

# Approach 4D Trajectory Planning based on the Aircraft Kinematic Model

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## Abstract

Trajectory based operation is an important part of future air traffic management research field, which needs Negotiation and Validation between Flight Management System and ground based 4D trajectory planning. Thus, ground based 4D trajectory planning is the foundation to implement trajectory based operation. And a combination of aircraft kinematics model and meteorological model based 4D trajectory planning method was presented. Firstly, historical radar data was analyzed by big data processing method upon which a horizontal path could be designed and altitude, speed ranges could be determined. Secondly, based on the selected horizontal path a 4D trajectory was planned with the application of combination of aircraft kinematics model and meteorological model based 4D trajectory planning method. Finally, taking arrival flights to Shanghai Pudong Airport from PINOT entry fix as examples, a simulation was conducted. And the simulation result indicates that the proposed 4D trajectory planning method can generate reasonable 4D trajectory, provide reference trajectory for airborne flight management system (FMS), accelerate the implementation of trajectory based operation, reduce flight delays, and improve the operational efficiency in terminal airspace.

## Keywords

Intelligent Transportation; Air Traffic Management; 4D Trajectory Planning; Trajectory based Operation.

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## 1. Introduction

The global air traffic flow doubles every 10 to 14 years, and the International Civil Aviation Organization (ICAO) predicts that the world's air traffic flow will grow at a rate of 5% per year until 2020. The rapid growth of traffic directly affects flight efficiency and imposes a huge burden on air traffic controllers. In order to cope with the continuous and rapid growth of traffic flow, the air traffic management system urgently needs to be upgraded, so the concept of aircraft trajectory-based operation mode (TBO) is proposed by NextGen. The implementation of trajectory-based operation mode (TBO) requires interactive collaboration between onboard flight management system (FMS) and ground 4D trajectory planning, so ground 4D trajectory planning is the basis for implementing trajectory-based operation mode (TBO). 4D trajectory planning needs to consider various factors such as aircraft kinematics model, performance model, meteorological model, flight plan, flyable horizontal path, planned arrival time (STA), etc. to plan the optimal flight trajectory.

Research on 4D trajectory planning has yielded a lot of results in the field of unmanned aerial vehicles [1], [2]. The research on trajectory planning in the civil aviation field was first proposed by NASA, the National Aeronautics and Space Administration of the United States, with an algorithm for computer-oriented terminal area guidance [3], [4]. In 1980, C. E. Knox et al. proposed a time-based fuel-saving 4D trajectory management algorithm [5]. In 1985, A. Chakracarty et al. proposed a four-dimensional guidance strategy for optimal fuel consumption in the presence of wind [6]. In 1998,

Claire et al. regarded aircraft models and maneuvering models as a hybrid control system to achieve conflict resolution through speed and heading adjustments [7]. In China, research on basic algorithms for generating altitude profiles and velocity profiles in terminal area 4D navigation started relatively late. Wang Dahai, Su Bin, Yang Jun et al. studied the generation of 3D flight profiles and velocity profiles in the terminal area by combining horizontal trajectory calculation methods[8], [9], and then generated 4D navigation commands. Using the generated 4D navigation commands, aircraft flying along straight or curved paths can arrive at specified locations within a specified time in the terminal area, thus achieving 4D guidance in the terminal area. These methods use only one parameter to guide the 4D trajectory without considering the impact of wind on the aircraft. Although the models are simple and efficient, the accuracy of 4D trajectory planning is low, so they do not meet the requirements of NextGen, the next generation air traffic management system. Subramanian et al. proposed a new onboard flight management system (FMS) based on aircraft kinematics model for real-time 4D trajectory operation[10], but it needs to interact and collaborate with ground 4D trajectory planning to determine an optimal 4D trajectory and guide the aircraft to run along the planned trajectory. In order to better match the ground-planned 4D trajectory with the 4D trajectory planned by the flight management system, it is convenient to implement trajectory-based operation mode.

This paper proposes a 4D trajectory planning method based on the combination of aircraft kinematics model and meteorological model. Firstly, historical radar tracks are analyzed by using dynamic time warping algorithm (DTW) [11] and hierarchical clustering method based on ward minimum variance criterion [12]. Combined with standard approach routes, multiple horizontal approach paths are designed. Secondly, statistical analysis is conducted on the historical radar data of incoming aircraft to determine the altitude and speed range of each waypoint. Thirdly, the 4D trajectory planning method combining aircraft kinematics model and meteorological model is used to complete the 4D trajectory planning. Finally, the proposed method is simulated and verified by selecting incoming aircraft at PINOT point of Pudong International Airport.

## 2. Speed Profile Planning

During the aircraft approach process, the Approach Management System (AMAN) optimizes and sorts the aircraft based on trajectory prediction, and then assigns a planned arrival time (STA) to each aircraft. In order to enable the aircraft to arrive at the specified waypoints at the given planned arrival time, it is necessary to plan a 4D trajectory that meets the planned arrival time. The speed profile directly determines the arrival time of the aircraft, so it is necessary to plan the speed profile to meet the planned arrival time. The determination of the speed profile requires interaction and collaboration between the trajectory generator and the speed adjustment parameter generator.

According to the planned arrival time, an appropriate horizontal path is selected from the horizontal path library. Then, the trajectory generator generates a nominal speed profile based on the selected horizontal path. Based on the nominal speed profile, an estimated arrival time (ETA) can be obtained. If the difference between the estimated arrival time and the planned arrival time exceeds a threshold, the speed adjustment parameter generator is triggered. Based on the above time difference (ETA-STA), the speed adjustment parameter generator generates corresponding adjustment parameters, which are used to adjust the speed profile. This process is repeated until the difference between the estimated arrival time and the planned arrival time is less than a certain threshold, and then the loop stops, resulting in the desired speed profile.

When the horizontal path of the aircraft is determined, the trajectory generator generates a nominal speed profile based on historical radar data, which corresponds to the estimated arrival time (ETA). To ensure that the aircraft arrives at the specified waypoint at the given planned arrival time (STA), we assume that the speed adjustment parameter SAP can be used to adjust the nominal speed profile to obtain the required speed profile.

$$ETA(SAP + SAPChange) \cong ETA(SAP) + \frac{\partial ETA}{\partial SAP} SAPChange = STA \quad (1)$$

$$SAPChange = \frac{-TimeError}{\frac{\partial ETA}{\partial SAP}}, TimeError = ETA - STA \quad (2)$$

According to the ground speed, divide the route into several segments, then the estimated arrival time:

$$ETA = \sum_{WPTS} \frac{LegDist}{GndSpd} + Current\_GMT \quad (3)$$

$$\frac{\partial ETA}{\partial SAP} = \sum_{WPTS} \frac{-LegTime}{GndSpd} \frac{\partial GndSpd}{\partial SAP} \quad (4)$$

$$SAP = 1 - \frac{GS_{nom}}{GndSpd} \quad (5)$$

$$SAP_{min} = 1 - \frac{GS_{nom}}{GndSpd_{min}} \quad (6)$$

$$SAP_{max} = 1 - \frac{GS_{nom}}{GndSpd_{max}}$$

$$\frac{\partial ETA}{\partial SAP} = \sum_{WPTS} \frac{-LegDist}{GS_{nom}} = \sum_{WPTS} -LegTime \quad (7)$$

$$SAPChange = \frac{TimeError}{\sum_{WPTS} LegTime} \quad (8)$$

$SAPChange$  is the variation of speed adjustment parameter,  $\frac{\partial ETA}{\partial SAP}$  is the impact of SAP variation on ETA,  $LegDist$  is the distance of leg,  $GndSpd$  is the ground speed,  $LegTime$  is the flight time of leg,  $Current\_GMT$  is the current time,  $GS_{nom}$  is the nominal ground speed. The solution of SAP is not a one-time process, but a cyclic iteration until the difference between the time required to generate the speed profile and the planned arrival time meets a certain threshold (3s), and then the cycle will terminate and generate the final required speed profile.

### 3. Trajectory Generator

In order to achieve fast solution of 4D trajectory planning, in the design process of trajectory generator, this paper uses aircraft kinematics model to model the aircraft. The kinematics model does not consider the elements such as thrust, lift and drag acting on the aircraft, so the control method of the kinematics model of the aircraft's particle is relatively simple, which can accelerate the solution process of 4D trajectory.

### 3.1 Aircraft Kinematics Model

The dynamic model of the aircraft is simplified into a three-degree-of-freedom particle model, and the motion equations of the three degrees of freedom describe its state and control its movement in longitude, latitude and vertical direction:

$$\frac{d\varphi}{dt} = \frac{V \cos \gamma \sin \chi + W_\varphi}{R_M + h} \quad (9)$$

$$\frac{d\lambda}{dt} = \frac{V \cos \gamma \cos \chi + W_\lambda}{\cos \varphi (R_T + h)} \quad (10)$$

$$\frac{dh}{dt} = V \sin \gamma + W_h \quad (11)$$

$$\frac{dV}{dt} = \frac{T(P, V, h) - D(L, V, h)}{m} - g \sin \gamma \quad (12)$$

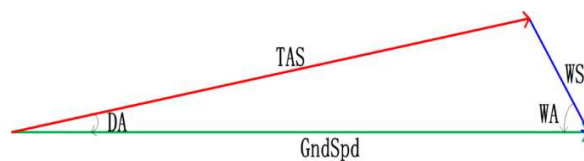
$$\frac{d\gamma}{dt} = \frac{g(n \cos \phi - \cos \gamma)}{V} \quad (13)$$

$$\frac{d\chi}{dt} = \left(\frac{g}{V} \times n\right) \left(\frac{\sin \phi}{\cos \gamma}\right) \quad (14)$$

$\varphi$  is the latitude,  $V$  is the vacuum speed,  $\gamma$  is the track angle,  $\chi$  is the heading,  $W_\varphi$  is the wind component in the latitude direction,  $R_M$  is the meridional radius of curvature,  $h$  is the flight altitude,  $\lambda$  is the longitude,  $W_\lambda$  is the wind component in the longitude direction,  $R_T$  is the transverse radius of curvature,  $W_h$  is the wind component in the vertical direction,  $T$  is the thrust,  $P$  is the power setting of the engine,  $D$  is the drag,  $L$  is the lift,  $g$  is the acceleration due to gravity,  $m$  is the mass,  $n$  is the load factor,  $\phi$  is the bank angle.

### 3.2 Meteorological Model

In the actual operation process, meteorological factors have a great impact on the movement of aircraft, especially the wind has a particularly obvious impact on the aircraft. The relationship between the ground speed, vacuum speed and wind speed of the aircraft is shown in Figure 1, TAS is the vacuum speed, GndSpd is the ground speed, WS is the wind speed, DA is the yaw angle, and WA is the wind angle.



**Figure 1.** Diagram of the relationship between ground speed, vacuum speed, and wind speed

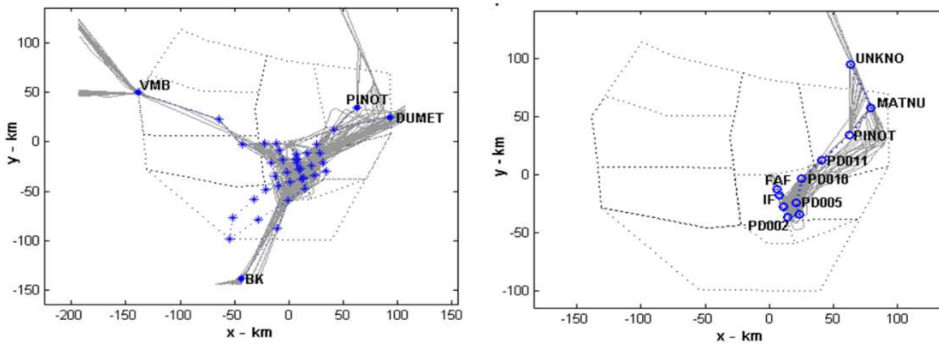
To improve the accuracy of aircraft 4D trajectory planning, historical meteorological information is obtained from the European Centre For Medium Range Weather Forecasts in this paper. The meteorological information is stored in files in GRIB data type, which is an effective tool for large-

scale gridded data. The global positioning of meteorological information is determined based on grid points, and the location of grid points is uniquely represented by longitude and latitude.

### 4. Case Study Analysis

The 4D trajectory planning proposed in this paper follows the following steps: Firstly, historical radar trajectory data is clustered and analyzed using dynamic time warping and hierarchical clustering. Based on the clustering results and standard instrument approach routes, multiple horizontal paths are planned. Secondly, the altitude and velocity ranges of each waypoint are statistically analyzed based on historical radar data. According to the designed horizontal path and the calculated altitude range, a corresponding altitude profile is designed. Finally, according to the planned arrival time given by AMAN, a reasonable horizontal path is selected, and the 4D trajectory planning method combining the aircraft kinematics model and meteorological model is used to complete the 4D trajectory planning.

#### 4.1 Approach Horizontal Path Planning



**Figure 2.** Schematic diagram of Shanghai terminal area and PINOT entry point entry radar trajectory and standard entry route

According to the prevailing traffic flow of aircraft approaching from the PINOT approach point and the standard approach route, 6 horizontal approach paths are planned, as shown in Table 1.

**Table 1.** Horizontal path of aircraft entering from PINOT port

Path1	Path2	Path3	Path4	Path5	Path6
UNKNO	UNKNO	UNKNO	UNKNO	UNKNO	UNKNO
PINOT	MATNU	PINOT	MATNU	MATNU	MATNU
PD011	PINOT	PD011	PINOT	PINOT	PINOT
PD010	PD011	PD010	PD011	PD011	PD011
IF	PD010	PD005	PD010	PD010	PD010
FAF	IF	PD001	PD005	PD005	PD005
	FAF	IF	PD001	PD004	PD004
		FAF	IF	PD002	PDA
			FAF	PD001	PDB
				IF	PD002
				FAF	PD001
					IF
					FAF

Select the PINOT approach point of Pudong International Airport as the simulation scenario (the approach trajectory, sector division and standard approach route of Shanghai terminal area are shown in Figure 2) for trajectory planning. Use dynamic time warping (DTW) algorithm and hierarchical clustering method based on ward minimum deviation square sum to perform cluster analysis on the historical radar trajectories of aircraft approaching from the PINOT approach point, and then reproduce the prevailing traffic flow from the PINOT approach point.

#### 4.2 Planning of Altitude Profile for Aircraft Approaching from PINOT Approach Point

Utilize big data processing method to analyze the historical radar data of aircraft approaching from PINOT approach point during January 2-8, 2013, and determine the altitude and velocity range of each waypoint.

The altitude of the aircraft at each waypoint obtained by statistical analysis of historical radar data is the actual altitude flown by the aircraft, so it can be considered that the calculated altitude and velocity range are reasonable and meet the performance requirements of the aircraft. Since flying at a higher altitude reduces fuel consumption and noise pollution on the ground, this paper follows the principle of maintaining a higher altitude flight while ensuring the safety and conflict-free of the aircraft in the planning of altitude profile, and the altitude profile is within the calculated altitude range. Taking PINOT standard approach route as an example, the altitude profile is planned, as shown in Figure 3.

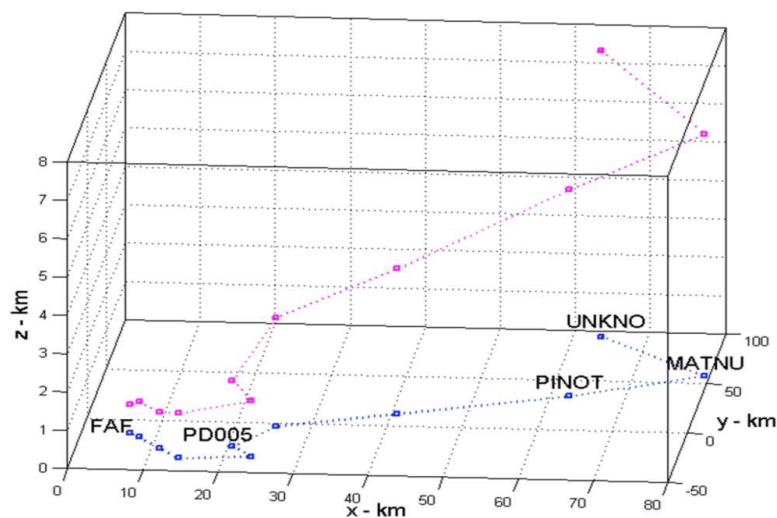


Figure 3. PINOT standard approach route altitude profile design diagram

#### 4.3 4D Trajectory Planning for Aircraft Approaching from PINOT Approach Point

According to the designed 6 horizontal approach paths, the total flight distance and corresponding nominal speed profile time for each horizontal approach path can be calculated, as shown in Table 2.

Table 2. The total range and nominal speed profile time corresponding to different horizontal entry paths at PINOT port entry points

Horizontal path	Total flight distance(km)	Time-consuming(s)
1	140	946
2	149	994
3	160	1112
4	169	1152
5	189	1352
6	202	1478

For any planned arrival time (STA) given by AMAN, select the horizontal path with the closest nominal speed profile to the required time (assumed STA is 1410s). The trajectory generator generates a nominal speed profile based on the selected horizontal path (horizontal path 5) and obtains its corresponding estimated time of arrival (ETA). Then, calculate the difference between ETA and STA (Time Error = 58s). If the Time Error exceeds the threshold (3s), activate the speed adjustment parameter generator to adjust the speed profile until the Time Error is less than the threshold (3s). The loop stops and the required 4D trajectory is generated, as shown in Figure 4.

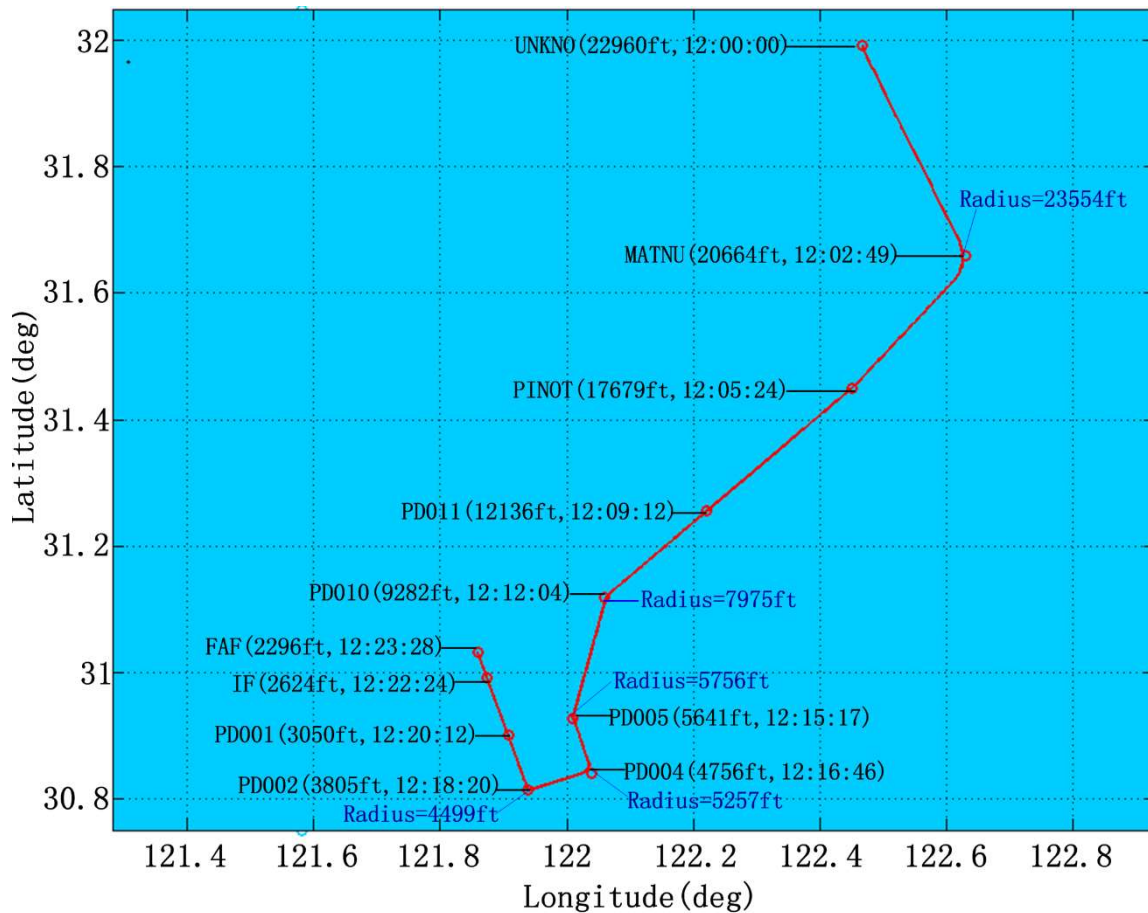


Figure 4. 4D trajectory planning diagram

As can be seen from the above, for any planned arrival time (STA) within a reasonable range given by AMAN, the trajectory generator can plan a speed profile and altitude profile corresponding to it, that is, complete the 4D trajectory planning that meets the planned arrival time (STA). In the process of planning the speed profile, the smaller the threshold set for Time Error, the slower the convergence speed. When the threshold is set to 3s, the convergence speed is relatively fast. When it is less than 3s, the convergence speed will slow down significantly. Therefore, in this case, the threshold is set to 3s, which not only ensures computational efficiency, but also ensures the accuracy of 4D trajectory planning.

## 5. Conclusion

The 4D trajectory planning method based on the kinematic model of aircraft proposed in this paper, during the planning of the horizontal approach path and the determination of the altitude and speed range at each waypoint, uses big data processing methods to analyze historical radar data. This ensures the rationality of the planned horizontal path and the determined altitude and speed range. The application of the 4D trajectory planning method that combines the kinematic model of the

aircraft with the meteorological model to plan the 4D trajectory is consistent with the 4D trajectory planning method of the onboard flight management system, which can greatly improve the fit between the planned 4D trajectory of the onboard and ground systems, facilitate interaction and collaboration between them, and accelerate the implementation of the 4D trajectory operation mode. This helps air traffic controllers to achieve on-time, safe, and efficient landing of incoming flights, reduce flight delays, and effectively improve the operational efficiency of terminal airspace. How to introduce fuel consumption, noise pollution or conflict resolution as optimization indicators into 4D trajectory planning is an important direction for future research.

## References

- [1] LI Shixiao, ZHU Fan, ZHANG Jian, et al. Multi-Restriction Path Planning Based on Improved A\* Algorithm [J]. *Electronics Optics & Control*, 2014, 21(7): 36-40.
- [2] Feng You, Ronghui Zhang, Guo Lie, Haiwei Wang, Huiyin Wen, Jianmin Xu. Trajectory planning and tracking control for autonomous lane change maneuver based on the cooperative vehicle infrastructure system[J]. *Expert Systems with Applications*, 2015,42(14):5932-46.
- [3] H. Erzbrwger, H. Q. Lee. Terminal Area Guidance Algorithm for Automated Air Traffic Control. In *Proceedings of NASA. TN-D 6773*, 1972.
- [4] Pecsvaradi T. Four-dimensional guidance algorithm for aircraft in an air traffic control environment[R]. *NASA TN-D7829*,1975.132.
- [5] Knox C E, Cannon D G. Development and test results of a flight management algorithm for fuel conservative descents in a time-based metered traffic environment[J]. 1980.
- [6] Chakravarty A. Four-dimensional fuel-optimal guidance in the presence of winds[J]. *Journal of Guidance, Control, and Dynamics*, 1985, 8(1): 16-22.
- [7] Tomlin C, Pappas G J, Sastry S. Conflict resolution for air traffic management: a study in multiagent hybrid systems[J]. *Automatic Control IEEE Transactions on*, 1998, 43(4):509-521.
- [8] Wang Dahai. *Optimal Flight Profile and 4D Navigation Research in Advanced Flight Management System*. Beijing: Beihang University, 1996.
- [9] Wang Dahai, Su Bin, Yang Jun. Research on altitude profile and velocity profile of terminal area 4D guidance. *Flight Mechanics*. 2000, 18(1): 35~40.
- [10] Subramanian Ramasamy, Roberto Sabatini, et al. Novel Flight Management System for Real-Time 4-Dimensional Trajectory Based Operations[C]. Boston, MA: AIAA Guidance, Navigation, and Control (GNC) Conference,2013.
- [11] Muller, M., *Information Retrieval for Music and Motion*, Springer-Verlag, Berlin, 1999.
- [12] Ward, J. H., "Hierarchical Grouping to Optimize an Objective Function," *Journal of the American Statistical Association*, Vol. 58, No. 301, 1963, pp. 236-244.