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Intermittent Pinning Synchronization for Complex Values Markovian Cohen-Grossberg Neural Networks with Reaction-Diffusion by A Nonseparation Approach

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Abstract

A complex Cohen-Grossberg dynamic network model with time-varying delay, Markov chain, parameter switching, and reaction-diffusion terms is established. In order to reduce the design cost of the controller, a method combining aperiodic intermittence and pinning control is proposed, which makes the master-slave neural network realize exponential synchronization. Under the framework of non-separation, by using Lyapunov stability theory and the complex inequality technique, the master-slave system synchronization condition is obtained. Finally, the validity of the theorem is verified by a numerical simulation.

Keywords

Complex Valued Markov Cohen-Grossberg Neural Network; Aperiodic Intermittent Pinning; Non-separation; Exponential Synchronization.

1. Introduction

Cohen-Grossberg neural network (CGNNs) was first proposed[1], and then different types of CGNNs were widely studied, such as memristor CGNNs, reaction-diffusion CGNNs [2–3], and so on. However, this literature did not take into account that many actual systems would switch between different models during operation due to factors such as device failure or external environment changes, which can be described by the Markov chain [4]. In addition, complex valued neural networks have superior computational power and performance in symmetry detection [5]. In recent years, many scholars have divided complex value systems into real part and imaginary part systems for research, meanwhile achieved good research results [6]. Although the separation method is feasible, dividing a complex-valued system into two real-valued systems will greatly increase the complexity of theoretical analysis. Therefore, Feng et al. [7] used a non-separation method to analyze complex valued systems.

On the other hand, aperiodic intermittent pinning control has excellent performance in the number of controlled nodes and control time, which can effectively reduce the amount of data transmission [8]. At present, some scholars have obtained good research results through intermittent pinning control [9-10]. However, non-periodic intermittent pinning control of Markovian reaction-diffusion complex CGNNs with parameter switching has not been studied. Therefore, this article explores this topic, and the innovation points and main achievements are as follows:

1) This paper constructs a complex CGNNs with Markovian chains, and parameter-dependent state switching. It is more universal and more widely used.

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2) A non-periodic intermittent pinning controller based on non-separation is designed, which can reduce the number of controllers to be designed and the control cost and make the analysis process simpler.

Notations: In this article, $\mathfrak{N}=\{1,2,\cdots,N\}$ with N is a positive integer, R,C represents real and complex numbers respectively. $\forall\lambda\in\mathrm{C},\mathrm{Re}(\lambda),\mathrm{Im}(\lambda)$ represents the real and imaginary parts of λ, $|\lambda|=\sqrt{\lambda\bar{\lambda}}=\sqrt{\bar{\lambda}\lambda}$ represents the module of λ, accordingly $\bar{\lambda}$ is λ conjugate complex number. $*\in\mathbb{R},|*|;*\in\mathbb{C},|*|$ represents the absolute value and module value value of $*$ respectively. And the space position $\Omega=\left\{\mathfrak{x}=(\mathfrak{x}_1,\mathfrak{x}_2,\ldots,\mathfrak{x}_m)^T\right\}\|\mathfrak{x}_\sigma|\le\ell_\sigma,\sigma=1,2,\ldots,m$. Symbolic function: $\mathrm{sgn}(\lambda)=\mathrm{sgn}(\lambda^R)+\mathrm{sgn}(\lambda^I)\mathrm{i}\in\mathbb{C}$. abbreviations as follows: $\begin{cases}\mathfrak{L}_i=\mathfrak{L}_i(\mathfrak{x},t)\\ \mathfrak{L}_{\tau j}=\mathfrak{L}_j(\mathfrak{x},t-\tau(t))\\ \mathfrak{L}=\mathfrak{p},\mathfrak{q},\mathfrak{w},\mathfrak{U}\end{cases}\begin{cases}\nu_{\mathfrak{pq}}^{\eta\mu}=\eta_{\varepsilon i}(\mathfrak{p}_i)\mu_{\varepsilon i}(\mathfrak{p}_i)-\eta_{\varepsilon i}(\mathfrak{q}_i)\mu_{\varepsilon i}(\mathfrak{q}_i)\\ \nu_{\mathfrak{pq}}^{J\eta}=J_i(\eta_{\varepsilon i}(\mathfrak{p}_i)-\eta_{\varepsilon i}(\mathfrak{q}_i))\end{cases}$.

2. Problem Description and Related Preparations

2.1 Markovian Process and Connection Weight Switching Mechanism Description

Consider a continuous-time Markovian chain $\{r_t:t\ge 0\}$ with the following transition probabilities:

$$\Pr\{r_{t+\Delta t}=\iota|r_t=\varepsilon\}=\begin{cases}\kappa_{\varepsilon\iota}s+o(s), & \varepsilon\ne\iota\\ 1+\kappa_{\varepsilon\varepsilon}s+o(s), & \varepsilon=\iota\end{cases},\varepsilon,\iota\in S=\{1,2,\ldots,N\}$$

Where: s represents the residence time and satisfies $s>0,\lim_{s\to 0}[o(s)/s]=0$, $\kappa_{\varepsilon\iota},\varepsilon\ne\iota$ is the transfer rate , and $\forall\varepsilon,\iota\in S,\kappa_{\varepsilon\varepsilon}=-\sum_{\iota\in S,\iota\ne\varepsilon}\kappa_{\varepsilon\iota}$. The state switching rules corresponding to the complex domain model:

$$\underline{\Delta}_{\varepsilon ij}(\Gamma_i),(\underline{\Delta}=\mathfrak{a},\mathfrak{b};\Gamma=\mathfrak{p},\mathfrak{q}),\underline{\Delta}_{\varepsilon ij}(\Gamma_i)=\begin{cases}\hat{\underline{\Delta}}_{\varepsilon ij}, & |\Gamma_i|\le\mathfrak{I}\\ \breve{\underline{\Delta}}_{\varepsilon ij}, & |\Gamma_i|>\mathfrak{I}\end{cases}$$

Where: $\mathfrak{I}>0$ is switching constant, $\hat{\mathfrak{a}}_{\varepsilon ij},\breve{\mathfrak{a}}_{\varepsilon ij},\hat{\mathfrak{b}}_{\varepsilon ij},\breve{\mathfrak{b}}_{\varepsilon ij}\in\mathrm{C}$ satisfy $\hat{\mathfrak{a}}_{\varepsilon ij}\ne\breve{\mathfrak{a}}_{\varepsilon ij},\hat{\mathfrak{b}}_{\varepsilon ij}\ne\breve{\mathfrak{b}}_{\varepsilon ij}$ .Markovian process definition: $r_t=\varepsilon\in S$, switching process definition: $\tilde{\underline{\Delta}}_{\varepsilon ij}=\max\left\{|\hat{\underline{\Delta}}_{\varepsilon ij}|,|\breve{\underline{\Delta}}_{\varepsilon ij}|\right\}$.

2.2 Closed-loop Network Construction

In this paper, consider the following CGNNs as the driving system:

$$\frac{\partial\mathfrak{p}_i}{\partial t}=-\eta_{\varepsilon i}(\mathfrak{p}_i)\times\left[\mu_{\varepsilon i}(\mathfrak{p}_i)-\sum_{j=1}^{n}\mathfrak{a}_{\varepsilon ij}(\mathfrak{p}_i)f_j(\mathfrak{p}_j)-\sum_{j=1}^{n}\mathfrak{b}_{\varepsilon ij}(\mathfrak{p}_i)f_j(\mathfrak{p}_{\tau j})+J_i\right]+\theta_i\Delta\mathfrak{p}_i\ ,i,j\in\mathfrak{N} \tag{1}$$

Where:, $\mathfrak{p}_i\in\mathrm{C}$ is the state variable of the i th neuron at time t and space $\mathfrak{x}$, $\eta_i(\mathfrak{p}_i),\mu_i(\mathfrak{p}_i),f_j(\cdot)\in\mathrm{C}$ represent the amplification function, the behavior function and the activation function of the system, $a_{ij},b_{ij}\in\mathrm{C}$ represents the neuron connection weight, $\tau(t)$ represents a time-varying discrete delay and satisfy $\tau(t)\le\tau$, $\Delta=\partial^2/\partial\mathfrak{x}^2$ is the laplace diffusion operator defined on Ω , $\theta_i>0$ represents the diffusion coefficient of transmission along the i ineuron. $J_i\in\mathrm{C}$ is external interference and conforms to the boundedness $|J_i|\le\tilde{J}_i$.

In addition, formula (1) satisfies the following conditions:

$$\mathfrak{p}_i(\mathfrak{x},t)=0,(\mathfrak{x},t)\in\partial\Omega\times[-\tau,+\infty)\quad \mathfrak{p}_i(\mathfrak{x},s)=\varphi_{\mathfrak{p}i}(\mathfrak{x},s),(\mathfrak{x},s)\in\Omega\times[-\tau,0) \tag{2}$$

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Obviously, (1) is a discontinuous system, and the traditional continuous solution cannot be defined. By set-valued mapping and differential inclusion theory, such as:

$$\frac{\partial \mathfrak{p}_i}{\partial t} \in -\eta_{\varepsilon i}(\mathfrak{p}_i) \times \left[\mu_{\varepsilon i}(\mathfrak{p}_i) - \sum_{j=1}^{n} co\left[\mathfrak{a}_{\varepsilon ij}(\mathfrak{p}_i)\right] f_j(\mathfrak{p}_j) - \sum_{j=1}^{n} co\left[\mathfrak{b}_{\varepsilon ij}(\mathfrak{p}_i)\right] f_j(\mathfrak{p}_{\tau j}) + J_i \right] + \theta_i \Delta \mathfrak{p}_i \tag{3}$$

Where: co is closed convex hull, $co\left[\mathfrak{a}_{\varepsilon ij}(\mathfrak{p}_i)\right], co\left[\mathfrak{b}_{\varepsilon ij}(\mathfrak{p}_i)\right]$ as follows:

$$co\left[\mathfrak{a}_{\varepsilon ij}(\mathfrak{p}_i)\right] = \begin{cases} \hat{\mathfrak{a}}_{\varepsilon ij}, & |\mathfrak{p}_i| \le \Im \\ \left[\hat{\mathfrak{a}}_{\varepsilon ij}, \breve{\mathfrak{a}}_{\varepsilon ij}\right], & |\mathfrak{p}_i| = \Im \\ \breve{\mathfrak{a}}_{\varepsilon ij}, & |\mathfrak{p}_i| > \Im \end{cases}, \quad co\left[\mathfrak{b}_{\varepsilon ij}(\mathfrak{p}_i)\right] = \begin{cases} \hat{\mathfrak{b}}_{\varepsilon ij}, & |\mathfrak{p}_i| \le \Im \\ \left[\hat{\mathfrak{b}}_{\varepsilon ij}, \breve{\mathfrak{b}}_{\varepsilon ij}\right], & |\mathfrak{p}_i| = \Im \\ \breve{\mathfrak{b}}_{\varepsilon ij}, & |\mathfrak{p}_i| > \Im \end{cases} \tag{4}$$

According to measurable theory $\exists \Delta^*_{\varepsilon ij}(\mathfrak{p}_i) \in co\left[\Delta_{\varepsilon ij}(\mathfrak{p}_i)\right]$, (3) be rephrased as:

$$\frac{\partial \mathfrak{p}_i}{\partial t} = -\eta_{\varepsilon i}(\mathfrak{p}_i) \times \left[\mu_{\varepsilon i}(\mathfrak{p}_i) - \sum_{j=1}^{n} \mathfrak{a}^*_{\varepsilon ij} f_j(\mathfrak{p}_j) - \sum_{j=1}^{n} \mathfrak{b}^*_{\varepsilon ij}(\mathfrak{p}_i) f_j(\mathfrak{p}_{\tau j}) + J_i \right] + \theta_i \Delta \mathfrak{p}_i \tag{5}$$

The response system corresponding to (1) is as follows:

$$\frac{\partial \mathfrak{q}_i}{\partial t} = -\eta_{\varepsilon i}(\mathfrak{q}_i) \times \left[\mu_{\varepsilon i}(\mathfrak{q}_i) - \sum_{j=1}^{n} \mathfrak{a}_{\varepsilon ij}(\mathfrak{q}_i) f_j(\mathfrak{q}_j) - \sum_{j=1}^{n} \mathfrak{b}_{\varepsilon ij}(\mathfrak{q}_i) f_j(\mathfrak{q}_{\tau j}) + J_i \right] + \theta_i \Delta \mathfrak{q}_i + \mathfrak{U}_i \tag{6}$$

In addition, formula (6) satisfies:

$$\mathfrak{q}_i(\mathfrak{x}, t) = 0, (\mathfrak{x}, t) \in \partial\Omega \times [-\tau, +\infty) \quad \mathfrak{q}_i(\mathfrak{x}, s) = \varphi_{\mathfrak{q}i}(\mathfrak{x}, s), (\mathfrak{x}, s) \in \Omega \times [-\tau, 0) \tag{7}$$

Similar to the above process:

$$\frac{\partial \mathfrak{q}_i}{\partial t} \in -\eta_{\varepsilon i}(\mathfrak{q}_i) \times \left[\mu_{\varepsilon i}(\mathfrak{q}_i) - \sum_{j=1}^{n} co\left[\mathfrak{a}_{\varepsilon ij}(\mathfrak{q}_i)\right] f_j(\mathfrak{q}_j) - \sum_{j=1}^{n} co\left[\mathfrak{b}_{\varepsilon ij}(\mathfrak{q}_i)\right] f_j(\mathfrak{q}_{\tau j}) + J_i \right] + \theta_i \Delta \mathfrak{q}_i + + \mathfrak{U}_i \tag{8}$$

Where $co\left[\mathfrak{a}_{\varepsilon ij}(\mathfrak{q}_i)\right], co\left[\mathfrak{b}_{\varepsilon ij}(\mathfrak{q}_i)\right]$ as follows:

$$co\left[\mathfrak{a}_{\varepsilon ij}(\mathfrak{q}_i)\right] = \begin{cases} \hat{\mathfrak{a}}_{\varepsilon ij}, & |\mathfrak{q}_i| \le \Im \\ \left[\hat{\mathfrak{a}}_{\varepsilon ij}, \breve{\mathfrak{a}}_{\varepsilon ij}\right], & |\mathfrak{q}_i| = \Im \\ \breve{\mathfrak{a}}_{\varepsilon ij}, & |\mathfrak{q}_i| > \Im \end{cases}, \quad co\left[\mathfrak{b}_{\varepsilon ij}(\mathfrak{q}_i)\right] = \begin{cases} \hat{\mathfrak{b}}_{\varepsilon ij}, & |\mathfrak{q}_i| \le \Im \\ \left[\hat{\mathfrak{b}}_{\varepsilon ij}, \breve{\mathfrak{b}}_{\varepsilon ij}\right], & |\mathfrak{q}_i| = \Im \\ \breve{\mathfrak{b}}_{\varepsilon ij}, & |\mathfrak{q}_i| > \Im \end{cases} \tag{9}$$

similarly $\exists \Delta^{**}_{\varepsilon ij}(\mathfrak{q}_i) \in co\left[\Delta_{\varepsilon ij}(\mathfrak{q}_i)\right]$, (8) be rephrased as:

$$\frac{\partial \mathfrak{q}_i}{\partial t} = -\eta_{\varepsilon i}(\mathfrak{q}_i) \times \left[\mu_{\varepsilon i}(\mathfrak{q}_i) - \sum_{j=1}^{n} \mathfrak{a}^{**}_{\varepsilon ij} f_j(\mathfrak{q}_j) - \sum_{j=1}^{n} \mathfrak{b}^{**}_{\varepsilon ij}(\mathfrak{q}_i) f_j(\mathfrak{q}_{\tau j}) + J_i \right] + \theta_i \Delta \mathfrak{q}_i + \mathfrak{U}_i \tag{10}$$

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Set error signal: $\mathfrak{w}_i = \mathfrak{q}_i - \mathfrak{p}_i$, The error dynamic behavior is as follows:

$$\frac{\partial \mathfrak{w}_i}{\partial t} = \nu_{\mathfrak{pq}}^{\eta\mu} + \theta_i \Delta \mathfrak{w}_i + \nu_{\mathfrak{pq}}^{J\eta} + \mathfrak{U}_i + \eta_{\varepsilon i}(\mathfrak{q}_i)\sum_{j=1}^{n} a_{\varepsilon ij}^{**}(\mathfrak{q}_i) f_j(\mathfrak{q}_j) - \eta_{\varepsilon i}(\mathfrak{p}_i)\sum_{j=1}^{n} a_{\varepsilon ij}^{*}(\mathfrak{p}_i) f_j(\mathfrak{p}_j) + \eta_{\varepsilon i}(\mathfrak{q}_i)\sum_{j=1}^{n} b_{\varepsilon ij}^{**}(\mathfrak{q}_i) f_j(\mathfrak{q}_{\tau j}) - \eta_{\varepsilon i}(\mathfrak{p}_i)\sum_{j=1}^{n} b_{\varepsilon ij}^{*}(\mathfrak{p}_i) f_j(\mathfrak{p}_{\tau j}) \tag{11}$$

The necessary assumptions, lemmas, and definitions are given before the main result is obtained:

Assumption 1 [11]: If $\mathcal{N}_{\varepsilon i}^{\eta}, \mathcal{N}_j^{f}, \mathcal{I}_{\varepsilon i}^{\eta}, \mathcal{I}_{\varepsilon i}^{\eta\mu}, \mathcal{I}_j^{f} > 0$ such that:

$$\left|\eta_{\varepsilon i}(\tilde{\lambda}_2) - \eta_{\varepsilon i}(\tilde{\lambda}_1)\right| \le \mathcal{I}_{\varepsilon i}^{\eta}\left|\tilde{\lambda}_2 - \tilde{\lambda}_1\right|, \left|\eta_{\varepsilon i}(\tilde{\lambda}_i)\right| \le \mathcal{N}_{\varepsilon i}^{\eta}, \left|f_j(\tilde{\lambda}_2) - f_j(\tilde{\lambda}_1)\right| \le \mathcal{I}_j^{f}\left|\tilde{\lambda}_2 - \tilde{\lambda}_1\right|$$
$$\left|f_j(\tilde{\lambda}_i)\right| \le \mathcal{N}_j^{f}, \left|\eta_{\varepsilon i}(\tilde{\lambda}_2)\mu_{\varepsilon i}(\tilde{\lambda}_2) - \eta_{\varepsilon i}(\tilde{\lambda}_1)\mu_{\varepsilon i}(\tilde{\lambda}_1)\right| \le \mathcal{I}_{\varepsilon i}^{\eta\mu}\left|\tilde{\lambda}_2 - \tilde{\lambda}_1\right|$$

Where: $\forall \tilde{\lambda}_i, \tilde{\lambda}_1, \tilde{\lambda}_2 \in \mathrm{C}, \tilde{\lambda}_1 \ne \tilde{\lambda}_2$.

Lemma 1 [7]: $\forall \mathfrak{h} : \mathrm{R} \to \mathrm{C}$: $\overline{\mathrm{sgn}(\mathfrak{h})}\mathfrak{h} + \bar{\mathfrak{h}}\mathrm{sgn}(\mathfrak{h}) \ge 2|\mathfrak{h}|$.

Lemma 2 [3]: A real-valued functions $\mathfrak{H}(x)$ defined $C^1(\Omega)$ above that satisfy $\mathfrak{H}(\mathfrak{x})|_{\partial\Omega} = 0$, then:

$$\int_{\Omega} \mathfrak{H}^2(\mathfrak{x}) d\mathfrak{x} \le \ell_{\sigma}^2 \int_{\Omega} \left|\frac{\partial \mathfrak{H}(\mathfrak{x})}{\partial \mathfrak{x}_{\sigma}}\right|^2 d\mathfrak{x}$$

Where: $\Omega = \left\{\mathfrak{x} = (\mathfrak{x}_1, \mathfrak{x}_2, \ldots, \mathfrak{x}_m)^T\right\} \|\mathfrak{x}_{\sigma}| \le \ell_{\sigma}, \sigma = 1, 2, \ldots, m$.

Definition 1 [8]: If $\mathrm{m} > 0, \varsigma > 1$ such as:

$$\|\mathfrak{q}_i - \mathfrak{p}_i\| \le \varsigma \sup_{-\tau \le s \le 0} \|\psi_i(\mathfrak{x}, s) - \varphi_i(\mathfrak{x}, s)\| e^{-\mathrm{m}t}$$

The response system (6) is exponentially synchronized with the drive system (1).

3. Main Results

3.1 Controller Design

In order to synchronize the error system (11), intermittent pinning control is adopted:

$$\mathfrak{U}_i = \begin{cases} -\mathrm{sgn}(\mathfrak{w}_i)\left(k_i \dfrac{\sum_{i=1}^{n}|\mathfrak{w}_i|}{\sum_{i=1}^{n_1}|\mathfrak{w}_i|}\sum_{i=1}^{n}|\mathfrak{w}_i| + \phi_i \dfrac{\sum_{i=1}^{n}|\mathfrak{w}_i|}{\sum_{i=1}^{n_1}|\mathfrak{w}_i|}\right), & t_l \le t < t_l + \mathcal{X}_l w_l \\ -\mathrm{sgn}(\mathfrak{w}_i)\phi_i \dfrac{\sum_{i=1}^{n}|\mathfrak{w}_i|}{\sum_{i=1}^{n_1}|\mathfrak{w}_i|}, & t_l + \mathcal{X}_l w_l \le t < t_{l+1} \end{cases}, i \in \mathfrak{N}_1 = \{1, 2, \ldots N_1\}(1 \le N_1 < \mathrm{N}) \tag{12}$$

Where: $k_i > 0, \phi_i > 0; w_l = t_{l+1} - t_l, w_0 = 0, 0 < \mathcal{X}_l < 1, \mathcal{X}_0 = 0, t_1 = 0$ and $\sum_{i=1}^{n_1}|\omega_i| \ne 0$.

3.2 Criteria of Exponential Synchronization

In order to express theorem 1 clearly, the following related symbolic definitions are given:

$$\Pi_{\varepsilon ij}=2\mathcal{N}_{\varepsilon i}^{\eta}\mathcal{N}_{j}^{f}\left(\left|\hat{\mathfrak{a}}_{\varepsilon ij}-\breve{\mathfrak{a}}_{\varepsilon ij}\right|+\left|\hat{\mathfrak{b}}_{\varepsilon ij}-\breve{\mathfrak{b}}_{\varepsilon ij}\right|\right),\Pi_{\varepsilon i}=\underset{\max\{j\in\mathfrak{R}\}}{\Pi_{\varepsilon ij}},\mathcal{T}_{\varepsilon i}=\sum_{j=1}^{n}\mathcal{N}_{\varepsilon j}^{\eta}\tilde{\mathfrak{b}}_{\varepsilon ji}\mathcal{I}_{i}^{f},\mathcal{T}=\underset{\max\{\varepsilon\in S,i\in\mathfrak{R}\}}{\mathcal{T}_{\varepsilon i}},\mathcal{P}_{\varepsilon i}>\Xi_{\varepsilon i},\mathcal{P}=\underset{\max\{\varepsilon\in S,i\in\mathfrak{R}\}}{\mathcal{P}_{\varepsilon i}},$$

$$\mathcal{P}^{+}=\max\{0,\mathcal{P}\},\ \Xi_{\varepsilon i}=\sum_{j=1}^{n}\left(2\tilde{\mathfrak{a}}_{\varepsilon ij}\left(\mathcal{N}_{\varepsilon i}^{\eta}\mathcal{I}_{j}^{f}+\mathcal{N}_{j}^{f}\mathcal{I}_{\varepsilon i}^{\eta}\right)+2\tilde{\mathfrak{b}}_{\varepsilon ij}\mathcal{N}_{j}^{f}\mathcal{I}_{\varepsilon i}^{\eta}+\tilde{\mathfrak{b}}_{\varepsilon ij}\mathcal{N}_{\varepsilon i}^{\eta}\mathcal{I}_{j}^{f}\right)+2\left|J_{i}\mathcal{I}_{\varepsilon i}^{\eta}\right|+2\left|\mathcal{I}_{\varepsilon i}^{\eta\mu}\right|-2\frac{\theta_{i}}{\ell_{\sigma}^{2}},$$

$$\mathcal{H}_{\eta\mathfrak{a}(sub)f}=\eta_{\varepsilon i}(\mathfrak{q}_{i})\left(\mathfrak{a}_{\varepsilon ij}^{**}(\mathfrak{q}_{i})-\mathfrak{a}_{\varepsilon ij}^{*}(\mathfrak{p}_{i})\right)f_{j}(\mathfrak{q}_{j}),\mathcal{H}_{\eta\mathfrak{b}(sub)f}=\eta_{\varepsilon i}(\mathfrak{q}_{i})\left(\mathfrak{b}_{\varepsilon ij}^{**}(\mathfrak{q}_{i})-\mathfrak{b}_{\varepsilon ij}^{*}(\mathfrak{p}_{i})\right)f_{j}(\mathfrak{q}_{\tau j}),$$

$$\mathcal{A}_{\eta\mathfrak{a}f(sub)}=\eta_{\varepsilon i}(\mathfrak{q}_{i})\mathfrak{a}_{\varepsilon ij}^{*}(\mathfrak{p}_{i})\left(f_{j}(\mathfrak{q}_{j})-f_{j}(\mathfrak{p}_{j})\right)\mathcal{A}_{\eta\mathfrak{b}f(sub)}=\eta_{\varepsilon i}(\mathfrak{q}_{i})\mathfrak{b}_{\varepsilon ij}^{*}(\mathfrak{p}_{i})\left(f_{j}(\mathfrak{q}_{\tau j})-f_{j}(\mathfrak{p}_{\tau j})\right),$$

$$\mathcal{B}_{\eta(sub)\mathfrak{a}f}=\left(\eta_{\varepsilon i}(\mathfrak{q}_{i})-\eta_{\varepsilon i}(\mathfrak{p}_{i})\right)\mathfrak{a}_{\varepsilon ij}^{*}(\mathfrak{p}_{i})f_{j}(\mathfrak{p}_{j}),\mathcal{B}_{\eta(sub)\mathfrak{b}f}=\left(\eta_{\varepsilon i}(\mathfrak{q}_{i})-\eta_{\varepsilon i}(\mathfrak{p}_{i})\right)\mathfrak{b}_{\varepsilon ij}^{*}(\mathfrak{p}_{i})f_{j}(\mathfrak{p}_{\tau j})$$

Theorem 1: Under hypothesis 1- 3, there exists positive constants $\mathcal{W}>\mathcal{T}\geq 0$, $k_i,\phi_i>0$ such that:

$$2\phi_i>\max_{\varepsilon\in S}\{\Pi_{\varepsilon i}\}\tag{13}$$

$$2k_i>\max_{\varepsilon\in S}\{\Xi_{\varepsilon i}\}+\mathcal{W}\tag{14}$$

$$\varepsilon>\left(\mathcal{W}+\mathcal{P}^{+}\right)(1-\mathcal{X}_{\inf})\tag{15}$$

Where: $\inf_{l\in\mathbb{Z}^{+}}\{\mathcal{X}_l\}=\mathcal{X}_{\inf}>0$,and (11) is globally exponentially stable.

Prove: set $\phi(r)=r-\mathcal{W}+\mathcal{T}e^{r\tau}$, apparently $\phi(0)<0$, $\phi(+\infty)>0$, $\dot{\phi}(r)>0$, according to the intermediate value theorem, the function $\phi(r)$ must be a unique positive number $\varepsilon>0$, satisfy $\phi(\varepsilon)=\varepsilon-\mathcal{W}+\mathcal{T}e^{\varepsilon\tau}=0$. Construct the Lyapunov function:

$$\mathfrak{V}(t)=\int_{\Omega}\sum_{i=1}^{n}\mathfrak{w}_i\overline{\mathfrak{w}_i}d\mathfrak{x}\tag{16}$$

When $t\in[t_l,t_l+\mathcal{X}_l w_l),l\in\mathbb{Z}^{+}$, The expression for $\dot{\mathfrak{V}}(t)$ is as follows :

$$\begin{aligned}\dot{\mathfrak{V}}(t)=&\int_{\Omega}\sum_{i=1}^{n}\sum_{j=1}^{n}\overline{\mathfrak{w}_i}\left(\mathcal{H}_{\eta\mathfrak{a}(sub)f}+\mathcal{A}_{\eta\mathfrak{a}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{a}f}+\mathcal{H}_{\eta\mathfrak{b}(sub)f}+\mathcal{A}_{\eta\mathfrak{b}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{b}f}\right)d\mathfrak{x}\\&+\int_{\Omega}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathfrak{w}_i\left(\overline{\mathcal{H}_{\eta\mathfrak{a}(sub)f}+\mathcal{A}_{\eta\mathfrak{a}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{a}f}+\mathcal{H}_{\eta\mathfrak{b}(sub)f}+\mathcal{A}_{\eta\mathfrak{b}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{b}f}}\right)d\mathfrak{x}\\&+\int_{\Omega}\sum_{i=1}^{n}\overline{\mathfrak{w}_i}\left(\nu_{\mathfrak{pq}}^{\eta\mu}+\theta_i\Delta\mathfrak{w}_i+\nu_{\mathfrak{pq}}^{J\eta}\right)dx+\int_{\Omega}\sum_{i=1}^{n}\mathfrak{w}_i\left(\overline{\nu_{\mathfrak{pq}}^{\eta\mu}}+\theta_i\overline{\Delta\mathfrak{w}_i}+\overline{\nu_{\mathfrak{pq}}^{J\eta}}\right)dx+\int_{\Omega}\sum_{i=1}^{n1}\left(\mathfrak{U}_i\overline{\omega_i}+\overline{\mathfrak{U}_i}\omega_i\right)d\mathfrak{x}\end{aligned}\tag{17}$$

With the help of hypothesis 1 and the basic inequalities of some complex numbers, apparently:

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\overline{\mathfrak{w}_i}\left(\mathcal{A}_{\eta\mathfrak{a}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{a}f}+\mathcal{H}_{\eta\mathfrak{b}(sub)f}+\mathcal{A}_{\eta\mathfrak{b}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{b}f}\right)+\mathfrak{w}_i\left(\overline{\mathcal{A}_{\eta\mathfrak{a}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{a}f}+\mathcal{A}_{\eta\mathfrak{b}f(sub)}+\mathcal{B}_{\eta(sub)\mathfrak{b}f}}\right)\right)\\&\leq 2\sum_{i=1}^{n}\sum_{j=1}^{n}\mathcal{N}_{\varepsilon i}^{\eta}\tilde{\mathfrak{a}}_{\varepsilon ij}\mathcal{I}_{j}^{f}\overline{\mathfrak{w}_i}\mathfrak{w}_i+2\sum_{i=1}^{n}\sum_{j=1}^{n}\mathcal{I}_{\varepsilon i}^{\eta}\tilde{\mathfrak{a}}_{\varepsilon ij}\mathcal{N}_{j}^{f}\overline{\mathfrak{w}_i}\mathfrak{w}_i+\sum_{i=1}^{n}\sum_{j=1}^{n}\left(2\mathcal{I}_{\varepsilon i}^{\eta}\tilde{\mathfrak{b}}_{\varepsilon ij}\mathcal{N}_{j}^{f}+\mathcal{N}_{\varepsilon i}^{\eta}\tilde{\mathfrak{b}}_{\varepsilon ij}\mathcal{I}_{j}^{f}\right)\overline{\mathfrak{w}_i}\mathfrak{w}_i+\sum_{i=1}^{n}\mathcal{T}\overline{\mathfrak{w}_{\tau i}}\mathfrak{w}_{\tau i}\end{aligned}\tag{18}$$

Similarly:

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$$\sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\overline{\mathfrak{w}_i}\left(\mathcal{H}_{\eta\mathfrak{a}(sub)f}+\mathcal{H}_{\eta\mathfrak{b}(sub)f}\right)+\mathfrak{w}_i\left(\overline{\mathcal{H}_{\eta\mathfrak{a}(sub)f}+\mathcal{H}_{\eta\mathfrak{b}(sub)f}}\right)\Big)\le 2\sum_{i=1}^{n}\sum_{j=1}^{n}\mathcal{N}_{\varepsilon i}^{\eta}\mathcal{N}_j^{f}\left(\left|\hat{\mathfrak{a}}_{\varepsilon ij}-\breve{\mathfrak{a}}_{\varepsilon ij}\right|+\left|\hat{\mathfrak{b}}_{\varepsilon ij}-\breve{\mathfrak{b}}_{\varepsilon ij}\right|\right)|\mathfrak{w}_i| \tag{19}$$

By the same method and considering the boundedness of external disturbances, the following inequality relations can be analyzed:

$$\sum_{i=1}^{n}\overline{\mathfrak{w}_i}\mathcal{V}_{\mathfrak{pq}}^{J\eta}+\sum_{i=1}^{n}\mathfrak{w}_i\overline{\mathcal{V}_{\mathfrak{pq}}^{J\eta}}+\sum_{i=1}^{n}\overline{\mathfrak{w}_i}\mathcal{V}_{\mathfrak{pq}}^{\eta\mu}+\sum_{i=1}^{n}\mathfrak{w}_i\overline{\mathcal{V}_{\mathfrak{pq}}^{\eta\mu}}\ \le 2\sum_{i=1}^{n}\left|\mathcal{I}_{\varepsilon i}^{\eta\mu}\right|\overline{\mathfrak{w}_i}\mathfrak{w}_i+2\sum_{i=1}^{n}\left|J_i\mathcal{I}_{\varepsilon i}^{\eta}\right|\overline{\mathfrak{w}_i}\mathfrak{w}_i \tag{20}$$

Through the boundary conditions and Green's formula and considering lemma 2, we can find out:

$$\begin{aligned}&\int_\Omega\sum_{i=1}^{n}\overline{\mathfrak{w}_i}\left(\theta_i\Delta\mathfrak{w}_i\right)d\mathfrak{x}+\int_\Omega\sum_{i=1}^{n}\mathfrak{w}_i\left(\theta_i\overline{\Delta\mathfrak{w}_i}\right)d\mathfrak{x}\le\int_\Omega\sum_{i=1}^{n}\theta_i\frac{\partial}{\partial x}\left(\overline{\mathfrak{w}_i}\frac{\partial\mathfrak{w}_i}{\partial x}\right)d\mathfrak{x}-\int_\Omega\sum_{i=1}^{n}\theta_i\frac{\partial\mathfrak{w}_i}{\partial x}\frac{\partial\overline{\mathfrak{w}_i}}{\partial x}d\mathfrak{x}\\&+\int_\Omega\sum_{i=1}^{n}\theta_i\frac{\partial}{\partial x}\left(\mathfrak{w}_i\frac{\partial\overline{\mathfrak{w}_i}}{\partial x}\right)d\mathfrak{x}-\int_\Omega\sum_{i=1}^{n}\theta_i\frac{\partial\overline{\mathfrak{w}_i}}{\partial x}\frac{\partial\mathfrak{w}_i}{\partial x}d\mathfrak{x}=-2\int_\Omega\sum_{i=1}^{n}\theta_i\left|\frac{\partial\mathfrak{w}_i}{\partial x}\right|^2d\mathfrak{x}\le-2\int_\Omega\sum_{i=1}^{n}\frac{\theta_i}{\ell_\sigma^2}\overline{\mathfrak{w}_i}\mathfrak{w}_i d\mathfrak{x}\end{aligned} \tag{21}$$

Bring the controller (12) into equation (17) and consider lemma 1 and $\sum_{i=1}^{n}|\mathfrak{w}_i|\sum_{i=1}^{n}|\mathfrak{w}_i|\ge\sum_{i=1}^{n}|\mathfrak{w}_i|^2$:

$$\int_\Omega\sum_{i=1}^{n1}\left(\mathfrak{U}_i\overline{\mathfrak{w}_i}+\overline{\mathfrak{U}_i}\mathfrak{w}_i\right)d\mathfrak{x}\le-2k_i\int_\Omega\sum_{i=1}^{n}|\mathfrak{w}_i|^2d\mathfrak{x}-2\phi_i\int_\Omega\sum_{i=1}^{n}|\mathfrak{w}_i|d\mathfrak{x} \tag{22}$$

Considering equations (17) through (22), we get the following formula:

$$\dot{\mathfrak{V}}(t)\le\mathcal{T}\int_\Omega\sum_{i=1}^{n}\overline{\mathfrak{w}_{\tau i}}\mathfrak{w}_{\tau i}d\mathfrak{x}++\int_\Omega\sum_{i=1}^{n}\left(\Xi_{\varepsilon i}+\mathcal{W}-2k_i\right)\overline{\mathfrak{w}_i}\mathfrak{w}_i d\mathfrak{x}-\mathcal{W}\int_\Omega\sum_{i=1}^{n}\overline{\mathfrak{w}_i}\mathfrak{w}_i d\mathfrak{x}+\int_\Omega\sum_{i=1}^{n}\left(\Pi_{\varepsilon ij}-2\phi_i\right)|\mathfrak{w}_i|d\mathfrak{x} \tag{23}$$

According to conditions (13), (14), obviously:

$$\dot{\mathfrak{V}}(t)\le-\mathcal{W}\mathfrak{V}(t)+\mathcal{T}\sup_{t-\tau(t)\le s\le t}\mathfrak{V}(s)\quad,\quad t\in\left[t_l,t_l+\mathcal{X}_l w_l\right),l\in\mathbb{Z}^+ \tag{24}$$

When $t\in\left[t_l+\mathcal{X}_l w_l,t_{l+1}\right),l\in\mathbb{Z}^+$, $\exists\mathcal{P}^+=\max\{0,\mathcal{P}\}$ makes the following formula true:

$$\dot{\mathfrak{V}}(t)\le\mathcal{T}\int_\Omega\sum_{i=1}^{n}\overline{\mathfrak{w}_{\tau i}}\mathfrak{w}_{\tau i}d\mathfrak{x}+\int_\Omega\sum_{i=1}^{n}\Xi_{\varepsilon i}\overline{\mathfrak{w}_i}\mathfrak{w}_i d\mathfrak{x}+\int_\Omega\sum_{i=1}^{n}\left(\Pi_{\varepsilon ij}-2\phi_i\right)|\mathfrak{w}_i|d\mathfrak{x}\le\mathcal{P}^+\mathfrak{V}(t)+\mathcal{T}\sup_{t-\tau(t)\le s\le t}\mathfrak{V}(s) \tag{25}$$

Hence:

$$\dot{\mathfrak{V}}(t)\le\begin{cases}-\mathcal{W}\mathfrak{V}(t)+\mathcal{T}\sup\limits_{t-\tau(t)\le s\le t}\mathfrak{V}(s),\ t\in\left[t_l,t_l+\mathcal{X}_l w_l\right),l\in\mathbb{Z}^+\\\mathcal{P}_1^+\mathfrak{V}(t)+\mathcal{T}\sup\limits_{t-\tau(t)\le s\le t}\mathfrak{V}(s),\ \ t\in\left[t_l+\mathcal{X}_l w_l,t_{l+1}\right),l\in\mathbb{Z}^+\end{cases} \tag{26}$$

Next, it will be proved that the error system (11) can achieve global exponential synchronization, and the necessary definition is given first: $\tilde{N}_0 = \sup_{-\tau\le s\le 0}\mathfrak{V}(s), L(t)=e^{\varepsilon t}\mathfrak{V}(t), G(t)=L(t)-c\tilde{N}_0, c\to 1$, obviously: $G(t)<0, t\in[-\tau,t_1], t_1=0$, suppose $G(t)<0$, $t\in[t_1,t_1+\mathcal{X}_1 w_1)$, if this condition is not met, then:

$$\begin{cases} G(t^*)=0, \dot{G}(t^*)=\dfrac{dG(t^*)}{dt}\ge 0 \\ G(t)<0, t\in[-\tau,t^*) \\ \exists t^*\in[t_1,t_1+\mathcal{X}_1 w_1) \end{cases} \Rightarrow \begin{cases} L(t)<c\tilde{N}_0 \Rightarrow \mathfrak{V}(t)<c\tilde{N}_0 e^{-\varepsilon t} \Rightarrow \sup\limits_{t^*-\tau\le s\le t^*}\mathfrak{V}(s)<c\tilde{N}_0 e^{-\varepsilon t^*}<e^{\varepsilon\tau}c\tilde{N}_0 e^{-\varepsilon t^*} \\ \Rightarrow \begin{cases} e^{\varepsilon t^*}\sup\limits_{t^*-\tau\le s\le t^*}\mathfrak{V}(s)<e^{\varepsilon\tau}c\tilde{N}_0 \\ L(t^*)=c\tilde{N}_0=c\sup\limits_{-\tau\le s\le 0}\mathfrak{V}(s) \end{cases} \Rightarrow \mathcal{T}e^{\varepsilon t^*}\sup\limits_{t^*-\tau\le s\le t^*}\mathfrak{V}(s)<\mathcal{T}e^{\varepsilon\tau}L(t^*) \end{cases} \tag{27}$$

Tkae condition (26) into $\dot{G}(t^*)$ and considering condition (27): (note: $\phi(\varepsilon)=\varepsilon-\mathcal{W}+\mathcal{T}e^{\varepsilon\tau}=0$).

$$\begin{cases} \dot{G}(t^*)=\varepsilon L(t^*)+e^{\varepsilon t^*}\dot{\mathfrak{V}}(t^*)\le \varepsilon L(t^*)+e^{\varepsilon t^*}\left(-\mathcal{W}\mathfrak{V}(t^*)+\mathcal{T}\sup\limits_{t^*-\tau(t)\le s\le t^*}\mathfrak{V}(s)\right) \\ <(\varepsilon-\mathcal{W})L(t^*)+\mathcal{T}e^{\varepsilon\tau}L(t^*)=0\Rightarrow \dot{G}(t^*)<0 \end{cases} \tag{28}$$

Hence: $G(t)<0, t\in[-\tau,t_1+\mathcal{X}_1 w_1)\Rightarrow L(t)<c\tilde{N}_0$, $t\in[-\tau,t_1+\mathcal{X}_1 w_1)$.

Similarly, defined expression: $\mu=\mathcal{W}+\mathcal{P}_1^+, L(t)=e^{\varepsilon t}\mathfrak{V}(t), \hat{\Psi}(t)=L(t)-c\tilde{N}_0 e^{\mu(t-t_1-\mathcal{X}_1 w_1)}<0$ $t\in[t_1+\mathcal{X}_1 w_1,t_2)$,

If the above conditions are not true, then: $\begin{cases} \hat{\Psi}(t^{**})=0, \dot{\hat{\Psi}}(t^{**})\ge 0, \ \exists t^{**}\in[t_1+\mathcal{X}_1 w_1,t_2) \\ \hat{\Psi}(t)<0, t\in[t_1+\mathcal{X}_1 w_1,t^{**}) \end{cases}$.

On account of $\tau>0$, it is divided into the following two situations to discuss:

Case 1: $t^{**}-\tau\in[t_1+\mathcal{X}_1 w_1,t^{**}), e^{\varepsilon t^{**}}\sup\limits_{t^{**}-\tau\le s\le t^{**}}V(s)<e^{\varepsilon\tau}c\tilde{N}_0 e^{\mu(t^{**}-\tau-t_1-\mathcal{X}_1 w_1)}<e^{\varepsilon\tau}c\tilde{N}_0 e^{\mu(t^{**}-t_1-\mathcal{X}_1 w_1)}=e^{\varepsilon\tau}L(t^{**})$.

Case 2: $t^{**}-\tau\in[-\tau,t_1+\mathcal{X}_1 w_1), e^{\varepsilon t^{**}}\sup\limits_{t^{**}-\tau\le s\le t^{**}}V(s)<e^{\varepsilon\tau}c\tilde{N}_0\le e^{\varepsilon\tau}L(t^{**})$.

Consider the above: $\dot{\hat{\Psi}}(t^{**})=\varepsilon e^{\varepsilon t^{**}}V(t^{**})+e^{\varepsilon t^{**}}\dot{V}(t^{**})-\mu c\tilde{N}_0 e^{\mu(t-t_1-\mathcal{X}_1 w_1)}<(\varepsilon-\mathcal{W}+\mathcal{T}e^{\varepsilon\tau})L(t^{**})=0$.

That proves: $L(t)<c\tilde{N}_0 e^{\mu(t_2-t_1-\mathcal{X}_1 w_1)}=c\tilde{N}_0 e^{\mu(1-\mathcal{X}_1)w_1}$, $t\in[-\tau,t_2)$ hold.

In conclusion, the following conclusions can be summarized: (note: $t_1=0$).

$$\begin{aligned} &L(t)<c\tilde{N}_0, \ t\in[t_1,t_1+\mathcal{X}_1 w_1)\Rightarrow L(t)<c\tilde{N}_0 e^{\mu(t-t_1-\mathcal{X}_1 w_1)}=c\tilde{N}_0 e^{\mu(t-\mathcal{X}_1 w_1)} \ t\in[t_1+\mathcal{X}_1 w_1,t_2)\Rightarrow \\ &L(t)<c\tilde{N}_0 e^{\mu(1-\mathcal{X}_1)w_1}, \ t\in[t_2,t_2+\mathcal{X}_2 w_2)\Rightarrow L(t)<c\tilde{N}_0 e^{\mu(1-\mathcal{X}_1)w_1} * e^{\mu(t-t_2-\mathcal{X}_2 w_2)}=c\tilde{N}_0 e^{\mu(t-\mathcal{X}_1 w_1-\mathcal{X}_2 w_2)}, \ t\in[t_2+\mathcal{X}_2 w_2,t_3) \end{aligned} \tag{29}$$

Set $\forall m\in\mathbb{Z}^+, w_0=0, \mathcal{X}_0=0$, The following formula can be summarized by mathematical induction:

$$L(t)<\begin{cases} c\tilde{N}_0\exp\left\{\mu\left(\sum\limits_{l=0}^{m-1}(1-\mathcal{X}_l)w_l\right)\right\}\le c\tilde{N}_0\exp\left\{\mu(1-\mathcal{X}_{\inf})\sum\limits_{l=0}^{m-1}w_l\right\}, t\in[t_m,t_m+\mathcal{X}_m w_m) \\ c\tilde{N}_0\exp\left\{\mu\left(t-\sum\limits_{l=0}^{m}\mathcal{X}_l w_l\right)\right\}\le c\tilde{N}_0\exp\left\{\mu\left(t-\mathcal{X}_{\inf}\sum\limits_{l=0}^{m}w_l\right)\right\}, t\in[t_m+\mathcal{X}_m w_m,t_{m+1}) \end{cases} \Rightarrow \mathfrak{V}(t)<c\tilde{N}_0 e^{\left((\mathcal{W}+\mathcal{P}_1^+)(1-\mathcal{X}_{\inf})-\varepsilon\right)t}$$

Considering definition 1, (14) is globally exponentially stable.

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4. Simulation Verification

Before giving a simulation example, to briefly describe the selected system parameters, define the following formula: $\mathfrak{p}_i^R(x,t)=real(\mathfrak{p}_i),\mathfrak{p}_i^I(x,t)=imag(\mathfrak{p}_i)$ $\varphi_{\mathfrak{p}R}=2+\tanh(\mathfrak{p}_i^R),\varphi_{\mathfrak{p}I}=2+\tanh(\mathfrak{p}_i^I)$.

Consider 3-dimensional Markovian reaction-diffusion CGNNs with parameter switching (1) is used to verify the rationality of the developed theorem 1. Accordingly the transfer rate matrix of Markovian chain as: $\begin{bmatrix} -2.3 & 2.3 \\ 2.5 & -2.5 \end{bmatrix}$, select and calculate the following parameters:

$$\begin{cases} \eta_{11}(\mathfrak{p}_i)=2.11/\varphi_{\mathfrak{p}R}+2.02/\varphi_{\mathfrak{p}I}*i,\ \eta_{21}(\mathfrak{p}_i)=2.03/\varphi_{\mathfrak{p}R}+2.13/\varphi_{\mathfrak{p}I}*i,\ \eta_{12}(\mathfrak{p}_i)=2.53/\varphi_{\mathfrak{p}R}+2.61/\varphi_{\mathfrak{p}I}*i \\ \eta_{22}(\mathfrak{p}_i)=2.66/\varphi_{\mathfrak{p}R}+2.57/\varphi_{\mathfrak{p}I}*i,\ \eta_{13}(\mathfrak{p}_i)=2.03/\varphi_{\mathfrak{p}R}+2.09/\varphi_{\mathfrak{p}I}*i,\ \eta_{23}(\mathfrak{p}_i)=2.47/\varphi_{\mathfrak{p}R}+2.59/\varphi_{\mathfrak{p}I}*i \\ \mu_{11}(\mathfrak{p}_i)=1.01*\mathfrak{p}_i,\mu_{21}(\mathfrak{p}_i)=0.99*\mathfrak{p}_i,\mu_{12}(\mathfrak{p}_i)=0.98*\mathfrak{p}_i,\mu_{22}(\mathfrak{p}_i)=0.99*\mathfrak{p}_i,\mu_{13}(\mathfrak{p}_i)=1.26*\mathfrak{p}_i, \\ \mu_{23}(\mathfrak{p}_i)=0.91*\mathfrak{p}_i,\ f_j(\mathfrak{p}_j)=1.1*\tanh(\mathfrak{p}_i^R)+2.9*\tanh(\mathfrak{p}_i^I)*i \end{cases}$$

$$\Rightarrow \mathcal{N}_{11}^{\eta}=2.984,\mathcal{N}_{21}^{\eta}=3.012,\mathcal{N}_{12}^{\eta}=3.691,\mathcal{N}_{22}^{\eta}=3.762,\mathcal{N}_{13}^{\eta}=2.956,\mathcal{N}_{23}^{\eta}=3.663,\mathcal{I}_{11}^{\eta}=2.11,\mathcal{I}_{21}^{\eta}=2.13,$$
$$\mathcal{I}_{12}^{\eta}=2.61,\mathcal{I}_{22}^{\eta}=2.66,\mathcal{I}_{13}^{\eta}=2.09,\mathcal{I}_{23}^{\eta}=2.59,\mathcal{I}_{11}^{\eta\mu}=6.3130,\mathcal{I}_{21}^{\eta\mu}=6.2495,\mathcal{I}_{12}^{\eta\mu}=7.6149,\mathcal{I}_{22}^{\eta\mu}=7.8215,$$
$$\mathcal{I}_{13}^{\eta\mu}=7.8442,\mathcal{I}_{23}^{\eta\mu}=6.9862,\mathcal{N}_{j}^{f}=4.101,\mathcal{I}_{j}^{f}=2.9;$$

Discrete time delay, external interference, and reaction-diffusion coefficient are shown below:

$$\tau_i(t)=0.9+0.2*\sin(t)\Rightarrow\tau=1.1,\theta_i=0.01,[-1,1]\Rightarrow\ell=1,J_1=-0.36+1.98i,J_2=0.51-1.99i,J_3=-0.68-2.16i.$$

In this paper, ε,a are used to represent the Markovian process and the connection weight switching respectively. Define $\Phi_{\varepsilon a}(\Phi=A,B;\varepsilon=1,2;a=1,2)$, $\Im=8.5$ and specific values are as follows:

$$A_{11}=\begin{bmatrix} 0.78+0.65i & -0.32+0.51i & -0.53+0.61i \\ -0.66-0.62i & 1.05+1.83i & -0.84+1.43i \\ 0.53+0.62i & -0.77+0.86i & 0.71-0.62i \end{bmatrix} A_{12}=\begin{bmatrix} 0.82+0.67i & -0.52+0.61i & -0.35+0.21i \\ -0.76-0.63i & 0.99+0.97i & -0.65+0.61i \\ 0.65+0.64i & -0.68+0.93i & 0.61-0.78i \end{bmatrix}$$

$$A_{21}=\begin{bmatrix} 0.75+0.87i & -0.73+0.77i & -0.44+0.32i \\ -0.75-0.81i & 0.95+1.64i & -0.82+1.05i \\ 0.88+0.44i & -0.71+0.88i & 0.54-0.74i \end{bmatrix} A_{22}=\begin{bmatrix} 0.86+0.76i & -0.81+0.87i & -0.27+0.65i \\ -0.85+0.77i & 0.81+1.41i & -0.83+0.77i \\ 0.62+0.73i & -0.31+1.31i & 0.52-0.51i \end{bmatrix}$$

$$B_{11}=\begin{bmatrix} 0.71+0.64i & 1.35+1.23i & 0.28-0.35i \\ -0.69+0.83i & -1.72+1.64i & 1.72-1.64i \\ 1.54-1.41i & 2.43+2.01i & 1.64+1.33i \end{bmatrix} B_{12}=\begin{bmatrix} 0.73+0.84i & 1.28+1.51i & 0.21-0.44i \\ -0.88+0.91i & -1.79+1.78i & 1.93-1.87i \\ 1.66-1.52i & 2.23+2.27i & 1.45+1.12i \end{bmatrix}$$

$$B_{21}=\begin{bmatrix} 0.82+0.66i & 1.02+1.43i & 0.37-0.54i \\ -1.68+0.81i & -1.88+1.76i & 1.86-1.75i \\ 1.13-1.32i & 2.43+2.24i & 1.57+1.54i \end{bmatrix} B_{22}=\begin{bmatrix} 0.57+0.46i & 1.41+1.13i & 0.24-0.64i \\ -1.82+0.94i & -1.87+1.78i & 0.69-1.34i \\ 1.05-1.44i & 2.51+2.35i & 1.46+1.34i \end{bmatrix}$$

In order to show clearly the switching function of the connection coefficient of the model in this paper, the four possibilities of switching correspond to the numbers 1,2,3, and 4 respectively, and then the spatial position 0 is analyzed, and the partial coefficient switching can be obtained as shown in Figure 1:

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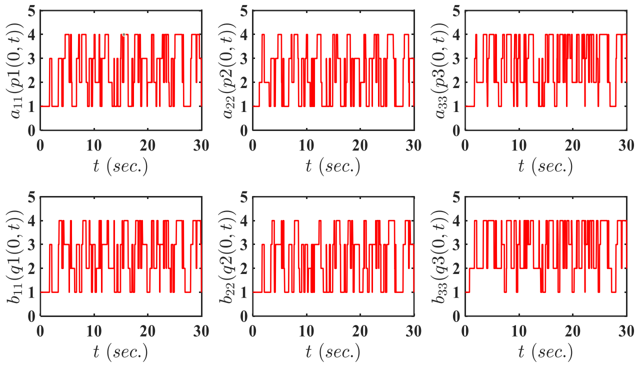

Figure 1. Partial connection weight switch

Figure 1 shows that there are frequent switching of system modes within 30 seconds, which is consistent with the actual modeling. Under the above parameter conditions, the real and imaginary parts trajectories and Markovian switching process as follow:

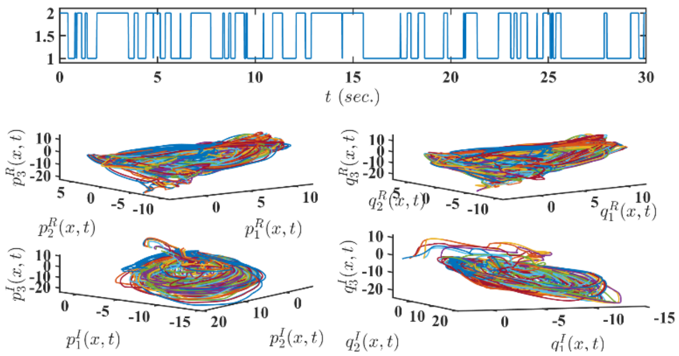

Figure 2. The Markovian process and chaotic attractors

Considering the response system (6), the error open-loop trajectories as shown in Figure 3:

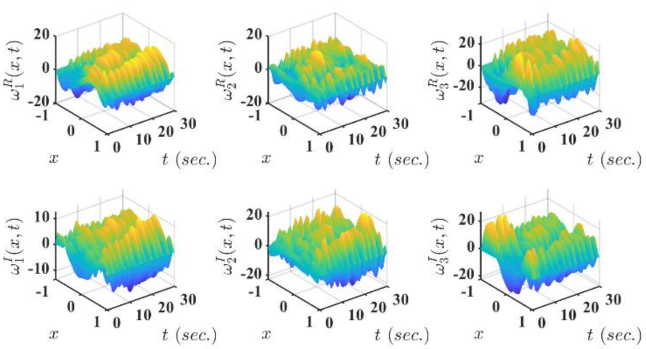

Figure 3. The Open-loop error trajectories

It can be obtained by calculating the parameter conditions in theorem 1:

$$\Pi_{\varepsilon i}=\begin{bmatrix}7.5 & 30.8\\ 43.8 & 22.3\end{bmatrix}\Rightarrow\begin{cases}\phi_1=22\\ \phi_2=16\end{cases}=,\mathcal{T}_{\varepsilon i}=\begin{bmatrix}4.1 & 8.1\\ 4.3 & 8.5\end{bmatrix}\Rightarrow \mathcal{T}=8.5,\Xi_{\varepsilon i}=\begin{bmatrix}30.9 & 40.8\\ 37.4 & 60.8\end{bmatrix}\Rightarrow\begin{cases}\Xi_{\varepsilon i}=\begin{bmatrix}38 & 61\end{bmatrix}\\ \mathcal{P}^{+}=62\end{cases}\Rightarrow\begin{cases}k_1=49\\ k_2=60.5\end{cases}$$

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The control time corresponding to the intermittent controller designed in this paper is not arbitrarily given,.However, it is decided by equation (15) and $\phi(\varepsilon)=\varepsilon-\mathcal{W}+\mathcal{T}e^{\varepsilon\tau}=0$ together. According to Figure 4 $\mathcal{W}=60,\mathcal{X}_{\inf}\approx 0.9857$. Finally, the trajectory evolution of (11) is as follows:

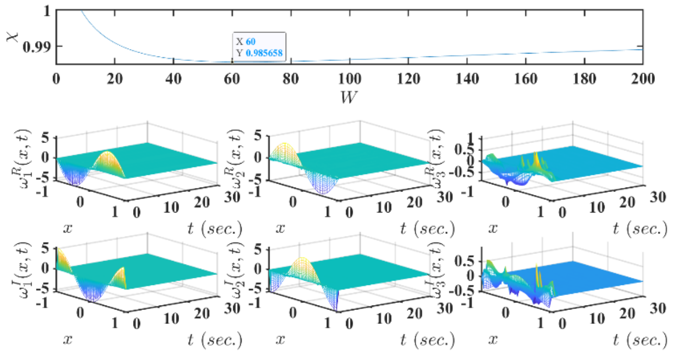

Figure 4. The Closed-loop error trajectory

It can be seen from the figure above that the design of the aperiential intermittent pinning controller (12) is effective, and the error system (11) can be stabilized in a short time by shortening a part of the controller time and selecting a part of the controlled nodes.

5. Conclusion

In this work, the Markovian reaction-diffusion complex value CGNNs with parameter switching is proposed, the drive response system is studied directly by the method of non-separation, and the aperiodic intermittent pinning controller is designed to ensure the stability of the system. Then a suitable Lyapunov function is selected, exponential synchronization condition of the system is obtained, and the rationality of the theorem is demonstrated by numerical calculation.

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