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Airline Crew Optimization Scheduling Problem

Jiacheng Wang*

School of Tianjin University of Commerce, Tianjin, China

*Corresponding author: 11445817@qq.com

Abstract

As people's standard of living continues to improve, air travel has become a widely chosen mode of transportation. To meet the demand for air travel, airlines are committed to improving operational efficiency, among which the optimization of airline crew scheduling is an important part of the entire airline's flight production planning process. Currently, the task of crew scheduling is primarily done manually by scheduling personnel, which requires high quality and capability of the personnel. Therefore, the scheduling work should evolve towards automation and intelligence. However, the existing automated scheduling methods still lack in refinement and cannot meet the increasingly complex management and operational requirements, and their dispatching and organizing capability cannot replace the existing manual scheduling mechanism. Thus, a more efficient automated scheduling method is the primary prerequisite for solving the optimization problem of airline crew scheduling. This paper aims to establish a linear programming model to assign crew members based on the principles of rationality and fairness.For the first problem, we start with data preprocessing, dividing the problem into two steps: the first is to pair 206 flights, and the second is to assign the paired flights to crew members. Given the large volume of flight and crew member data, the exhaustive method takes too long. Therefore, this paper adopts an innovative collaborative method of depth-first search algorithm and genetic algorithm to establish a linear programming model for this multi-objective optimization problem. In the first step, the depth-first search algorithm is used to explore all possible flight pairings, and in the second step, the genetic algorithm is used for the assignment of crew members. Without considering the cost of duty cycles and other tedious details, we conclude the flight pairing and crew member allocation under the basic constraints of aviation management regulations.For the second problem, this paper introduces the concept of "duty" and provides the duty cost per unit hour (DCP) for each crew member in the given data. Building on the first question, this paper pairs flights into duties based on further constraints, and selects the duty situations covering all flights with the minimum cost function based on the duty costs of the assigned crew members. This sub-problem introduces the concept of duty period (PDF) and calculates each crew member's flying time based on the duty schedule, combining the genetic algorithm from the first sub-question with the depth-first search algorithm to determine the flight pairings and crew member allocations that minimize duty costs.

Keywords

Genetic Algorithm; Depth-First; Crew Scheduling; Linear Programming Model.

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1. Introduction

In recent years, air travel has become the main travel option for people living in a fast-paced lifestyle. Data shows that with the increase in passenger traffic and the continuous growth of flight numbers, China's civil aviation system has leaped to the second largest air transport system in the world. Additionally, due to foreign airlines also competing for the domestic market, the competition among various airlines has become particularly fierce. Therefore, how to survive and grow in such a rapid development trend, achieving the optimal operational effect with the least cost, has become one of the important tasks that airlines need to solve in each work cycle[1]. Crew scheduling is a major part of the airline's operational plan organization, where each flight needs to be staffed with standard-compliant crew members to take off normally. It is also necessary to consider the airline's resource costs, crew members' vacations, and safety timeliness among other factors, to rationally arrange each crew member for flight tasks under these constraints. Since a large part of the airline's operating costs are generated by crew member costs, improving the efficiency of scheduling can significantly reduce the operational efficiency of airlines[2].

Since the 1970s, researchers have continuously explored the field of airline operations, such as Dennis Huisman and Albert P.M. Wagelmans from the Transportation Center at the Massachusetts Institute of Technology in the United States, who have conducted specific studies on airline resource allocation, revenue management, and crew scheduling, achieving notable results. The first company to use a large computer to solve scheduling problems was Sabre, which developed a scheduling algorithm that is a comprehensive solution. Compared to foreign countries, China's civil aviation industry started later, and research on crew scheduling is still at an immature stage[3]. The workflow is relatively rough, and in the early years, scheduling was still done manually, which was not only inefficient but also no longer suitable for the current surge in passenger traffic in the civil aviation system. Most civil aviation companies have introduced foreign scheduling systems to complete this work. Although directly adopting foreign scheduling systems can solve urgent problems, the specific conditions of different countries mean that such foreign algorithms cannot perfectly meet the operational requirements of domestic civil aviation[4]. Therefore, it is urgent to develop a crew scheduling system that suits the Chinese civil aviation system, and in recent years, domestic scholars have also made research contributions to this.

This paper mainly utilizes a linear programming model[5] with the goal of establishing the optimal crew scheduling scheme under a series of constraints. The operational management of airlines in actual work is very complex[6]. In this paper, by removing the intricate details and focusing on the core of the scheduling issue, we mainly concentrate on two aspects: how to generate Pairings from flights (Fight), and how to rationally and orderly match crew members (Crew) to various flights as well as scheduling work plans for them. Two research questions are proposed as follows:

Question 1: This sub-problem requires the assignment of flights to crew members under three constraints: 1) Each crew member must start from their base and ultimately return to it; 2) The departure airport of the next segment must be the same as the arrival airport of the previous segment; 3) The connection time between two adjacent segments must not be less than MinCT minutes, i.e., 40 minutes. At the same time, it must satisfy three "as much as possible" conditions: fulfill the minimum crew configuration for as many flights as possible, minimize the total number of flights taken overall, and use as few reserves as possible.

Question 2: This sub-problem introduces the concept of "Duty", which consists of multiple flights and interval connection times, while also strengthening the constraints: 1) Considering the balance of work and rest for crew members, each crew member can only perform one duty per day at most; 2) The flight time for each duty must not exceed MaxBlk minutes, i.e., 600 minutes; 3) The duration of each duty (including rest time) must not exceed MaxDP minutes, i.e., 720 minutes; 4) The starting airport of the next duty for each crew member must be consistent with the ending airport of the previous duty; 5) The rest time between two consecutive duties for crew members must not be less than MinRest minutes, i.e., 660 minutes. Additionally, this sub-problem needs to calculate the total

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duty cost for crew members, minimizing the overall cost and ensuring as balanced duty time as possible for each crew member to avoid any unfair situations[7].

2. Model Hypothesis

This paper proposes the following assumptions for the linear programming model:

1) In this model, the crew members only include the Captain and the First Officer, and they can be paired in any combination.

2) The model assumes that there is only one type of aircraft configuration.

3) The model allows for the existence of flights that cannot take off due to not meeting the minimum crew qualification configuration (Comp).

4) All crew members in this model cannot be assigned to flights that do not meet the minimum crew configuration.

5) Crew members are allowed to deadhead (Deadhead), where the segment time of deadheading crew members is not counted towards flying time but is included in duty time; each flight can have a maximum of MaxDH (MaxDH=5) deadheading passengers.

6) It is assumed that each crew member has a fixed base (Base), and they must start from and ultimately return to their base.

7) It is assumed that the arrival airport (ArrvStn) of a crew member's departing segment needs to be consistent with the departure airport (DptrStn) of the next leg.

8) It is assumed that the minimum connection time between adjacent segments for each crew member must be greater than or equal to MinCT minutes (MinCT=40 minutes).

9) It is assumed that each crew member can only perform duty once per day, and a single duty's flight time must be less than or equal to MaxBlk minutes (MaxBlk=600 minutes), with the duration of each duty not exceeding MaxDP minutes (MaxDP=720 minutes).

10) It is assumed that the arrival airport of a crew member's last duty must be consistent with the departure airport of the next duty.

11) It is assumed that the rest time between adjacent duties for each crew member is greater than or equal to MinRest minutes (MinRest=660 minutes).

12) The total duration of duty periods for each crew member during a scheduling period must be less than or equal to MaxTAFB minutes (MaxTAFB=14400 minutes).

13) The rest days between adjacent duty periods for each crew member must not be less than MinVacDay days (MinVacDay=2 days rest).

14) The consecutive duty days for each crew member must not exceed MaxSuccOn days (MaxSuccOn=4 days).

3. Symbol Description and Data Preprocessing

3.1 Description of Symbols

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Table 1. Description of symbols

sign	Meaning
N_1	Total number of mission cycles
N_2	Total number of flights
D	Total days required for flight missions
M	Total number of crew members
T	Total duration of flight missions over D days
i	Crew member ID
h	Flight number
d	Duty number
j	Mission cycle number
T_j	Working duration of the j-th mission cycle
z_h	Whether flight h is covered
C_{hf}	Whether flight h and f are in time conflict
x_{ih}	Whether the i-th crew member is assigned to flight h
x_{ij}	Whether the i-th crew member is assigned to mission cycle j
p_{ik}	Whether crew member i can perform flight tasks on day k
q_{jk}	Whether mission cycle j starts executing flight tasks on day k
s_{il}	Crew member's rank
c_{jl}	Whether mission cycle j requires crew members of rank l
t_h	Flight time of flight h
M'	Actual number of crew members required for all mission cycles
D_M	Available crew member days
a_i'	Ideal working time for each crew member
$D(t)$	Duty time

3.2 Data Preprocessing

This paper has obtained two sets of testing data. For Set A, we only considered flights and crew members from August 11, 2021, to August 25, 2021. After selection, a total of 206 flights and 21 crew members were subjected to crew-flight scheme allocation. For Set B, we only considered flights and crew members from August 1, 2021, to August 31, 2021. After selection, a total of 13,954 flights and 465 crew members were subjected to crew-flight scheme allocation. Additionally, to ensure the model's integrity, we also assigned attributes to flights that could not take off due to not meeting the minimum crew qualification configurations, facilitating the algorithm's operation.

This paper also conducted data preprocessing on the Excel spreadsheet of the flight schedule. The time data in the spreadsheet, such as departure and arrival times, were converted into numerical data using the formula HOUR(time)*60+MINUTE(time). For example, 10:10 was converted into 610 minutes. The given departure and arrival date data were not date data, so they were split using Excel's text-to-columns feature. Since the year was 2021 and the month was August for all data, only the day data was selected for calculation. This approach handled the time data, laying the groundwork for the subsequent operation of the algorithm. By encoding the departure and arrival times of flights, the

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variables' intended states could be efficiently compressed. Originally, the data provided in Excel required four variables to determine and optimize the flight path: departure date, departure time, arrival date, and arrival time. Now, it can be handled with just two variables. This approach of compressing states can reduce the number of variables, accelerating the model's solution.

4. Solution and Algorithm Design for Problem 1

4.1 Analysis of Problem 1

The first step involves data preprocessing. Problem 1 requires us to allocate processed crew member data to flights based on the constraints given in the problem statement. This process is divided into two steps: first, pairing the 206 flights in Data A-Flight, and second, assigning the paired flight groups to crew members. For the first step, we employ a depth-first search algorithm to traverse all flights and generate flight pairings under the condition of covering all flights as much as possible. For the second step of assigning crew members, we initially consider using the exhaustive method, arranging captains and co-pilots for each flight according to their requirements, listing all possible sets, and then eliminating combinations that do not meet the constraint conditions mentioned in the aviation management in the problem statement. However, due to the large volume of flight and crew member data, the time consumed by the exhaustive method is too long, making this inefficient method impractical in real-world applications. Therefore, we consider using a heuristic algorithm suitable for large-scale data that can be truly applied in the working environment—the genetic algorithm—to implement the crew scheduling process[8].

4.2 Model Construction for Problem 1

Considering the need to search for flights with shorter flight durations and earlier departure times, the aim is to successfully pair duty flights within the same day as much as possible to reduce accommodation costs for overnight stays. Since in Set A data, the number of bases for crew members is 1, with all captains and co-pilots having NKX as their fixed base, our flight pairing approach can be seen as having a start and end at NKX. Given the regulation that the rest time between adjacent duties must not be less than 660 minutes, if crew members finish their duty after a short flight, they will enter an interval exceeding 11 hours, thereby reducing the number of flights that meet the minimum crew configuration. Therefore, when seeking the optimal flight crew configuration, the duty hours of crew members should be as close as possible to the maximum duty hours, which are 720 minutes. We use a depth-optimized algorithm to solve the routing problem, with the idea being: for ease of program output, we number the flights on different dates with sequential numbers like 1, 2, 3, ... 206. For example, the FA680 flight on 8/12/2021 is numbered 3, the FA680 flight on 8/13/2021 is numbered 4, ... and the FA680 flight on 8/25/2021 is numbered 17, and so on. The departure airport is set as set 1, and the arrival airport as set 2. Starting with flight number 3 from NKX in set 1 with duty time TD=0, sequentially determine the data in set 2; if it equals NKX, continue matching; if not, keep the flight number in a subset, and at this time, subtract the initial departure time from the arrival time and add to TD; if TD is less than or equal to 720, continue operation, otherwise delete the number and stop computing. If continuing, sequentially determine the data in set 1; if they do not match, continue matching; if they match, calculate the departure time minus the arrival time—if it's greater than 40, stop computing; otherwise, continue matching until the result is output. If TD exceeds 720 during operation, output the result and then enter the next cycle, until all data in set 1 have been processed, then stop operating[9]. The results of our calculations are presented in 4.3:

Below is the construction of the personnel assignment model: $x_{ih} = 1$ indicates that the i-th crew member is assigned to the h-th flight; $x_{ih} = 0$indicates that the i-th crew member is not assigned to the h-th flight.

Here, we need to introduce the meaning of flight coverage: a flight is considered covered when the required crew members for the flight are met.

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$$\sum_{i=1}^{M} x_{ih} = d_h \ ,\forall h$$

When a flight's required crew members are not met at all, we say the flight is completely uncovered. In this case, the equation is:

$$\sum_{i=1}^{M} x_{ih} = 0 \ ,\forall h$$

Introduce z_h as a new decision variable, where$z_h = 1$ indicates that the h-th flight is covered, and $z_h = 0$ indicates that the h-th flight is completely uncovered. Thus, the overall coverage rate is:

$$\sum_{h=1}^{N_2} z_h$$

Set the objective function as:

$$\max f = \sum_{h=1}^{N_2} z_h$$

Based on the above formula, for the scheduling plan to be executable, it must satisfy:

$$\sum_{i=1}^{M} x_{ih} = d_h \, z_h \ ,\forall h = 1,2, \dots N_2$$

When the h-th flight is completely uncovered, that is, $z_h = 0$, any x_{ih} must also equal 0; when the h-th flight is covered, that is, $z_h = 1$, the equation simplifies to:

$$\sum_{i=1}^{M} x_{ih} = d_h \ ,\forall h$$

Moreover, since crew members cannot serve two flights at the same time, to prevent the algorithm from assigning the same crew member to overlapping flights, we introduce a conflict matrix. We use C_{hf} to denote whether flights h and f are in time conflict, i.e., $C_{hf} = 1$ indicates that flights h and f conflict in time; otherwise, it indicates that the two flights do not conflict in time, and flights h and f can be assigned to the same crew member. Therefore, the constraints of this model can be divided into:

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$$\max f = \sum_{h=1}^{N_2} z_h$$
$$\sum_{i=1}^{M} x_{ih} = d_h z_h$$
$$x_{ih} + x_{if} + C_{hf} \leq 2$$

Model construction is complete.

4.3 Experimental Results for Problem 1

Generation of Flight Pairings.

D1: 3--18--33--48; D2: 4--19--34--49; D3: 5--20--35--50; D4: 6--21—36--51; D5: 7--22--37--52; D6: 8--23--38--53; D7: 9--24--39--54; D8: 10--25--40--55; D9: 11--26--41--56; D10: 12--27--42--57; D11: 13--28--43--58; D12: 14--29--44--59; D13: 15--30--45--60; D14: 16--31--46--61; D15: 17--32--47--62; D16: 63--78; D17: 64--79; D18: 65--80; D19: 66--81; D20: 67--82; D21: 68--83; D22: 69--84; D23: 70--85; D24: 71--86; D25: 72--87; D26: 73--88; D27: 74--89; D28: 75--90; D29: 76--91; D30: 77--92; D31: 93--108; D32: 94--109; D33: 95--110; D34: 96--111; D35: 97--112; D36: 98--113; D37: 99--114; D38: 100--115; D39: 101--116; D40: 102--117; D41: 103--118; D42: 104--119; D43: 105--120; D44: 106--121; D45: 107--122; D46: 123--137--152--167; D47: 124--138--153--168; D48: 125--139--154--169; D49: 126--140--155--170; D50: 12--141--156--171; D51: 128--142--157--172; D52: 129--143--158--173; D53: 130--144--159--174; D54: 131--145--160--175; D55: 132--146--161--176; D56: 133--147--162--177; D57: 134--148--163--178; D58: 135--149--164--179; D59: 136--150--165--180; D60: 181--182; D61: 183--195; D62: 184--197; D63: 185--198; D64: 186--199; D65: 187--200; D66: 188--201; D67: 189--202; D68: 190--203; D69: 191--204; D70: 192--205; D71: 193--206.

Among them, the number of flights that meet the crew configuration requirements is 170.

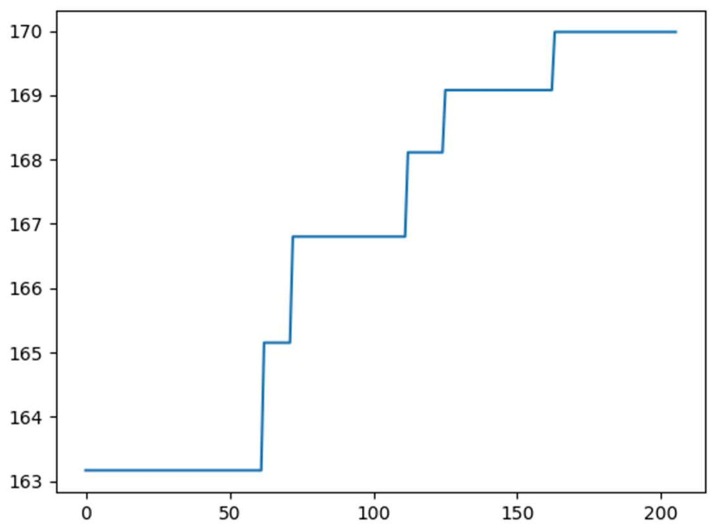

Fig. 1 Genetic Algorithm for Flights Meeting Crew Configuration

Number of flights not meeting crew configuration: 36.

Total number of flights taken by crew members: 61.

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5. Solution and Algorithm Design for Problem 2

5.1 Analysis of Problem 2

The second sub-problem introduces the concept of "Duty," and the given data includes the duty cost per unit hour (DCP) for each crew member. Since the first question, assigning crew members to various flights, has been completed, the second question requires us to further pair flights into duties based on additional constraints. Moreover, we need to select duties covering all flights that minimize the cost function, based on the duty costs of the assigned crew members. The first strategy is to primarily consider crew members capable of serving both as captain and co-pilot, without considering factors such as flight duration. The second strategy optimizes flight pairings, including considerations of flight and rest times, to reduce crew members' duty costs. This sub-problem employs genetic algorithms and depth-first search algorithms, outlining penalty functions, mutation operators, and selection operators to construct the model and solve it[10].

5.2 Model Construction for Problem 2

Let PDT represent the duty period, indicating the total duration from the start to the end of a duty. Based on the constraints, the flying time for the i-th pilot can be represented by the following formula:

$$\sum_{j=1}^{m} x_{ij} PDT_j$$

Assuming the average flying time per crew member is denoted as AVG, it can be expressed as:

$$\text{AVG}=\frac{\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} PDT_j}{n}$$

Assuming the scheduling cycle in days is CYD, since each crew member's number of flying days within a YD varies, it's necessary to calculate the flying days relative to the current crew member's AVG ratio by proportion. That is, the relative AVG for a crew member = (AVG / CYD) * flying days. Let FMi represent the flying days for the i-th crew member; hence, the objective function can be defined as:

$$w_c \sum_{j=1}^{m} z_j + w_f \sqrt{\frac{\sum_{i=1}^{n}\left(\sum_{j=1}^{m} x_{ij} PDT_j - \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} PDT_j}{n}\frac{FMi}{CYD}\right)^{\wedge}2}{n}}$$

In the above formula, w_c and w_f are two constants used to adjust the influence of different objectives on the optimization goal. Then, based on the duty cost per unit hour for crew members with different functions, supplemented by the pairing results of flights and crew members calculated through the genetic algorithm from the first question, we use each pilot's flying time $\sum_{j=1}^{m} x_{ij} PDT_j$ multiplied by the duty cost per hour at different pilot ranks-680, 640, 600-to seek a more optimized pairing of crew members and flights.

This paper introduces a Depth-First Search algorithm model (DFS), which falls under graph algorithms. Simply put, the process involves deeply calculating every possible path established in the model until the algorithm can no longer proceed, then outputs the result[11].

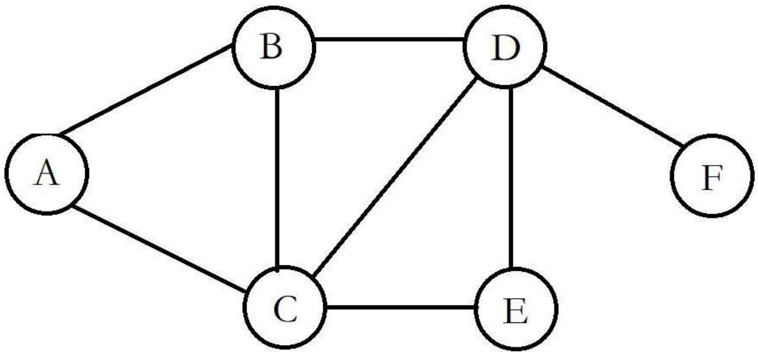

Fig. 2 Node Diagram

As illustrated in Fig.2, assuming node A is the starting node, place the starting node A into a set, then search for A's children nodes, nodes B and C, and add them to the set. Next, remove node B from the set, search for B's child node D. After that, take node D out, search for its child node F, and include it in the set. Continue repeating the above operations until the entire graph has been searched. When a node has multiple child nodes, the order in which the child nodes are added to the set is random, and the same algorithm may yield different results. When reaching node F, if it is found that there are no child nodes on branch F, the search will return to F's parent node and look for other unexplored child nodes. If the parent node has no other child nodes, the search will continue to return to the parent node of the upper level until the entire process is completed.

5.3 Experimental Results for Problem 2

To streamline the algorithm, this paper has further compressed variables by processing the time data that had already been preprocessed. By calculating the difference between arrival and departure times, the flight duration for each flight, i.e., the duty flight time, was determined. This approach allows for a clearer presentation of the thought process, a more concise algorithm formulation, and an increased likelihood of finding solutions closer to the optimal. The results of the run are shown in the following figure:

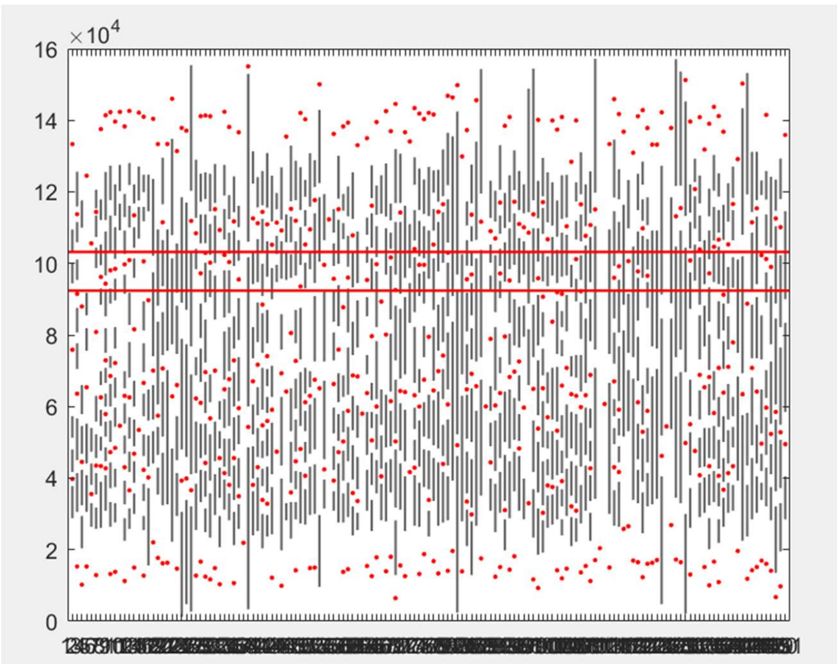

Fig. 3 Depth Optimization for the Lowest Duty Cost Plan

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6. Summary

This paper analyzes a classic problem in operations research through modeling-the crew scheduling problem. Given the current state of aviation crew scheduling, which primarily relies on manual planning due to difficulties with automated scheduling, this paper proposes a mathematical model for aviation crew scheduling. The main contributions of this paper include:

(1) The research status and journal papers on aviation crew scheduling and related topics were collected through platforms such as CNKI and CSDN. It was found that under the competitive pressure of the domestic and international airline market, airlines are compelled to seek solutions that achieve optimal operational effects at the lowest cost. Therefore, optimizing crew scheduling is an urgent need. To solve the crew scheduling problem, this paper established a mathematical model using genetic algorithms and depth-first search algorithms.

(2) In Problem 1, the paper preprocesses data, allocating processed crew member data to flights based on the constraints given in the problem. This was divided into two steps: first, pairing the 206 flights in Data A-Flight, and second, assigning the paired flights to crew members. For the first step, a depth-first search algorithm was used to traverse all flights and generate flight pairings under the condition of covering all flights as much as possible. For the second step, considering the exhaustive method initially to assign captains and co-pilots to each flight based on their requirements, and then eliminating combinations that do not meet the constraints set by aviation management. However, due to the large volume of flight and crew member data, and the extensive time required by the exhaustive method, this inefficient method is impractical for real-world application, thus the genetic algorithm was employed to calculate the final data.

(3) In Problem 2, the concept of duty was introduced, along with the given duty cost per unit hour for each crew member. Building on the first question, further constraints were applied to pair flights into duties and select duties that cover all flights with the minimum cost function based on the duty costs of assigned crew members. This sub-problem introduces duty time and expresses the flying time for each pilot based on it, combining the genetic algorithm from the first sub-question with a depth-first search algorithm to identify the flight pairings and crew member assignments that minimize duty costs.

Efficient crew scheduling is one of the urgent problems to be solved in the civil aviation industry. This paper optimizes the crew scheduling problem using algorithms such as the genetic algorithm, effectively narrowing the decision space for multi-objective optimization under multiple constraints to bring the model's output closer to the optimal solution. In future optimizations of the model, it is possible to consider further expanding the time unit span and designing more comprehensive and reasonable constraints to increase the possibility of obtaining higher quality scheduling plans.

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