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An Many-Objective Optimization Algorithm with Reference Point and Angle based on Non-Dominated Sorting Approach

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Abstract

It is critical to improve convergence and diversity in many-objective evolutionary algorithms. It is found that NSGA-III uses reference points to improve the diversity of the population but the convergence is poor. An evolutionary many-objective optimization algorithm with reference point and angle based on non-dominated sorting approach (NA-NSGA-III) is proposed. It utilizes the core framework of NSGA-III. NA-NSGA-III uses a case-by-case discussion strategy to enhance the convergence of solutions, and uses the angular penalty distance method to improve algorithm performance. On 16 well-known benchmark problems, NA-NSGA-III outperforms four state-of-the-art algorithms.

Keywords

Many-objective Optimization Algorithm; Non-Dominated Sorting; Reference Point; Case-by-case Discussion; Angle Penalty Distance.

1. Introduction

MULTIOBJECTIVE optimization problems (MOPs) contains two or three competing objectives to be optimized simultaneously [1]. Many-objective problems (MaOPs) are optimization problems with more than three objectives. In recent years, MaOPs have received extensive attention from many researchers [2][3]. The contradiction of MaOPs means that it is difficult to find a single solution to optimize all problems. Therefore, a set of Pareto optimal solutions can be obtained to illustrate the trade-off between the objectives.

At present, a lot of multi-objective evolutionary algorithms (MOEAs) have been developed into many-objective evolutionary algorithms (MaOEAs) to solve MaOPs. The large number of objectives leads to almost all solutions being non-dominated, which greatly increases the difficulty of population convergence [4]. In MaOEAs, some research attempt to distinguish excellent solutions by strengthening convergence, such as fuzzy domination [5], grid domination [6] and L-optimality [7]. In literature [8], a non-dominated sorting method is proposed to enhance the convergence to a certain extent. In literature [9], a displacement-based density estimation (SDE) strategy is proposed, which liberated the regions with poor convergence by coordinate transformation. In literature [10], KnEA uses a knee-point-based neighbor penalty density estimation scheme.

In contrast to the previous algorithms, the indicator-based algorithm employs indicators value to guide population evolution, such as the hypervolume indicator [12], the R2 indicator in R2-EMOA [21], and so on. Unfortunately, some indicators' computing complexity grows exponentially, making MaOP difficult to employ.

It has been found that decomposition-based MOEA has been found to be very promising for solving MaOPs [22]. The typical algorithm is MOEA/D [14], which uses the vector to decompose a MOP into a set of single-objective problems, and uses the aggregation function to compare individuals that can obtain good convergence. Now, many variants based on MOEA/D have been designed for MaOPs, such as RVEA [16] and MOEA/DD [23]. Due to the use of reference points, NSGA-III proposed by Deb and Jain [18] also belongs to this category. NSGA-III helps maintain diversity by providing reference points. But the convergence of NSGA-III is not ideal.

For the problems and difficulties mentioned above, a new Algorithm, that is, an evolutionary many-objective optimization algorithm with reference point and angle based on non-dominated sorting approach (NA-NSGA-III) is proposed. It uses the basic framework of NSGA-III. NA-NSGA-III discusses the situation, and uses reference point and angle for solution selection to solve MaOPs. The main contributions of NA-NSGA-III are summarized:

(1) Different from most decomposition-based algorithms that use the penalty-based boundary intersection (PBI) method as the criterion for evaluating solutions, NA-NSGA-III uses the angle penalty distance (APD) to correlate the weight vector, and as a selection criterion, it helps to improve convergence.

(2) Different from the method of selecting solutions by NSGA-III, this paper uses angle and APD to select solutions. The use of the angle-based principle can ensure that the algorithm is still effective in solving MaOPs and effectively improve diversity.

(3) Different from the direct solutions selection method of NSGA-III, this paper considers the majority of non-dominated solutions. Firstly, the case-by-case discussion strategy is used, and the reference point is used to select the excellent solution. The new method accelerates the convergence rate of the solution.

Section I is introduction. Section II introduces the preparation work. Section III expounds the research motivation of this paper. Section IV presents the details of NA-NSGA-III. Section V introduces the experimental setup. In Section VI, the performance comparison results of NA-NSGA-III and four MaOEAs are discussed. Finally, Section VII gives the conclusion and future work.

2. Preliminary

2.1 NSGA-III

NSGA-III [18] is improved on the basis of NSGA-II [11]. The crowding distance used by NSGA-II has no obvious effect in the high-dimensional objectives space. NSGA-III has made significant adjustments to the selection method by introducing reference points to maintain diversity.

The parent population is Pt with N solutions, and the generated offspring population is Qt with N solutions. The combination of Pt and Qt becomes a new population Ct, and Ct divides into level (F1, F2, ..., Fl) by non-dominated sorting, as shown in Fig.1.

Solutions are selected from the F1 layer, followed by F2... Put them in St until the size of St is N, as the parent population of the next iteration Pt+1.

When the population reaches the last layer, only part $K = N - |Pt+1|$ is selected. The method adopted by NSGA-III is to specify the reference line in advance and calculate the distance between the solution in St and the reference line.

The steps are as follows:

1) Adaptive normalization of the values in St.

2) Define the reference line connecting the reference points on the hyperplane.

3) Calculate the vertical distance between the individual in St and the reference line.

4) Each solution is associated with a reference point according to the minimum vertical distance.

5) Calculation of the number of solutions associated with each reference point.

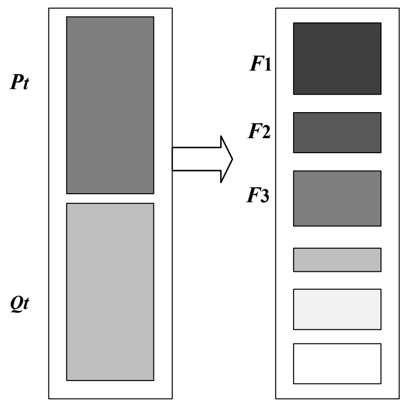

Fig. 1 Non-dominated sorting of populations and offspring.

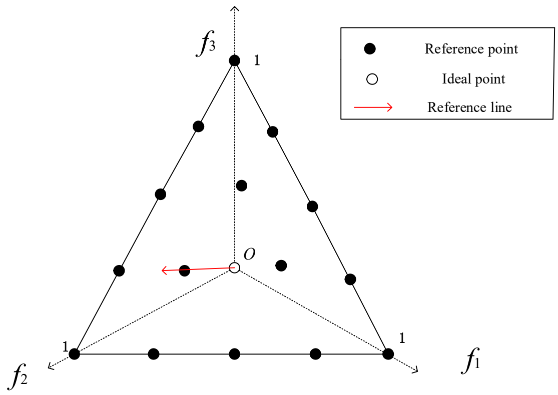

Fig. 2 For the three objectives problem with p = 4, 15 reference points are created.

6) Select K solutions according to the number of the fifth step.

NSGA-III uses the method of Das and Dennis [19] to locate points on a normalized (M-1)-dimensional hyperplane (line 14). If there are p partitions for each objective, then the number of reference points (H) can be given by:

$$H = \binom{M + p - 1}{p} \tag{1}$$

To illustrate more clearly, as shown in Fig. 2, in 3 objectives problem (M = 3), if four partitions (p = 4) are selected for each objective axis, 15 reference points be created.

2.2 Angle-penalized Distance

Angle-penalized distance (APD) [26] is a distance metric commonly used in MaOEAs. When calculating APD, the angle difference of solutions will be considered. The calculation of APD can be achieved by the following steps.

Firstly, the distance between solution Ix and Iy is calculated, which is expressed as d (Ix, Iy).

Then, the angle difference between solution Ix and Iy is calculated and expressed as θ (Ix, Iy). The cosine similarity can be used to measure the angle difference. The specific calculation Eq. Is:

$$\theta(I_x, I_y) = \arccos(\frac{f(I_x) \times f(I_y)}{\|f(I_x)\| \times \|f(I_y)\|}) \tag{2}$$

Among them, f(Ix) and f(Iy) represent the target vectors of solution Ix and Iy in the objectives space, respectively.

Finally, the d (Ix, Iy) is multiplied by an angle penalty factor and expressed as P(θ(Ix, Iy)). The angle penalty factor can be defined according to specific problems, and the common forms are linear penalty, exponential penalty and so on.

$$P(\theta(I_x, I_y)) = d(I_x, I_y) \times g(\theta(I_x, I_y)) \tag{3}$$

Here, g (θ (Ix, Iy)) is the angle penalty function, which is defined according to the specific problem. Through the above steps, the APD value between solution Ix and Iy can be obtained.

3. Motivation of This Work

NSGA-III uses reference points to solve MaOPs. NSGA-III can not only effectively solve the problem of a large number of objectives, but also has better scalability. Nevertheless, NSGAIII has the following problems.

Convergence should be enhanced. Although NSGA-III generated non-dominated solutions, excellent convergence cannot be ensured. The use of non-dominated sorting in NSGA-III can promote population convergence, however the convergence is not ideal when there are a lot of non-dominated solutions.

Diversity should be maintained. Although NSGA-III introduces reference points to maintain diversity, however, when used, it involves Euclidean distance, in the case of high dimensions, the calculation equation will fail due to ' curse of dimensionality '. The algorithm may not be able to effectively maintain the distribution of solutions, and it is difficult to ensure good population diversity.

In summary, the following points are proposed.

First, in the framework of NSGAIII, the case-by-case discussion strategy of population after non-dominated sorting is added. When all non-dominated solutions are located in F1, excellent solutions are selected based on a certain indicator and reference points. If the number of excellent solutions satisfies the required size N, excellent solutions are selected from the remaining solutions based on a certain indicator and vector angle until the number of solutions satisfies N. When the non-dominated solutions are not all located in F1, the required solutions are selected by using a certain indicator combined with the NSGA-III environment selection method.

Then, importantly, the selection of the evaluation indicator of the solution determines the quality of the diversity. Therefore, the indicator that performs well in the high-dimensional environment should be selected as the evaluation indicator of the solution.

4. Proposed Algorithm

An evolutionary many-objective optimization algorithm with reference point and angle based non-dominated sorting approach (NA-NSGA-III) is introduced in detail. Fig.3 depicts the process of NA-NSGA-III.

4.1 Main Framework

The framework of NA-NSGA-III is given in Algorithm 1.

First, population Pt with N solutions and reference points Zs are generated (lines 1&2). The main loop of the evolution process is executed until the stop condition is met (line 3). The simulated binary crossover[13] and polynomial mutation [25] is adopted to generate offspring populations Qt from Pt (line 4). Qt is combined with Pt to generate Ct for elite selection (line 5). Classification of Ct into Non-dominated Levels and the level (FrontNo) and maximum level (MaxFNo) of all individuals are obtained (line 6). When MaxFNo is 1, use Environmental-Selection-I, otherwise use Environmental-Selection-II (lines 7-11).

Next, two environment selections of Algorithm 1 will be introduced, i.e., environmental selection I (Algorithm 2) and environmental selection I (Algorithm 3).

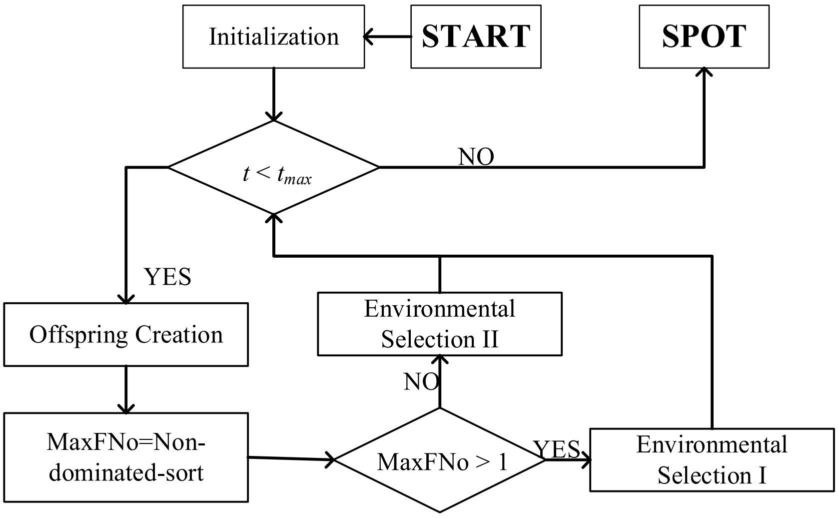

Fig. 3 Framework of NA-NSGA-III.

Algorithm **1 Main Framework**
Input: Maximal number of generations t_{max}
Output: Final population P_{tmax}.

```
1   Create population P_t with N solutions;
2   Generate reference points Z^s;
3   while t < t_max do
4       Q_t = Offspring-Creation (P_t);
5       C_t = P_t ∪ Q_t
6       [FrontNo, MaxFNo] = non-dominated-sort (C_t);
7       if MaxFNo ~= 1
8           P_{t+1} = Environmental-Selection-II (R_t, FrontNo, MaxFNo, Z^s);
9       else
10          P_{t+1} = Environmental-Selection-I (R_t, Z^s);
11      End if
12  End while
```

4.2 Environmental Selection I

When the maximum level exceeds 1, the environment selection I is selected. Algorithm 2 describes the environment selection I of NA-NSGA-III.

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First, the solutions before the maximum level are put into Next, and the maximum level is liberated into Last (lines 1&2). The number of solutions in Next associated with each reference point is calculated and put into AssNumber (line 3). When the size of Next is less than N, the APD of the solutions in Last are calculated (line 5), and the reference point current with the least correlation number in AssNumber is selected (line 6). If current can be correlated with the solution in Last, the solution with the smallest APD is selected to Next (line 8), otherwise any one is selected to Next (line 10).

***Algorithm* 2 Environmental Selection I in the Proposed NA-NSGA-III**

Input: P_t, Z^s, FrontNo, MaxFNo;

Output: P_{t+1}.

1 *Next* = FrontNo < MaxFNo;
2 *Last* = find (FrontNo==MaxFNo);
3 Calculates the number of solutions in *Next* associated with each reference point and places them in the *AssNumber*;
4 **While** length (*Next*) < *N*
5   Calculate the APD of the solutions in *Last*;
6   *current* = min (*AssNumber*);
7   **If** *current* associated with *Last*
8   The smallest APD in the associated candidate solution is selected into the *Next*;
9   **else**
10   Choose any one;
11   **End if**
12 **End while**

4.3 Environmental Selection II

When the maximum level is 1, the environment selection II is selected. Algorithm 3 describes the environment selection II of NA-NSGA-III.

Firstly, the angle between the reference point Zs and the solution in Pt is calculated and stored in Angle (line 1). Initialize Next that stores excellent solutions and allAPD that stores all solutions APD (lines 2&3). Then, for each reference point i, find the solution that has the smallest angle with i and put it in current, calculate the APD of the solution in current and store them in allAPD, find the solution with the smallest APD and put it in Next (lines 4-10). Finally, when the number of Next is less than N (line 11), calculate the angle between the solution of Next and the remaining solution (line 12). For each solution j in Next, find the solution with the largest angle and the smallest APD with j, and put it in Next (lines 14-16).

***Algorithm* 3 Environmental Selection II in the Proposed NA-NSGA-III**

Input: P_t, Z^s.

Output: P_{t+1}.

1 Calculate the angle between the reference points Z^s and the P_t and store in *Angle*;

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```
    Next = 0;
    allAPD = 0;
    For i = 1: N
        current = min (Angle);
        Calculate the APD of the solutions in current;
        solution = current (min (APD));
        Next = Next ∪ solution;
        allAPD = allAPD ∪ APD;
    End for
    While length (Next)< = N
        Calculate the angle between the solutions in Next and other solutions and store in Angle;
        For j = 1: length (Next)
            current = max (Angle);
            solution = min (allAPD (current));
            Next = Next ∪ solution;
        End for
    End while
```

4.4 Computational Complexity

The computational complexity of NA-NSGA-III considers the non-dominated sorting and environment selection.

The non-dominated sorting of populations with M-dimensional target vectors of size 2N requires O (NlogM-2N) calculations.

In Algorithm 2, the normalization of up to 2N population members requires O (N) calculations. It takes O (MN2) calculations to associate up to 2N population members with N reference points. Algorithm 2 requires a total of O (MN2) calculations.

In Algorithm 3, the computational complexity of APD calculation and solution selection in the worst case is O (MN2) and O (N2), respectively.

Considering all the above factors and the amount of calculation, the complexity of NA-NSGA-III is O (N2logM-2N) or O (MN2), whichever is greater.

5. Experimental Setup

This section introduces the general parameter settings, the performance metric used in experiments and benchmark test problems. Compare NA-NSGA-III with four most advanced MaOEAs, NSGA-III [18], RVEA [16], VaEA [20] and MOEA/DD [23]. For each test problem, consider objectives M = 3,5,8,10.

5.1 Parameter Settings

5.1.1 Population Size

The population size of NA-NSGA-III, NSGA-III and VaEA is set according to literature [18][20] and the size is the same. The population size of MOEA/DD and RVEA is the same as that in literature [14][18].

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5.1.2 Specific Parameter Settings

For the parameters in MOEA/DD, according to literature [23], the specific situation is given in Table I.

There are only two parameters in the RVEA [16] that need to be defined in advance. The parameters are about controlling the rate of change and probability of the penalty function.

Table 1. The parameters in MOEA/DD.

Parameter	Setting
neighborhood size T	20
penalty parameter θ	5
neighborhood selection probability δ	0.9

5.2 Performance Metric

The performance of the five algorithms is evaluated by inverse generation distance (IGD) and hypervolume (HV) [24].

5.2.1 IGD

IGD calculates the distance between the algorithm's solution set and the true frontier. Specifically, for each point on the real frontier, IGD calculates the distance from the point to the solution generated by the nearest algorithm, and averages the distances of all points.

The advantage of the IGD indicator is simple and intuitive, and does not require the real Pareto front. However, it also has some limitations, such as the inability to distinguish the density and distribution of decomposition sets. Therefore, when using the IGD indicator to compare MOEAs, it is also necessary to comprehensively consider other indicators and problem characteristics.

5.2.2 HV

HV is extensively used to assess the quality of the solution set created by the algorithm in the search space.

HV calculates the hypervolume of the solution set surrounded by PF to determine the quality of the solution set. Specifically, it computes the non-dominated hypervolume between the searched solution set and the true Pareto front. HV can be used to compare the search ability and diversity of algorithms different algorithms.

5.3 Benchmark Test Problems

5.3.1 DTLZ

DTLZ (Deb-Thiele-Laumanns-Zitzler) [15] is a set of commonly used multi-objective optimization test function sets, which was proposed by Kalyanmoy Deb, Lothar Thiele, Eckart Zitzler and Klaus Laumanns in 2005.

The DTLZ test set is designed to simulate the problem of multiple conflicting targets in the real world. Each test function contains one or more decision variables and multiple objective functions. The decision variable is the variable that determines the solution of the problem, and the objective function is the indicator to minimize or maximize.

The functions in the DTLZ test set are divided into a series of functions such as DTLZ1 and DTLZ2 according to the different parameter settings. Each function has its own characteristics and difficulty level. In general, the goal of the DTLZ test set is to generate non-dominated solutions, these solutions form a reference set that is approximately a hyperplane in the target space.

These test functions are very useful in evaluating multi-objective optimization algorithms, which can help us understand the effect of the algorithm in solving MOPs and compare the performance differences between different algorithms. By using DTLZ, researchers can test and improve their multi-objective optimization algorithms to find better solutions.

5.3.2 MaF

The MaF (Many-Objective Benchmark Functions) [17] test set is a set of benchmark functions used to evaluate the performance of MaOEAs. MaF test set contains a series of functions with different characteristics and difficulty levels. These functions have different dimensions and functions.

By using MaF, researchers can compare the performance of different MaOPs in dealing with large-scale objective functions. This helps to improve the design of the algorithm and change the parameter settings to obtain a better solution.

In summary, the MaF test set provides a standardized evaluation platform for multi-objective optimization algorithms, helps researchers understand the advantages and disadvantages of algorithms, and promotes the development and innovation of MaOPs.

6. Results

The experiment aims to study the ability of NA-NSGA-III to solve different types of MaOPs. Five algorithms were tested on 16 benchmark problems in DTLZ [15] and MaF [17], and tested 20 times. Wilcoxon rank sum test was used to compare the results of NA-NSGA-III with others. ' - ' means that NA-NSGA-III is significantly better than the comparison algorithm, ' + ' is the opposite. ' = ' means that there is no significant difference in the results of several algorithms.

6.1 Performance on DTLZ1–DTLZ6

Table 2. The HV of DTLZ.

Problem	M	NSGAIII	RVEA	VaEA	MOEADD	NANSGAIII
DTLZ1	3	3.9475e-1=	8.5097e-2=	3.1091e-1=	6.6758e-2-	3.8733e-1
	5	1.2682e-1-	4.1286e-1=	7.4672e-2-	2.1872e-1-	6.5512e-1
	8	4.5572e-3-	1.6392e-1=	0.0000e+0-	4.3808e-1-	4.6323e-1
	10	7.2648e-2=	5.7718e-1=	1.2194e-3-	6.4330e-1=	2.9262e-1
DTLZ2	3	5.5597e-1+	5.5260e-1-	5.5332e-1=	5.5420e-1=	5.5397e-1
	5	7.5425e-1-	7.6517e-1-	7.3284e-1-	7.6074e-1-	7.6697e-1
	8	8.2130e-1-	8.5455e-1=	7.6180e-1-	7.2746e-1-	8.6014e-1
	10	8.5425e-1-	9.2457e-1=	6.6491e-1-	9.0372e-1=	9.2314e-1
DTLZ4	3	5.0125e-1=	5.5153e-1=	4.7594e-1=	5.0842e-1=	5.0792e-1
	5	7.0101e-1=	7.5676e-1=	7.2949e-1=	7.2106e-1=	7.1281e-1
	8	8.3364e-1=	8.3491e-1=	7.5240e-1-	8.4034e-1=	8.5534e-1
	10	8.4928e-1=	9.1087e-1=	7.1319e-1-	8.8929e-1=	8.9667e-1
DTLZ5	3	1.9326e-1+	1.4489e-1-	1.9838e-1+	1.8243e-1=	1.8442e-1
	5	8.8406e-2-	9.3101e-2-	7.4816e-2-	1.0791e-1=	1.1042e-1
	8	6.7386e-2-	9.1282e-2=	4.1274e-2-	8.6604e-2-	9.4396e-2
	10	5.2044e-2-	9.2071e-2-	4.6247e-2-	9.3689e-2=	9.3644e-2
DTLZ6	3	1.6691e-1-	1.2691e-1-	1.9937e-1+	1.7587e-1=	1.7531e-1
	5	0.0000e+0-	6.7092e-2=	0.0000e+0-	8.2179e-3-	7.8856e-2
	8	0.0000e+0-	4.6787e-2=	0.0000e+0-	8.3998e-3-	7.4623e-2
	10	0.0000e+0-	4.7464e-2-	0.0000e+0-	3.3213e-2-	9.3867e-2
+/-/=		2/12/6	0/7/13	2/14/4	0/9/11	

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The comparison of HV value and IGD value between NA-NSGA-III and other four algorithms is given in Table II and III, and the best data is highlighted.

For DTLZ1, NA-NSGA-III is as good as the other four algorithms. In both low and high dimensions, it has shown strong competitiveness.

For DTLZ2, NA-NSGA-III performed well in 8 and 10 objectives, while the HV value was slightly smaller than that of NSGA-III in 3 objectives, and the IGD value was slightly larger than that of MOEA/DD in 5 objectives.

For DTLZ3, the HV of NA-NSGA-III was larger than that of others and IGD was smaller than that of others in 3-8 objectives, but in 10 objectives, NA-NSGA-III performs worse than MOEA/DD.

For DTLZ4, NA-NSGA-III is as good as the other four algorithms. In both low and high dimensions, it has shown strong competitiveness.

For DTLZ5, NA-NSGA-III has the best performance on objectives other than 3 objectives. However, in 3 objectives, the HV of NA-NSGA-III is smaller than that of NSGA-III and VaEA, while the IGD value is larger than that of NSGA-III and VaEA.

For DTLZ6, except that the performance of NA-NSGA-III in 3 objectives is slightly worse than VaEA, its performance in other objectives is best.

On the whole, NA-NSGA-III algorithm has well performance on DTLZ cases, although it performs poorly on a few individual issues.

Table 3. The IGD of DTLZ.

Problem	M	NSGAIII	RVEA	VaEA	MOEADD	NANSGAIII
DTLZ1	3	1.9744e-1=	4.7459e-1=	3.3817e-1=	5.2472e-1=	3.4800e-1
	5	5.9275e-1-	3.5654e-1=	6.0579e-1-	4.1389e-1-	1.9265e-1
	8	1.6160e+0-	4.5053e-1=	1.3092e+0-	2.5785e-1=	2.2789e-1
	10	6.2530e-1=	3.6199e-1=	1.4883e+0-	3.5820e-1=	3.4943e-1
DTLZ2	3	5.4901e-2+	5.5813e-2=	5.5233e-2=	5.5352e-2=	5.5839e-2
	5	2.1646e-1=	2.1398e-1+	2.1691e-1=	2.1392e-1+	2.1685e-1
	8	4.2968e-1-	3.8851e-1=	4.4194e-1-	3.9222e-1-	3.8799e-1
	10	6.0797e-1-	5.2386e-1=	5.7691e-1-	5.0980e-1=	5.0811e-1
DTLZ3	3	1.3253e+1=	1.8366e+1-	1.1780e+1=	2.1581e+1-	1.0410e+1
	5	1.1569e+1-	1.2132e+1-	6.1891e+0=	7.7226e+0-	6.0034e+0
	8	2.6447e+1-	1.3979e+1=	7.4901e+1-	9.7205e+0=	1.0958e+1
	10	8.4077e+0=	1.5685e+1=	2.7630e+1=	4.8316e+0+	1.7295e+1
DTLZ4	3	1.5245e-1=	5.5792e-2=	2.3799e-1=	1.5314e-1=	1.5368e-1
	5	3.1432e-1=	2.4098e-1+	2.2024e-1=	3.0245e-1=	3.0565e-1
	8	4.6128e-1=	4.7358e-1=	4.5645e-1=	4.4634e-1=	4.5432e-1
	10	6.2083e-1=	5.7590e-1=	5.8508e-1=	6.1082e-1=	6.0492e-1
DTLZ5	3	1.2289e-2+	8.2790e-2-	5.6977e-3+	3.0357e-2-	2.4879e-2
	5	1.4356e-1-	2.3678e-1-	1.7210e-1-	9.0469e-2-	7.4333e-2
	8	2.6168e-1=	3.5590e-1=	3.9618e-1=	2.1541e-1=	2.6106e-1
	10	2.9906e-1-	6.7458e-1-	4.0393e-1-	1.4199e-1=	1.3711e-1
DTLZ6	3	1.9744e-1=	4.7459e-1=	3.3817e-1=	5.2472e-1=	4.0233e-2
	5	5.9275e-1-	3.5654e-1=	6.0579e-1-	4.1389e-1-	2.1900e-1
	8	1.6160e+0-	4.5053e-1=	1.3092e+0-	2.5785e-1=	5.0559e-1
	10	6.2530e-1=	3.6199e-1=	1.4883e+0-	3.5820e-1=	1.7357e-1
+/-/=		2/12/10	2/9/13	2/11/11	2/9/13	

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6.2 Performance on MaF5-MaF15

Table 4. The HV of MaF.

Problem	M	NSGAIII	RVEA	VaEA	MOEADD	NANSGAIII
MaF5	3	5.5579e-1+	5.5223e-1=	5.5237e-1=	4.9647e-1=	5.0958e-1
	5	7.0224e-1=	7.4637e-1=	7.3437e-1=	5.6706e-1-	7.3309e-1
	8	8.6843e-1+	7.4328e-1=	7.5556e-1=	3.9444e-1-	7.9255e-1
	10	9.3381e-1+	7.7345e-1=	7.0088e-1=	4.3626e-1-	7.8736e-1
MaF6	3	1.9211e-1=	1.4388e-1-	1.9622e-1=	1.7140e-1-	1.9446e-1
	5	1.2301e-1-	1.1163e-1-	1.2631e-1=	7.5685e-2-	1.2663e-1
	8	4.0120e-2-	7.2849e-2-	6.2423e-2-	8.8192e-2-	1.0491e-1
	10	0.0000e+0=	9.4035e-2=	0.0000e+0=	8.8157e-2-	9.5186e-2
MaF7	3	2.4636e-1=	2.0737e-1-	2.4958e-1=	2.0960e-1-	2.4981e-1
	5	1.4674e-1=	1.2670e-1=	1.5077e-1=	1.0758e-1=	1.5804e-1
	8	1.0581e-2=	3.9476e-2=	3.2324e-3=	2.2683e-2=	3.1548e-2
	10	3.6489e-2=	2.5623e-2=	1.1633e-3-	4.1633e-5-	4.4871e-2
MaF8	3	1.2497e-1=	2.1101e-1=	1.3915e-1=	9.8492e-2=	1.5216e-1
	5	4.2425e-2=	7.3161e-2=	9.5469e-3-	3.9376e-2=	3.6897e-2
	8	8.4689e-3=	7.7393e-3=	5.0410e-3=	1.3169e-2=	1.0604e-2
	10	2.6950e-3=	3.1775e-3=	2.4257e-3=	2.9996e-3=	4.5794e-3
MaF9	3	7.1652e-1=	7.2085e-1=	5.8971e-1=	5.6144e-1=	7.2788e-1
	5	1.3128e-1=	1.4846e-1=	0.0000e+0-	4.2041e-2-	1.5222e-1
	8	0.0000e+0=	1.0015e-2+	0.0000e+0=	1.5261e-2+	7.0682e-3
	10	2.5264e-3-	1.1924e-3-	8.6801e-3=	3.9719e-3=	6.7921e-3
MaF10	3	6.5513e-1=	5.6261e-1=	6.1913e-1=	4.0825e-1-	6.6768e-1
	5	5.5647e-1-	5.9209e-1-	4.8298e-1-	3.3515e-1-	6.5889e-1
	8	4.8668e-1=	4.7585e-1=	3.6399e-1-	3.1643e-1-	4.8726e-1
	10	5.9346e-1=	5.1441e-1=	3.2902e-1-	4.4097e-1=	4.9861e-1
MaF11	3	9.0371e-1=	8.8078e-1-	9.0557e-1=	8.9614e-1=	9.0686e-1
	5	9.5469e-1=	9.2583e-1-	9.4603e-1=	9.1907e-1-	9.5654e-1
	8	9.6109e-1=	8.9185e-1-	9.5692e-1=	8.2136e-1-	9.6128e-1
	10	9.5854e-1=	8.9442e-1-	9.4387e-1-	9.1507e-1-	9.6276e-1
MaF12	3	5.0775e-1=	4.9308e-1=	5.0033e-1=	4.8619e-1=	5.1741e-1
	5	6.2368e-1=	6.4489e-1=	5.6864e-1=	5.2270e-1=	5.8749e-1
	8	5.6421e-1-	5.7183e-1=	6.1858e-1=	4.8777e-1-	6.9865e-1
	10	6.6469e-1=	6.0835e-1=	6.0470e-1=	4.6865e-1-	6.8161e-1
MaF13	3	4.7334e-1=	3.6016e-1-	4.6862e-1=	4.8181e-1=	4.5953e-1
	5	1.6947e-1-	9.3807e-2-	2.0574e-1=	1.5192e-1-	2.0950e-1
	8	8.0269e-2-	7.2029e-2-	1.1341e-1=	4.3044e-2=	1.1462e-1
	10	4.0578e-2-	7.4746e-2=	9.4677e-2=	3.6654e-2=	7.7752e-2
+/-/=		3/8/24	1/13/22	0/8/28	1/19/16	

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Table 5. The IGD of MaF.

Problem	M	NSGAIII	RVEA	VaEA	MOEADD	NANSGAIII
MaF5	3	2.6186e-1=	2.6667e-1=	3.0718e-1=	5.4962e-1=	2.5782e-1
	5	3.4098e+0=	2.4914e+0=	2.4425e+0=	5.8601e+0-	2.3332e+0
	8	2.4604e+1=	2.8129e+1-	2.2606e+1=	8.0742e+1-	1.9092e+1
	10	1.1313e+2-	1.4425e+2-	8.4736e+1=	2.5572e+2-	7.5302e+1
MaF6	3	1.2996e-2=	9.7303e-2-	8.0104e-3=	3.3864e-2-	7.6152e-3
	5	4.4233e-2-	9.3973e-2-	7.8779e-3=	9.3013e-2-	7.3577e-3
	8	8.2975e-1-	2.4622e-1-	1.2717e+0=	8.6677e-2-	9.4299e-3
	10	1.3682e+0=	3.5054e-1-	4.2607e+0-	8.9907e-2-	7.3247e-2
MaF7	3	9.5958e-2=	1.7861e-1-	8.9186e-2=	4.6574e-1-	8.9864e-2
	5	5.9072e-1=	5.8414e-1=	5.3074e-1=	2.4239e+0-	5.1490e-1
	8	4.6921e+0-	1.2471e+0+	1.5302e+0=	2.1039e+0=	1.4902e+0
	10	5.3014e+0-	1.7940e+0=	2.0639e+0=	2.5304e+0=	1.7780e+0
MaF8	3	5.2028e-1=	2.2735e-1=	4.4081e-1=	5.8403e-1=	4.2121e-1
	5	8.7165e-1=	4.5337e-1=	1.6123e+0=	6.9511e-1=	5.9657e-1
	8	7.8373e-1=	1.0663e+0=	1.0897e+0=	7.8642e-1=	8.6927e-1
	10	8.9757e-1=	1.1327e+0-	1.3637e+0=	1.0227e+0-	5.8507e-1
MaF9	3	2.0959e-1=	1.9067e-1=	4.3026e-1=	3.9286e-1=	1.9650e-1
	5	1.0356e+0=	5.2657e-1=	1.9804e+0-	1.1589e+0=	5.2894e-1
	8	6.7345e+0-	1.0715e+0=	5.3385e+0=	1.1969e+0+	1.8452e+0
	10	2.0615e+0=	3.1577e+0-	6.0871e-1=	2.2258e+0=	7.3241e-1
MaF10	3	6.3131e-1=	8.2531e-1-	6.8972e-1=	1.1620e+0-	6.0063e-1
	5	1.2524e+0-	1.1352e+0=	1.3731e+0=	1.8037e+0-	9.9638e-1
	8	1.9894e+0=	1.7938e+0=	2.1905e+0-	2.3046e+0-	1.7811e+0
	10	1.9708e+0=	1.8904e+0=	2.5555e+0=	2.2209e+0=	2.0766e+0
MaF11	3	1.7182e-1=	2.1986e-1-	1.7313e-1=	1.8700e-1=	1.7961e-1
	5	5.0220e-1+	4.8979e-1+	5.1139e-1=	5.4048e-1-	5.2129e-1
	8	1.2681e+0=	1.1385e+0=	1.0142e+0+	1.3522e+0-	1.1013e+0
	10	1.5256e+0=	1.3195e+0=	1.3005e+0+	1.4821e+0=	1.4065e+0
MaF12	3	2.3330e-1=	2.6520e-1-	2.3534e-1=	2.6807e-1-	2.3926e-1
	5	1.2160e+0=	1.2043e+0=	1.2199e+0=	1.3939e+0-	1.2441e+0
	8	3.6233e+0=	3.5656e+0=	3.3603e+0+	3.9993e+0-	3.5582e+0
	10	5.4699e+0-	5.7086e+0-	4.9271e+0+	6.0759e+0-	5.2845e+0
MaF13	3	9.8335e-2+	1.7546e-1-	9.7095e-2=	8.5497e-2+	9.7826e-2
	5	2.2552e-1-	5.2556e-1-	1.8985e-1=	3.4556e-1-	1.7886e-1
	8	3.1673e-1-	6.9650e-1-	2.4181e-1=	4.4253e-1-	2.2945e-1
	10	3.9260e-1-	9.4614e-1-	2.0360e-1+	5.3169e-1-	2.6323e-1
MaF14	3	3.4600e+0=	6.7499e+0-	2.5021e+0=	3.7930e+0=	2.4592e+0
	5	1.0113e+1-	6.4589e+0=	1.0128e+1-	3.7828e+0=	3.4405e+0
	8	7.8817e+1-	9.0525e+0=	2.4398e+1-	1.0637e+1=	1.0848e+1
	10	1.9078e+1=	1.6439e+1=	2.1051e+1=	9.3450e+0=	1.5571e+1
MaF15	3	1.4054e+0-	1.0715e+0-	1.1069e+0-	4.5868e-1=	5.6641e-1
	5	4.4717e+0-	1.0924e+0+	3.7560e+0-	1.7218e+0=	1.2746e+0
	8	9.7004e+0-	4.6404e+0=	6.7023e+0-	1.0860e+1-	4.2576e+0
	10	2.1534e+1-	3.3189e+0+	9.0602e+0-	5.4744e+0=	4.7651e+0
+/-/=		2/17/25	4/19/21	5/9/30	2/23/19	

The comparison of HV value and IGD value between NA-NSGA-III and other four algorithms is given in Table IV and V, and the best data is highlighted.

For MaF5, although the HV of NA-NSGA-III is smaller than that of NSGA-III in some objectives, but the IGD of NA-NSGA-III is significantly the smallest, indicating that the performance of NA-NSGA-III is highly competitive.

For MaF6, NA-NSGA-III is as good as the other four algorithms. In both low and high dimensions, it has shown strong competitiveness.

For MaF7, NA-NSGA-III has the best performance on objectives other than 8 objectives, but the performance of NA-NSGA-III is not as good as RVEA in 8 objectives.

For MaF8-MaF10, in most cases, NA-NSGA-III has the largest HV and the smallest IGD, while in 8 objectives, the data of MOEA/DD data is more competitive than that of NA-NSGA-III.

For MaF11-MAF13, in the high-dimensional case, the IGD of NA-NSGA-III is only larger than that of VaEA, which means that its performance is worse than that of VaEA in the high-dimensional case.

For MaF14, NA-NSGA-III is as good as the other four algorithms. In both low and high dimensions, it has shown strong competitiveness.

For MaF15, in 5 objectives and 10 objectives, the IGD value of RVEA is smaller than that of NA-NSGA-III. In other objectives, the performance of NA-NSGA-III is best.

In summary, NA-NSGA-III has the best comprehensive performance on MaF cases, although it performs poorly on the high dimensions of individual problems.

7. Conclusion

An evolutionary many-objective optimization algorithm with reference point and angle based on non-dominated sorting approach (NA-NSGA-III) is proposed to solve many-objective optimization problems (MaOPs). NA-NSGA-III has the advantages of NSGA-III. The main feature of NA-NSGA-III is to discuss the selection environment selection strategy according to the situation, and use the angle-based principle and APD to select the excellent solution in the environment selection process. The case-by-case discussion strategy focuses on the stratification after non-dominated sorting. When there is only one level, the reference point correlation solution is directly used to select the excellent solution, and the remaining excellent solution is selected based on the angle principle and APD until the population size meets the requirements. For the case where the non-dominated solution is located in the multi-level, the reference point correlation solution and APD are used to improve diversity in the last level.

For the 17 problems of DTLZ and MaF, NA-NSGA-III is compared with four advanced MaOEAs. Experiments show that, in most cases, NA-NSGA-III has achieved an overwhelming performance on IGD and HV.

However, for the proposed algorithm, the performance of solving some MaF test set problems in high-dimensional environment is not very good, which may be caused by the single indicator of solution selection. Combining the excellent indicators of high-dimensional environment and improving the solution selection criteria, it is possible to make the proposed algorithm outstanding in high-dimensional environment.

NA-NSGA-III could be applied to solve the constrained MaOPs. Also, NA-NSGA-III can be applied to solve several practical engineering problems.

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