
An Image Fusion Algorithm Based on Non-subsampled Contourlet Transform and Compressed Sensing

Liangxue Huang ^a, Bohua Wang, Jianyong Yu, Yuanwei Zhang

Key Laboratory of Industrial Internet of Things & Network Control, MOE, Chongqing University of Posts and Telecommunications, Chongqing 400065, China

^ahuangliangxue@126.com

Abstract

For the problem of large amount of data and high computational complexity when fusing the high and low frequency coefficients respectively based on the traditional non-subsampled contourlet transform(NSCT), an image fusion algorithm based on non-subsampled contourlet transform and compressed sensing is proposed. Firstly, the registered images from the same scene are decomposed by NSCT transform in multi-scale and multi-direction. Then by taking different fusion rules for the low and high frequency coefficients: the low frequency coefficient is fused by adopting the combination fusion rules of regional energy and variance, while applying the compressed sensing(CS) technology in the sparse high frequency components, that is to say, the measured values of high frequency coefficients obtained use the weighted fusion program and should be reconstructed by orthogonal matching pursuit algorithm. Finally, the high and low frequency coefficients would be fused by inverse NSCT transform to obtain the final fusion image. The experimental results show that the fusion image in both subjective vision and objective evaluation indexes have good effect, at the same time, the experimental data verifies the effectiveness of the proposed method in this paper.

Keywords

Image fusion; NSCT transform; Compressed sensing; Fusion rule; Regional energy.

1. Introduction

Image fusion^[1] is to carry on the treatment of two or more images with different natures from the same scene after which the redundant and complementary information between the images shall be gained with the use of the adequate fusion rule for the final fusion of a new composite image of which the information is more complete and accurate compared with any original image.

With the development of information technology, the technology of image fusion is not only used in the military field like target detection, tracking and recognition etc. but also implemented in a wide range of civil fields like weather forecast, computer vision, robot, medical imaging, security monitoring etc. Multiscale image fusion is currently the hot spot of both domestic and foreign research of which many defects exist for more or less^[2]. Take NSCT like Da Cunha A L in 2006 as an example^[3], although it completed the decomposition of multiscale and multidirection for the image and solve the problem of translation invariance that is not possessed by contourlet and eliminate the phenomenon of Pseudo-Gibbs, subband coefficients of multiple directions will appear at the different levels during the decomposition of multiscale and multidirection of the NSCT for which the separate fusion in accordance to the fusion rule for each direction will be a very complicated process during the subsequent fusion with the data amount and the storage space being increased for which the efficiency of fusion is undoubtedly reduced. Recently, the emergence of a new theory, compressed sensing (CS)^[4] effectively

solved this kind of problem. The theory explains that when the signal is sparse or compressible, there is no need to follow the traditional Nyquist Sampling Theorem for the sampling^{[5][6]}, under the condition of not losing the information, the sampling frequency far lower than it is required in the sampling theorem shall be used for the sampling of the signal for which the signal can be recovered accurately.

With the integration of the advantages and characteristics of NSCT and CS respectively^{[7][8]}, this article provides a method of research on image fusion based on NSCT and compressed sensing for which the good fusion result can be gained even under the condition of less observation data for the image compressive sampling.

2. NSCT Theory

The transform of NSCT^[3] is composed by the bi-iteration structure of NSP (non-subsampled pyramid) and NSDFB (non-subsampled directional filter bank) as indicated in figure 1. This structure gains the achievement of dividing the two-dimensional frequency domain into wedge-shaped directional subband. First the image shall be decomposed in multiscale through NSP, and then the high frequency subband gained from the decomposition shall be decomposed in multidirection with the use of NSDFB. After the decomposition of NSCT, 1 low frequency subband image and $\sum_{j=1}^J 2^{l_j}$ bandpass directional subband images shall be finally gained of which the size is the same as the size of the original input image.

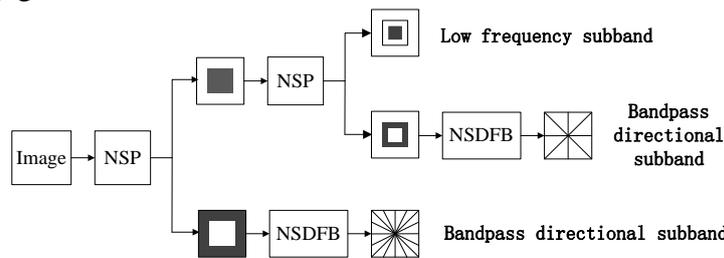


Fig. 1 Schematic Diagram of the Transform of NSCT

3. CS Theory

The process on CS theory concludes three parts: sparse representations, measurement matrix and reconstruction algorithm^{[4][9]}.

3.1 Sparse Representation

Suppose x is the one-dimensional discrete time signal in the space of R^N , the signal can use the respective columns of the orthonormal basis $\Psi = [\varphi_1, \varphi_2, \dots, \varphi_N]$ for the indication of linear combination, the expression is as follows:

$$x = \Psi \alpha = \sum_{i=1}^N \varphi_i \alpha_i \tag{1}$$

Of which, Ψ is the matrix of $N \times N$, α is the column vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_N]$ of $N \times 1$ dimension which is the coefficient vector of sparse representation of signal x on the matrix of Ψ , when signal x has $K \ll N$ nonzero coefficient only on the matrix of Ψ , Ψ is called as the sparse matrix of signal x , K is the sparsity of the signal.

3.2 Measurement Matrix

To design a measurement matrix with high effectiveness plays a very important role to the accuracy during the restructuring of the signal later, the measurement matrix can catch the important information in the sparse signals with the compressed data as less as possible. You can get a measurement signal y of $M \times 1$ dimension by using a matrix Φ that is entirely not related to the sparse matrix Ψ which achieves the compressive sampling of the original signal x , the expression is as follows:

$$y = \Phi x \quad (2)$$

In the above expression, $y \in R^M$ ($M \ll N$) is the observed value, Φ is the observation matrix of $M \times N$, with the process of compression, the number of dimension for the signal is dropped from N to M which greatly reduces the data amount for transmission, storage and processing. Substitute expression (1) in expression (2), you can get the following expression:

$$y = \Phi x = \Phi \Psi \alpha = \Theta \alpha \quad (3)$$

Of which $\Theta = \Phi \Psi$ is the matrix of $M \times N$ which is called as the sensing matrix or projection matrix. Only the projection matrix Θ meets the condition of restricted isometry property (RIP), signal x can be accurately recovered by the restructuring algorithm with the measurement value y .

3.3 Restructuring Algorithm

The restructuring of the signal is the core part of the compressed sensing theory, it can convert the solution for the problem of formula groups $y = \Phi x$ to the problem of minimum l_0 norm, the optimization problem of l_0 norm is a non-polynomial intractable problem. Normally it will be converted into the minimum l_1 norm for the restructuring of the original signal. It can be expressed by the following formula:

$$\hat{x} = \arg \min \|x\|_1 \quad s.t. \quad y = \Phi x \quad (4)$$

Therefore, a nonconvex optimization problem is converted into a convex optimization problem in formula (4) which is very easy to get the solution, the current restructuring algorithm of regular use can be divided into the following three kinds: greedy tracing method, convex relaxation, combinatorial algorithm.

4. Improved Algorithm for Image Fusion Based on NSCT and CS

Before fusion, the images have been pretreated strictly such as spatial registration. The main fusion steps are as follows:

1. Firstly, the registered images from the same scene are decomposed by NSCT transform
2. Then by taking different fusion rules for the low and high frequency coefficients: the low frequency coefficient is fused by adopting the combination fusion rules of regional energy and variance, while applying the compressed sensing(CS) technology in the sparse high frequency components, that is to say, the measured values of high frequency coefficients obtained use the weighted fusion program and should be reconstructed by orthogonal matching pursuit algorithm.
3. Finally, the high and low frequency coefficients would be fused by inverse NSCT transform to obtain the final fusion image.

4.1 Fusion rule of Low Frequency Coefficient

The method of traditional image fusion based on the regional characteristics focuses only on the fusion of characteristics like the single regional energy or regional variance etc. It is very likely to have a phenomenon of which the big energy has small variance and the small energy has big variance at the same region of the two images for which the fusion result will be impacted. Therefore, this article greatly improves the fusion effect of the image by the integration of the two fusion methods.

This article selects the window with the size of 3×3 , the regional energy of $M \times N$ centered with (i, j) for the corresponding two images is:

$$E_{j_0}^x(i, j) = \sum_{m \leq M, n \leq N} |C_{j_0}^x(i+m, i+n)|^2 \quad (5)$$

The regional variance is:

$$\sigma_{j_0}^x(i, j) = \frac{1}{M \times N} \sum_{m \leq M, n \leq N} |C_{j_0}^x(i+m, j+n) - \overline{C_{j_0}^x(i, j)}|^2 \quad (6)$$

Of which $C_{j_0}^X(i, j)$ indicates the low frequency coefficient of image A, B , $\overline{C_{j_0}^X(i, j)}$ indicates the partial mean value. And the formula of low frequency fusion is as follows:

$$C_{j_0}^F(i, j) = \begin{cases} C_{j_0}^A(i, j) & \sigma_{j_0}^A(i, j) \geq \sigma_{j_0}^B(i, j), E_{j_0}^A(i, j) \geq E_{j_0}^B(i, j) \\ C_{j_0}^B(i, j) & \sigma_{j_0}^A(i, j) \leq \sigma_{j_0}^B(i, j), E_{j_0}^A(i, j) \leq E_{j_0}^B(i, j) \end{cases} \quad (7)$$

When $\sigma_{j_0}^A(i, j) \geq \sigma_{j_0}^B(i, j), E_{j_0}^A(i, j) \leq E_{j_0}^B(i, j)$

$$C_{j_0}^F(i, j) = \begin{cases} C_{j_0}^A(i, j) & \frac{\sigma_{j_0}^A(i, j) - \sigma_{j_0}^B(i, j)}{\sigma_{j_0}^A(i, j) + \sigma_{j_0}^B(i, j)} \geq \frac{E_{j_0}^B(i, j) - E_{j_0}^A(i, j)}{E_{j_0}^A(i, j) + E_{j_0}^B(i, j)} \\ C_{j_0}^B(i, j) & otherwise \end{cases} \quad (8)$$

When $\sigma_{j_0}^A(i, j) \leq \sigma_{j_0}^B(i, j), E_{j_0}^A(i, j) \geq E_{j_0}^B(i, j)$

$$C_{j_0}^F(i, j) = \begin{cases} C_{j_0}^A(i, j) & \frac{\sigma_{j_0}^B(i, j) - \sigma_{j_0}^A(i, j)}{\sigma_{j_0}^A(i, j) + \sigma_{j_0}^B(i, j)} \geq \frac{E_{j_0}^A(i, j) - E_{j_0}^B(i, j)}{E_{j_0}^A(i, j) + E_{j_0}^B(i, j)} \\ C_{j_0}^B(i, j) & otherwise \end{cases} \quad (9)$$

4.2 Fusion Rules for the Measurement Value of High Frequency Coefficient

This article gains the measurement value of fusion with the method of weighted fusion for the measurement value of high frequency coefficient from the images of two origins. After the decomposition, the high frequency coefficient includes the large amount of detailed information of the images, the observation process of CS on the image is a linear process, and it can be considered that the measurement value after the observation also has the existence of linear relationship. It can be considered that the bigger the observation value is, the larger the image information included is. Therefore, we use strategy of weighted fusion for the measurement value of high frequency coefficient. The detailed calculating formulas are as follows:

$$Y_{j,l}^F(i, j) = W_{j,l}^A Y_{j,l}^A(i, j) + W_{j,l}^B Y_{j,l}^B(i, j) \quad (10)$$

$$W_{j,l}^A = \frac{Y_{j,l}^A(i, j)}{Y_{j,l}^A(i, j) + Y_{j,l}^B(i, j)} \quad (11)$$

$$W_{j,l}^B = 1 - W_{j,l}^A \quad (12)$$

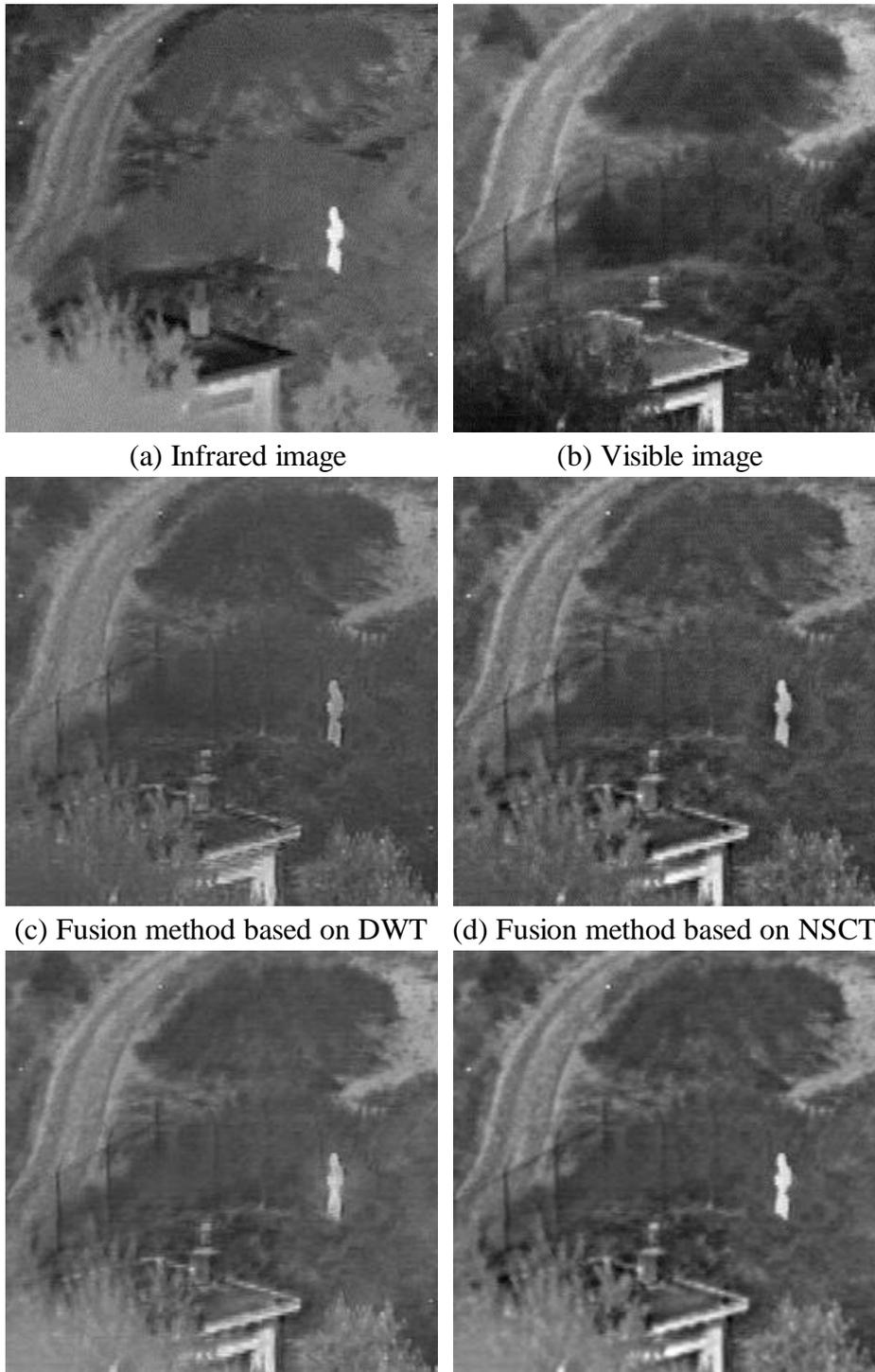
In the above formula, $Y_{j,l}^F(i, j)$, $Y_{j,l}^A(i, j)$ and $Y_{j,l}^B(i, j)$ are the measurement value of the image after fusion, image A and B respectively, $W_{j,l}^A$ and $W_{j,l}^B$ are the weighted coefficient for the fusion of the observation value at the respective high frequency directions.

5. Results and Analysis of the Experiment

In order to verify the effectiveness and correctness of the algorithm mentioned in this article, we have carried on the experiment by using the different fusion method on the simulation platform of MATLAB7.11 with the selection of one group of infrared and visible images with the same size which is 256×256 . In accordance to this article, we have selected the image fusion method based on discrete wavelet transform DWT and NSCT transform and discrete wavelet transform and compressed sensing for the comparison with the selection of 3 as the decomposition order for all the multiscale transform of which the directional decomposition number of Contourlet and NSCT transform are all 2, 8 and 16. The fusion result is indicated in figure 2.

From the perspective of subjective evaluation, the visual perception can give out the direct comparison, the method mentioned in this article can keep the important information in the source image into the fused image in a good way. From the image, it can be seen that both the information of fences, bushes and roof etc. in the visible images and the heat sensitive information of human shadow in the infrared image can be reflected in the fused image in a good way. It has been improved obviously compared with

the figure 2(c) and figure 2(e), compared with figure 2(d), the fusion results are very close which is very difficult to judge by the visual sense.



(a) Infrared image

(b) Visible image

(c) Fusion method based on DWT

(d) Fusion method based on NSCT

(e) Fusion method based on DWT and CS (f) Method mentioned in this article

Fig. 2 The Source Image of the Experiment and Its Fusion Result

When it cannot be distinguished by the visual sense, in order to get the objective assessment, this article carries on the analysis for the fused images by several indexes which are information entropy(IE), average gradient(AG), standard deviation(SD), spatial frequency(SF), definition(DE) and edge retention($Q^{AB/F}$). The result is shown on table 1.

From table 1 of respective assessment indexes, it can be gained that the effect of the fusion image decomposed by the NSCT transform is far better than it is based on DWT transform. All the scores for the method of this article is higher than it is for the other methods on information entropy, standard

deviation, spatial frequency and edge retention, although it is lower than the fusion method based only on the NSCT transform on the average gradient and definition, the algorithm of this article can greatly reduce the data amount of fusion which decreases the complexity of calculation and the storage space for data in the meantime and meets the visual sense of human.

Table 1 Index of Objective Assessment for the Fused Images

Index	Assessment	IE	AG	SD	SF	DE	$Q^{AB/F}$
Fusion method							
	DWT	6.2907	4.9859	23.2463	10.0550	4.9470	0.3319
	NSCT	6.7301	6.0551	29.5934	12.1016	5.9585	0.4349
	DWT_CS	6.6030	5.0639	27.0788	10.0931	4.8752	0.3017
	Method of this article	6.7664	5.8762	30.3739	12.3648	5.7781	0.4432

6. Conclusion

This article puts forward an image fusion method based on NSCT and compressed sensing. After the NSCT transform, you can directly use the fusion rules with the integration of regional energy and variance for low frequency coefficient, you can use the method of weighted fusion for the measurement value after the measurement of high frequency coefficient of which the fused measurement value can be used for the restructuring of high frequency fusion coefficient with the method of orthogonal matching pursuit. By the analysis of the experiment, you can see that the fusion image effectively keeps the detailed textures, shape and profile information of the source image, achieves higher visual fusion effect and reduces the data amount of the fusion in the meantime and decreases the complexity of calculation and improves the efficiency of fusion.

References

- [1] POHL, Van Genderen J L. Multisensor image fusion in remote sensing: concepts, methods and applications. *International Journal of Remote Sensing*. 1998, 19(5): 823-854.
- [2] Jinglei Zhang, Eying Zhao. Fusion method for infrared and visible light images based on NSCT. *Laser & Infrared*, 2013, 43(3):319-323.
- [3] Da Cunha A, Zhou J P, Do M N. The nonsubsamped contourlet transform: Theory, design, and Applications. *IEEE Transactions on Image Processing*, 2006, 15(10): 3089-3101.
- [4] Donoho D L. Compressed sensing. *Information Theory. IEEE Transactions on*. 2006, 52(4): 1289-1306.
- [5] Xiaosheng Huang, Qiufang Dai, Yiqin Cao. Compressive sensing image fusion algorithm based on wavelet sparse basis. *Application Research of Computers*. 2012, 29(9): 3581-3583.
- [6] Zhang Q, Maldague X. An infrared-visible image fusion scheme based on NSCT and compressed sensing. *SPIE Defense + Security. International Society for Optics and Photonics*, 2015: 94740Y-94740Y-7.
- [7] Zhou X, Wang W. Liu R. Image Fusion in Compressed Sensing Based on Non-subsampled Contourlet Transform. *Proceedings of the Third international Conference on Communications. Signal Processing, and Systems*. Springer International Publishing, 2015: 645- 652.
- [8] Xiaoxue Xing, Fu Liu, Weiwei Shang, et al. Medical Image Fusion in Compressed Sensing Based on Non-subsampled Contourlet Transform. *Mobile Ad-hoc and Sensor Networks (MSN)*, 2013 IEEE Ninth international Conference on IEEE. 2013: 490-493.
- [9] Li X, Qin S Y. Efficient fusion for infrared and visible images based on compressive sensing principle. *Image Processing, IET*, 2011, 5(2): 141-147.