Research on Multi-beam Line Measurement based on Geometric Mathematical Model

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Abstract

Ocean sounding is an important work to determine the depth of sea water and explore the seabed topography. Nowadays, there are two main groups of ocean sounding techniques: single-beam sounding system and multi-beam sounding system. The multi-beam sounding system overcomes the shortcomings of the single beam sounding system, which is suitable for simple exploration tasks, and can provide more accurate and effective terrain data. In this paper, a geometric mathematical model is established according to the triangle formed between the beam emitted by the transducer and the submarine slope at different survey line positions of the survey ship, and the data of the central sea area is calculated by using the triangular function relationship. The relevant result data is calculated by using the similar ratio characteristics and coverage formula of each triangle.

Keywords

Ocean Sounding; Multi-beam Line Measurement; Trigonometric Relationship; Mathematical Modeling; Exhaustive Method.

1. Introduction

There are two main techniques in seabed sounding to explore the seabed topography and seawater depth: single-beam sounding system using the characteristics of sound waves in water to measure the depth of water, and multi-beam sounding system based on single-beam sounding technology. In a multi-beam sounding system, the coverage width W varies with the opening Angle and water depth D, and the limit of overlap rate is given. The topography of the seabed is complicated, so it is necessary to design the survey line interval according to the actual situation to ensure that the survey results avoid missing measurement and data redundancy. It is required to establish a mathematical model of the overlap rate between the coverage width W and adjacent strips in multi-beam sounding technology, and calculate the seabed depth, coverage width and coverage rate of different parallel and equally spaced survey line positions, which are collectively referred to as sea area characteristic data. After analysis, since all survey lines are parallel to each other and have the same interval, it can be obtained that the beam emitted by the transducer at different survey line positions has the same similar ratio with the triangle formed by the submarine slope under the condition that the submarine slope is constant and the opening Angle of the transducer is constant. Therefore, the sea feature data of the central sea area can be calculated using the geometric relationship of the trigonometric function. The similar ratio is used to calculate the characteristic data of other sea areas to be measured, and MATLAB software is used to calculate the solution.

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2. Model Assumption

In order to build a more accurate mathematical model of ocean sounding and obtain more reasonable and accurate data results, the following reasonable assumptions and constraints for model establishment are made according to the actual situation:

(1) Assuming that the velocity of the vessel and the velocity of sea water are uniform and the opening Angle is fixed, the spacing of each strip is not affected by the velocity of the vessel and the velocity of sea water.

(2) It is assumed that the propagation speed of acoustic signal is much higher than that of the hull, so that the time of transmitting acoustic wave to echo can be ignored, and the multi-beam sounding system can simultaneously emit sounding acoustic wave in an open Angle , that is, the influence of the time from transmitting acoustic wave to receiving echo on the research results is ignored, so the model is simplified.

(3) Assume that the data given in the attached ocean depth data table is true and reliable.

(4) It is assumed that the topography of the seabed is continuously changing, and there is no sudden change in the height of the seabed topography.

(5) It is assumed that sound waves propagate uniformly in seawater, and the refraction of sound waves in water is not considered.

The corresponding symbols in the paper are shown in Table 1.

Table 1. Description of symbols

symbol	Symbol specification
	Overlap rate
	Submarine slope
	The Angle at which the normal of a seafloor slope is projected on a horizontal plane
	The opening Angle of a multi-beam transducer
W	Covering width
d	Line spacing
D	Sea depth
A	Measure the distance matrix of the line from the center point
D_0	Depth of water in the central sea area
W_0	Central sea area coverage width
d'	The distance of the measuring ship from the center of the sea area
D	*Q*'*Q*''' distance

3. Model Building

For problem 1, a mathematical model is first established for a single sounding, as shown in Figure 1.

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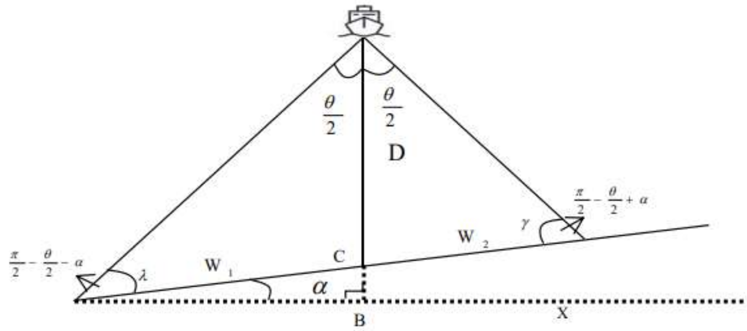

Figure 1. Single probe scenario

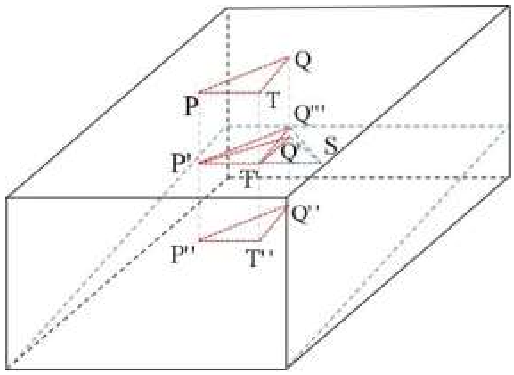

Figure 2. Projection of line measurement

According to the meaning of the question, the known quantity is the multi-beam transducer open Angle , The depth of the water at the center of the ocean D0, slope . Construct A = [-800, -600, -400, -200, 0, 200, 400, 600, 800] each line center distance matrix, The coverage width of the central sea area is W0, The horizontal projection of the submarine slope is X, An Angle bisector with an easy open Angle intersects a slope at a point C, Extend its angular bisector to the X boundary at point B, It can be seen that the angular bisector is perpendicular to the X boundary. By analyzing the relationship between the size of each Angle and each boundary to establish a mathematical plane geometry model, the following trigonometric function relationship can be obtained.

$$\begin{cases} \lambda = \frac{\pi}{2} - \frac{\theta}{2} - \alpha \\ \lambda = \frac{\pi}{2} - \frac{\theta}{2} + \alpha \end{cases} \tag{1}$$

According to the trigonometric relationship in the figure and the bottom depth of the central sea area, the following formula can be obtained to calculate the corresponding bottom depth of different lateral positions.

$$\mathrm{D} = \mathrm{D}_0 - 200\tan\alpha * \left(\frac{\mathrm{A}}{200}\right) \tag{2}$$

The beam emitted by the transducer on different survey lines is similar to the triangle formed by the submarine slope, and the similarity ratio is the same. The coverage width of the sea area to be measured can be calculated according to the similar ratio and the triangle data formed by the beamforming in the central sea area.

$$\begin{cases} \frac{W_1}{D_0} = \frac{sin\frac{\theta}{2}}{sin\left(\frac{\pi}{2}-\frac{\theta}{2}+\alpha\right)} \\ \frac{W_2}{D_0} = \frac{sin\frac{\theta}{2}}{sin\left(\frac{\pi}{2}-\frac{\theta}{2}-\alpha\right)} \end{cases} \tag{3}$$

$$W_0 = W_1 + W_2 \tag{4}$$

From the above formula:

$$W_0 = D_0 * \left(\frac{sin\frac{\theta}{2}}{sin\left(\frac{\pi}{2}-\frac{\theta}{2}+\alpha\right)} - \frac{sin\frac{\theta}{2}}{sin\left(\frac{\pi}{2}-\frac{\theta}{2}-\alpha\right)}\right) \tag{5}$$

$$W = W_0 * \frac{D}{D_0} \tag{6}$$

Because the slope Angle α is small, the effect on the result is negligible, so the known coverage formula is combined:

$$\eta = 1 - \frac{d}{W} \tag{7}$$

The corresponding coverage rate can be calculated.

4. Model solving

According to the above formula, the data of the central sea area =70 was substituted and calculated using MATLAB software. The results of the first question were obtained as shown in Table 2.

Table 2. Results of model calculation

Measure the distance of the line from the center point/m	-800	-600	-400	-200	0	200	400	600	800
Sea depth/m	91	86	81	76	70	65	60	54	49
Coverage width/m	316	298	279	261	243	225	207	188	170
The overlap rate with the previous line/%	——	33	28	28	28	11	3	-6	-17

5. Comprehensive Evaluation of the Model

5.1 Advantages of the Model

(1) Mainly analyzes the relationship between mathematical plane and three-dimensional geometric models, and the formulas obtained are simple and intuitive, easy to understand.

(2) Problem 2 Model discusses the magnitude change law of submarine slope Angle α involved in the survey line with the change of the direction of the survey line, which makes the model more accurate and reliable.

(3) The assumptions of this model are reasonable, and the results obtained have practical reference value.

5.2 Deficiency of Model

Due to the small Angle of the seafloor slope, the models of problem 1 and 2 are approximately regarded as horizontal and flat terrain, and are directly calculated by using equation, which leads to small errors in the data, but does not affect the reliability of the data.

6. Conclusion

(1) The mathematical model established in this paper is a typical layout optimization problem of ocean sounding survey. In a rectangular sea area to be measured, it is required to design a group of survey lines with the shortest length, which can cover the whole sea area to be measured and the coverage rate meets the requirements. Based on the mathematical model of the second question, under the same depth, when the opening Angle is 90 degrees or 270 degrees, the coverage width is the largest, so the question can be divided into east and west parts from north to south from the center point of the sea, and the survey line positions meeting the requirements are gradually taken from both sides by using the exhaustive method. Firstly, the coverage width of the central sea area is calculated using the seawater depth of the central sea area, and then the interval between the adjacent survey line and the adjacent side is calculated according to the optimal survey line with the coverage of 10%~20% and as close as possible to 10%, and the position of the adjacent survey line is obtained, and the seawater depth at this position is calculated. The sum of the positions and lengths of all survey lines is obtained recursively according to the law. MATLAB software is used to calculate the result in meters.

(2) In this paper, a geometric mathematical model is established according to the triangle formation between the beam emitted by the transducer and the submarine slope at different position of the survey line. The advantages and disadvantages of the mathematical model are objectively evaluated. The results of this study are considered to be of certain reference value in the layout design of multi-beam survey line system in ocean sounding work.

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