respectively. The corresponding $S_{X,X,X,3}(\omega_1, \omega_2)$ is determinted through Eq. (6) and plotted in Fig. 5(b). The 3rd order statistical moment from the estimation $S_{X,X,X}(\omega_1, \omega_2)$ and approximation $S_{X,X,X,3}(\omega_1, \omega_2)$ equal to -0.079 and -0.075, respectively. The results of the approximation $S_{X,X,X,3}(\omega_1, \omega_2)$ demonstrate the validity of the proposed approximate model.



(a) The estimation 3rd order moment spectrum (b) The approximation 3rd order moment spectrum

Fig. 5 The comparison of 3rd order moment spectrum.

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3. Conclusion

This article proposes the accurate and efficient approximation method of the high-order moment spectra for the non-Gaussian wind pressure. In this paper, the approximation 2nd order moment spectrum model is constructed by the polynomial form, and the number of truncations is determined by the variance convergence criterion. Subsequently, the approximation 3rd order moment spectrum is constructed by the approximation 2nd order moment spectrum model. The results show that the proposed method is efficient for determining the high-order moment spectrum model.

It is worth pointing out that the parameters of the approximation high-order moment spectrum model are calculated by the corresponding estimation high-order moment spectrum from time-history sample. The determination of the relationship between coefficients and non-Gaussian probability information will be studied in future work.

Acknowledgments

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