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# An Improved Two-stage Constrained Optimization Algorithm based on Indicator

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## Abstract

**Convergence, feasibility and diversity are three important indexes in constrained multi-objective optimization problems (CMOPs). It is very important to balance the relationship among them. In order to balance these three indexes well, an index-based two-stage constrained optimization algorithm (TSTA) is proposed in this paper. In stage I, non-dominated sorting is used to obtain individuals with good distribution, which can avoid the population convergence stagnation. Currently, most of the individuals are scattered near the feasible region. The main task of the stage II is to bring the the optimal solutions close to the Pareto front (PF). The proposed TSTA is evaluated against five other advanced algorithms on 23 CMOPs. The final test data confirm that TSTA presents some advantages in processing CMOPs.**

## Keywords

**Convergence; Feasibility; Diversity.**

## 1. Introduction

Usually, a CMOP is described as follows:

$$\begin{aligned} &\min F(x) = (f_1(x), f_2(x), \ldots, f_M(x))^T, \\ &s.t\ X = (x_1, x_2, \ldots, x_D)^T \in W, \\ &g_i(x) \le 0, i = 1, \ldots, p, \\ &h_j(x) = 0, j = p+1, \ldots, q, \end{aligned} \tag{1}$$

At present, many types of constrained multi-objective optimization algorithms (CMOEAs) is used to reconcile constraints and objective values, which include multi-stage, multi-group, mixed and so on [1]. However, when the search space and feasible region of constrained optimization problems are complex, these algorithms have some shortcomings. The balance between the target value and the constraint conditions is likely to reduce the convergence rate of the population. Using two-stage technology, different stages introduce different strategies to better balance constraints and target values can achieve good results [2]. The two-stage algorithm proposed in this paper becomes TSTA. The following are described:

Because diversity and convergence need to be maintained, stage I can contain infeasible solutions, and the individuals are gathered to the PF. Based on stage I, stage II selects feasible solutions to

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accelerate the rate of convergence. The fitness values of the two stages are calculated in different ways. The stage I adopts the sorting method, and the second stage adopts the Euclidean distance between solutions.

The work of this paper is described as follows: the related work and motivation are introduced in chapter II. The details of the proposed TSTA is stated in chapter III. The process and results of the experiment are presented in chapter IV, and the chapter V is the summary of the whole paper.

# 2. Work and Motivations

## 2.1 Section Existing CMOEAs

### 2.1.1 Fitness-based CMOEA

The main purpose of the fitness-based CMOEAs is to transform the CMOP into an unconstrained problem, and each objective related to the solution will be punished for the degree of violation of constraints. Not all solutions will receive constraint penalties. Constraint violation is for a specific solution, and the penalty term is for all solutions. Therefore, the fitness-based method is very extensive, and its performance depends on the penalty parameters. In 2021, a new fitness method is used by Ma et al., which consists of two parts, the Constraint dominance Principle (CDP) and the Pareto dominance Principle (PDP) are introduced [3], with different added values assigned to each individual. CDP is used to represent the feasibility, PDP is used to represent the optimality, the ratio between the two parts according to the number of feasible solutions set [4].

### 2.1.2 Multi-population CMOEAS

In [5], a co-evolutionary method with two populations is proposed by Tian et al., one population searches for PF, the other population searches for CPF, and the two populations co-evolve together. In the whole process, the two populations have the same priority, one of which completely ignores the existence of constraints, and the other considers the existence of constraints and achieves balance.

## 2.2 Motivation

Convergence, diversity and feasibility are considered at the same time in the balanced ranking method, which avoids the phenomenon of discontinuity and stagnation of convergence in feasible regions [6][6]. Different factors need to be considered at different times. In the early stage, the unknown field should be explored, and later in evolution, the convergence rate of the solutions should be promoted [7]. On the premise of satisfying the feasibility, convergence and diversity will be meaningful [8]. If all the solutions can completely enter the feasible region, it indicates that the feasible domain is continuous [9]. A new two-stage CMOEA (TSTA) is proposed introduce the above methods in this paper. In the ***stage*** **I**, the three properties are considered comprehensively, and the purpose is to explore the heavy area. Infeasible individuals may be included in the population; in ***stage*** **II**, the ARMOEA is introduced for population evolution.

# 3. The proposed algorithm:TSTA

## 3.1 General Framework

| ***Algorithm* 1**: The framework of TSTA | |
|---|---|
| Input: $N$ (population size); $G_{\max}$ (maximal number of generation); $N_Z$ (Number of reference points and archive size) | |
| Output: $P$ (population) | |
| 1: | Initial Population $P$, create an uniform reference point ($Z$) |
| 2: | $g = 1$, $stage = 1$ |
| 3: | while $g \le G_{\max}$ do |
| 4: | if $stage == 1$ then |

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convergenceCalculation($P$)
convergenceCalculation($P$)
convergenceCalculation($P$)
Calculate the fitness $fit1$ of solutions in $P$
Progeny $Q$ is generated according to fitness $fit$
$R = P \cup Q$
$P \leftarrow$ Environmental Selection ***Stage* I** ($R$)
else
archive $A = P$
$Z' = Z$
Q = generate offspring ($P$, $Z'$)
[A, $Z'$] = Self-adaptation of reference point ($A \cup Q$, $Z$, $P$)
P = Environmental Selection ***Stage* II** ($P \cup Q$, $Z'$, $N$)
end if
$g = g + 1$
end while
return $P$

According to ***Algorithm* 1**, first, initialize the population and create a uniform reference point; when the population evolution generation is less than the maximum generation, if it is the ***stage* I**, the convergence, diversit, feasibility and fitness value of each solution are calculated respectively. In ***Stage* II**, the file $A$ and offspring $Q$ are generated , and then the environment selection stage is executed.

## 3.2 The Stage I of TSTA

Indicator of convergence:

$$fpr(p) = \sum_{j=1}^{M} fj(p), \qquad fj(p) = \frac{fj(p) - f_{\min,j}}{f_{\max,j} - f_{\min,j}} \tag{2}$$

where fmax,j and fmin,j denote the maximum and minimum results in the j-th objective values of the population.

Indicator of diversity:

$$fcd(p) = \sqrt{\sum_{q \in X, q \neq p} share(p,q)} \tag{3}$$

Where share(p,q) is defined as a shared function between individuals p and q.

Indicator of feasibility:

The feasibility of solution p is measured by the degree of constraint violation fcv(p).

$$CVi(x)=\max(0,gi(x)),\quad i=1,\ldots,p \tag{4}$$

And the overall constant violation (CV) is calculated as follow:

$$CV(x)=\sum_{i=1}^{q} CVi(x) \tag{5}$$

hich is the degree of CV on all the constrains.

| ***Algorithm* 2**: Environmental Selection for ***Stage* I** | |
|---|---|
| Input: $R$ | |
| Output: $P$ | |
| 1: | for $a \in R$ do |
| 2: | $fpr(x)$ = convergence Calculation($x$) |
| 3: | $fcd(x)$ = diversity Calculation($x$) |
| 4: | $fcv(x)$ = feasibility Calculation($x$) |
| 5: | end for |
| 6: | $fm(x)$ = Pareto Non-dominated Sort($fpr(x)$, $fcd(x)$) |
| 7: | $frank(x)$ = unbiasedBiObjectiveModel($fm(x)$, $fcv(x)$) |
| 8: | $fit_1(x) = frank(x) + fcv(x)/(fcv(x) + 1)$ |
| 9: | $P = N$ individuals are selected according to $fit1$ |
| 10: | return $P$ |

### 3.3 The *Stage* II of TSTA

When the stage I is completed, the final solutions obtained is well distributed, but the number of feasible solutions is insufficient. Therefore, the stage II is to let all individuals advance to the feasible region and converge quickly. The ARMOEA is introduced in the stage II. When facing the different shape of the feasible region, it will be uniformly sampled in the unit hyperplane [10]. At the same time, the location of reference points is adaptively updated based on the non-dominated solutions in the file.

| ***Algorithm* 3**: Environmental Selection for ***Stage* II** | |
|---|---|
| Input: $P$, $Z'$, $N$ | |
| Output: $P$ | |
| 1: | *Front* = efficient non-dominated sorting ($P$) |
| 2: | $k$ = minimum number satisfies $\lvert \cup_{i=1}^{k} \text{Front}_i \rvert \geq N$ |
| 3: | $Q = \cup_{i=1}^{k} \text{Front}_i$ |
| 4: | while $\lvert \text{Front}_k \rvert > N - \lvert Q \rvert$ do |

5: $p$ = $\text{argmin}_{p \in \text{Frontk}}$ IGD-NS (Frontk\{$p$}, $Z'$)

6: $\text{Front}_k = \text{Front}_k \setminus \{p\}$

7: end while

8: $Q = Q \cup \text{Front}_k$

9: $P = Q$

10: return $P$

---

Algorithm 3 gives the environment selection process based on IGD-NS in stage II. Before using the IGD-NS index for candidate screening, the efficient non-dominated sorting (ENS) is introduced to rank MOPs, and the tree-based ENS (T-ENS) is introduced to rank MaOPs. The candidate solutions located in the top (k – 1) fronts have priority for promotion, and the IGD-NS index that represents the contribution value of the solutions is used to select the remaining solutions in the Frontk, where k is the minimum number satisfying $\left|\bigcup_{i=1}^{k} Front_i\right| \geq N$ [11]. The IGD-NS index value of each solution in Frontk is calculated, and the solutions with the smallest result will be eliminated. Once the candidate set is updated, the contribution value will be calculate again. This procedure is continuously executed until the number of selected solutions in $\bigcup_{i=1}^{k} Front_i$ up to N.

### 3.4 Complexity Analysis

In stage I, the complexity of calculating the evaluation index value of N solutions is about O(MN2 + qN). The complexity of non-dominated sorting is O(MN2). Among them, M is the dimension of the objective space; and q denotes the number of constraints. In stage II, Computing the fitness of N solutions requires about O(MN2) complexity. The execution of the environment selection phase requires a maximum of O(N3) complexity. To sum up, taking into account the above considerations and calculations, the overall worst-case complexity of one generation of TSTA is O(MN2 + qN + N3).

## 4. Results and Discussion

### 4.1 Experiment Setup

In the experiment, the parameters of each CMOEA are set as follows:

(1) The population size is set as 100.

(2) The algorithm runs independently 30 times.

(3) Functional evaluation: MW, C-DTLZ, DC-DTLZ a total of 30,000; 6 million in Eq-DTLZ and Eq-IDTLZ.

Other parameter settings refer to the original settings of other comparison algorithms, and all experiments are completed on platEMO.

### 4.2 Experimental Results

#### 4.2.1 The Comparison Results on the MW Benchmark Test Set

The feasible region of the MW benchmark suite has diversity characteristics, such as the feasible region is continuous; the feasible area is narrow and interrupted; the feasible area is discontinuous. The data of TSTA and other five comparison CMOEAs on MW1-MW14 are presented in Table 1.

From the results, TSTA generally performed well on the MW benchmark. Specifically, TSTA performs best on seven problems, TSTI performs best on two problems, MOEADDAE performs best on four problems, PPS performs best on one problem, and IDBEA performs worst.

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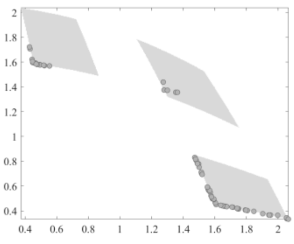
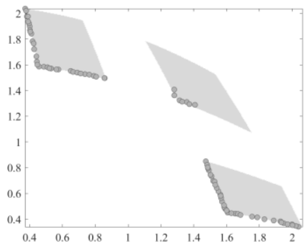
**Table 1.** The results of six CMOEAs in MW-CMOPs

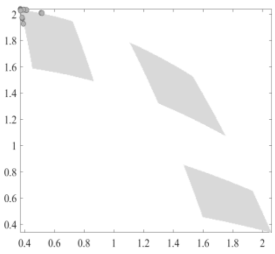
| Problem | M | D | MOEADDAE | PPS | IDBEA | CMOEA_MS | TSTI | TSTA |
|---|---|---|---|---|---|---|---|---|
| MW1 | 2 | 15 | 2.3919e-2 (0.00e+0) - | NaN (NaN) | 0.0000e+0 (0.00e+0) = | 2.5761e-1 (0.00e+0) - | 2.5924e-1 (0.00e+0) - | 3.1555e-1 (5.32e-2) |
| MW2 | 2 | 15 | 5.0020e-1 (6.39e-2) - | 5.0895e-1 (1.43e-2) - | 1.5464e-1 (6.76e-2) - | 5.0640e-1 (5.11e-2) - | 5.6090e-1 (4.31e-3) = | 5.5253e-1 (1.48e-2) |
| MW3 | 2 | 15 | 5.3410e-1 (2.71e-3) = | 5.3462e-1 (2.21e-3) = | 1.1489e-1 (4.01e-2) = | 5.3071e-1 (7.16e-3) = | 5.3306e-1 (3.95e-3) = | 4.8414e-1 (3.94e-2) |
| MW4 | 3 | 15 | NaN (NaN) | NaN (NaN) | 1.7135e-1 (1.53e-1) - | 8.0357e-1 (0.00e+0) - | 8.1071e-1 (1.84e-2) = | 8.1306e-1 (1.20e-2) |
| MW5 | 2 | 15 | 2.9048e-1 (9.58e-3) = | NaN (NaN) | 2.4730e-2 (2.00e-2) = | 2.3604e-1 (6.47e-2) = | 2.1538e-1 (1.15e-1) = | 1.8373e-1 (1.40e-1) |
| MW6 | 2 | 15 | 2.7634e-1 (6.46e-2) - | 0.0000e+0 (0.00e+0) = | 7.0731e-2 (2.05e-2) - | 2.5942e-1 (7.54e-2) - | 2.3976e-1 (5.97e-2) - | 2.9891e-1 (8.00e-3) |
| MW7 | 2 | 15 | 4.0801e-1 (9.75e-4) = | 4.0463e-1 (6.22e-4) = | 1.1478e-1 (2.86e-2) = | 4.0250e-1 (6.57e-4) = | 4.0191e-1 (3.59e-3) = | 4.0208e-1 (1.54e-3) |
| MW8 | 3 | 15 | 4.2535e-1 (6.71e-2) - | 1.4431e-1 (8.94e-2) - | 4.9131e-2 (4.27e-2) - | 5.0885e-1 (8.81e-3) - | 5.0218e-1 (2.92e-2) - | 5.1224e-1 (6.73e-3) |
| MW9 | 2 | 15 | 0.0000e+0 (0.00e+0) = | NaN (NaN) | 8.5710e-2 (7.57e-2) - | 2.2964e-1 (2.00e-1) - | 8.0291e-2 (7.25e-2) - | 3.2537e-1 (4.01e-2) |
| MW10 | 2 | 15 | 3.1974e-1 (3.21e-2) - | NaN (NaN) | 1.1888e-1 (1.60e-2) - | 3.8640e-1 (8.01e-2) - | 3.8733e-1 (1.25e-2) - | 4.1197e-1 (1.42e-2) |
| MW11 | 2 | 15 | 4.3695e-1 (2.31e-3) = | 4.3943e-1 (2.28e-3) = | 1.7544e-1 (8.59e-2) - | 4.0764e-1 (5.63e-2) - | 4.4120e-1 (1.31e-3) = | 4.4243e-1 (1.14e-3) |
| MW12 | 2 | 15 | 2.6453e-1 (3.74e-1) = | NaN (NaN) | 1.5865e-2 (2.75e-2) = | 5.5648e-1 (2.81e-2) = | 5.8187e-1 (1.41e-2) = | 2.8030e-1 (2.80e-1) |
| MW13 | 2 | 15 | 4.2709e-1 (1.46e-2) = | 2.6382e-1 (9.62e-2) = | 1.6306e-1 (3.48e-3) = | 2.6081e-1 (1.03e-1) = | 3.6131e-1 (6.97e-3) = | 4.0668e-1 (4.29e-2) |
| MW14 | 3 | 15 | 4.4022e-1 (2.85e-2) = | 1.0609e-1 (6.10e-2) = | 8.9831e-3 (1.97e-4) = | 3.8719e-1 (8.77e-2) = | 4.2317e-1 (1.70e-2) = | 3.2474e-1 (9.63e-2) |
| +/-/= | | | 0/5/9 | 0/2/12 | 0/7/7 | 0/8/6 | 0/5/9 | |



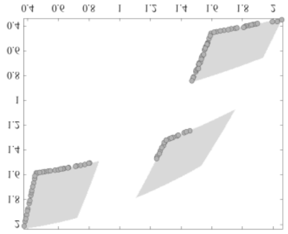
(a)MOEADDAE in MW11



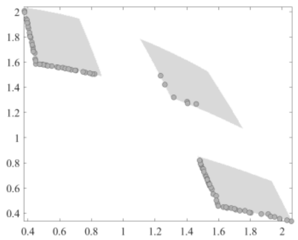
(b)PPS in MW11



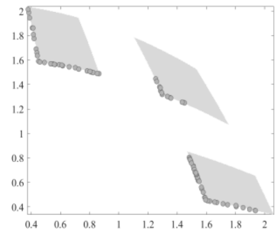
(c)IDBEA in MW11



(d)CMOEAMS in MW11



(e)TSTI in MW11



(f)TSTA in MW11

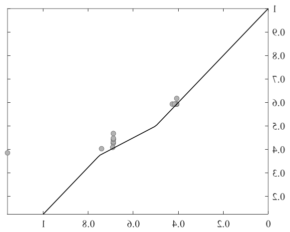
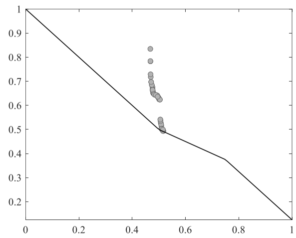
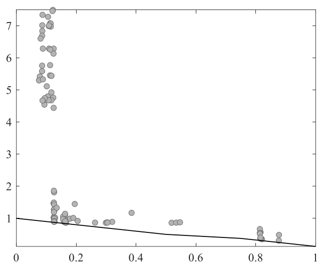
**Figure 1.** The results of six algorithms on MW11

Figure 1 shows that TSTA performs best on MW11, and all solutions converge towards true PF. The optimal solutions are evenly distributed, and the convergence and feasibility are good. However, other algorithms cannot simultaneously balance the above properties.

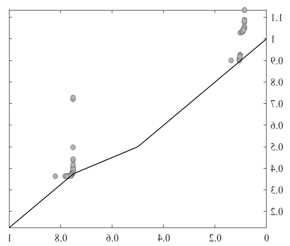
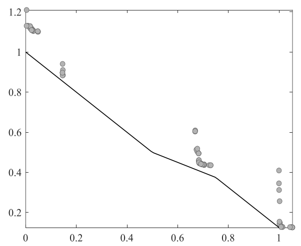
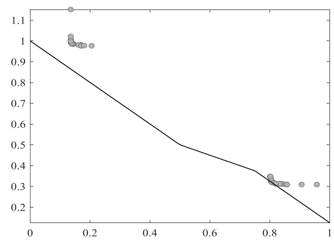
### 4.2.2 The Comparison Results on the MW Benchmark Test Set

**Table 2.** The results of six CMOEAs in CF-CMOPs

| Problem | M | D | MOEADDAE | PPS | IDBEA | CMOEA_MS | TSTI | TSTA |
|---|---|---|---|---|---|---|---|---|
| CF1 | 2 | 10 | 5.1813e-1 (1.24e-2) = | 5.3985e-1 (6.69e-3) = | 4.3428e-1 (7.65e-3) = | 4.6836e-1 (1.42e-2) = | 4.4162e-1 (6.59e-3) = | 4.6187e-1 (9.34e-3) |
| CF2 | 2 | 10 | 5.5640e-1 (4.30e-2) = | 6.3898e-1 (1.87e-2) = | 5.4452e-1 (2.82e-2) = | 5.4864e-1 (1.20e-2) = | 5.6804e-1 (1.33e-2) = | 5.5762e-1 (5.55e-2) |
| CF3 | 2 | 10 | 1.3281e-1 (1.67e-2) = | 6.4984e-2 (4.70e-2) = | 1.0377e-2 (1.80e-2) = | 1.1846e-1 (2.86e-2) = | 8.3207e-2 (2.55e-2) = | 1.1282e-1 (5.70e-2) |
| CF4 | 2 | 10 | 3.0333e-1 (2.10e-2) - | 3.3129e-1 (6.75e-2) - | 2.7031e-1 (3.48e-2) - | 3.3821e-1 (3.66e-3) - | 3.6154e-1 (8.35e-2) - | 3.7528e-1 (2.08e-2) |
| CF5 | 2 | 10 | 2.5901e-1 (2.75e-2) = | 2.1459e-1 (9.50e-2) = | 8.1618e-2 (7.13e-2) = | 2.1171e-1 (1.55e-1) = | 2.3387e-1 (4.85e-2) = | 2.0217e-1 (4.26e-2) |
| CF6 | 2 | 10 | 5.3286e-1 (5.31e-2) = | 6.1791e-1 (2.62e-2) = | 4.5915e-1 (4.26e-2) = | 6.0761e-1 (2.97e-2) = | 5.8273e-1 (2.92e-2) = | 5.7824e-1 (2.39e-2) |
| CF7 | 2 | 10 | 3.0562e-1 (1.94e-1) - | 3.8942e-1 (4.17e-2) - | 1.6696e-2 (2.89e-2) - | 3.4507e-1 (1.21e-1) - | 3.3930e-1 (1.96e-1) - | 4.0882e-1 (8.18e-2) |
| CF8 | 3 | 10 | 2.6601e-1 (2.50e-2) = | 2.4422e-1 (1.42e-2) = | 3.7163e-3 (5.26e-3) = | 1.8728e-1 (4.55e-2) = | 1.1610e-1 (3.44e-2) = | 1.5068e-1 (1.42e-2) |
| CF9 | 3 | 10 | 3.5328e-1 (4.06e-2) = | 3.8809e-1 (1.59e-2) = | 8.0371e-2 (1.73e-2) = | 3.1240e-1 (2.94e-2) = | 2.8862e-1 (2.27e-2) = | 3.1173e-1 (3.82e-2) |
| +/-/= | | | 0/2/7 | 0/2/7 | 0/2/7 | 0/2/7 | 0/2/7 | |

(a)MOEADDAE in CF4 (b)PPS in CF4 (c)IDBEA in CF4

(d)CMOEAMS in CF4 (e)TSTI in CF4 (f)TSTA in CF4

**Figure 2.** The results of six algorithms on CF4

Table 2 presents the obtained PF on CF of the six MOEAs compared. TSTA performs well on two problems, while MOEADDAE and PPS perform well on three and four problems, respectively. The DAE mechanism in MOEADDAE can help infeasible solutions enter the feasible region, so it performs well on the test set. Because of the complexity of the CF test set feasible domain [12], the TSTA algorithm cannot step into the infeasible region, so it don't perform well .

## 5. Conclusion

An improved two-stage constrained optimization algorithm based on indicator is proposed in this paper, named TSTA, for dealing with CMOPs. TSTA includes two stages. In the ***stage* I**, Balanced sorting is introduced to maintain convergence, feasibility and diversity, by which more unknown areas can be explored. In the ***stage* II**, the ARMOEA is adopted, reference points are adaptively adjusted based on non-dominant solutions in external files. It must be pointed out that TSTA still has a lot of room for improvement in solving CMOPs.

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