# An algorithm for the estimation of PMSM position based on the new sliding-mode observer

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## Abstract

In the sensorless control of PMSM, the estimate of rotor's position and speed need to be achieved. Firstly, the effect of temperature on the stator resistance is analyzed in this paper. Then,the influence of the variation of stator resistance on the vector control system is discussed.In order to improve the function of the system and the precision of rotor position,A new-style siliding mode observer was designed in this paper.This siliding mode observer can measure stator resistance real-timely using Lyapunovhy perstability theory.Finally,this algorithms was proved with simulink Simulation results show that this algorithms can overcome the harmful effect of the change of motor parameters and it has superior capability.

## Keywords

PMSM;siliding mode observer;stator resistance;vector control.

## 1. Introduction

As with the development of permanent magnet materials and control technologies, the application of permanent magnet synchronous motor (PMSM) has aroused a wide attention. Compared to the conventional motor, PMSM is featured with the advantages such as high torque-to-inertia ratio and high efficiency, which has been extensively applied to aerospace, robotics and other high-end technical sectors.

In the control system of PMSM, the rotor position should be detected to ensure the stability of system operation. To this end, the conventional method is to acquire the rotor position by mounting the position sensor on the rotor shaft, however, the mounting method of position sensor is found with many defects in the actual applications, e.g.: inapplicable to the high-temperature setting or adding the rotational inertia, it should be also noted that the high-precision position sensor is too costly. Therefore, the sensorless detection method of rotor position has been widely focused and researched[1].

The existing sensorless detection methods of PMSM rotor position mainly include: ① The open-loop estimation algorithm based on the ideal model of motor; ② The estimation algorithm based on various observers; ③ The estimation algorithm based on the high-frequency injection[2].Among these methods, the sliding-mode observer has aroused more and more attention due to its superior performance, simple control algorithm and good robustness and other advantages[5-8].

Generally, the stator resistance R is determined as the fixed constant value in the establishment of motor model, but the stator resistance R will vary with the influence of ambient environment in the actual operation of PMSM, mainly including the temperature of motor operation and the change of stator current value. The error of the stator resistance R is up to 23％ at the temperature change of 60℃ [9]. The variation of the stator resistance will affect the accuracy of the decoupling of vector control system and also cause an error to the design of the closed-loop PI controller. In this concern, it is necessary to detect the stator resistance in real time to improve the estimation accuracy of stator position and further enhance the overall performance of system. This work designs a sliding-mode observer, utilizes a simple and operable method to dynamically detect the stator resistance in real time based on Lyapunov hyperstability theory, feedback the instantaneous value of stator resistance into the sliding-mode observer to alleviate the effects of motor parameters on the system. Through the simulated verification, this designed sliding-mode observer improves the accuracy of position detection with strong robustness.

## 2. Effects of temperature on the stator resistance value

The temperature in the operation of motor is one of the key factors for the change of stator resistance. The dependence of temperature on the stator resistance can be expressed as[10]:

$$R' = \frac{235 + t_2}{235 + t_1} R \tag{1}$$

Wherein, $R'$ is the stator resistance value at $t_2$, $R$ is the stator resistance value at $t_1$. The above equation shows that the stator resistance value will show a linear change with temperature. However, the temperature of motor mainframe will rise due to a variety of losses in the actual operation of motor and these losses are finally converted to heat. So the final stator resistance value under the influence of temperature will show the change as indicated in Fig.1.

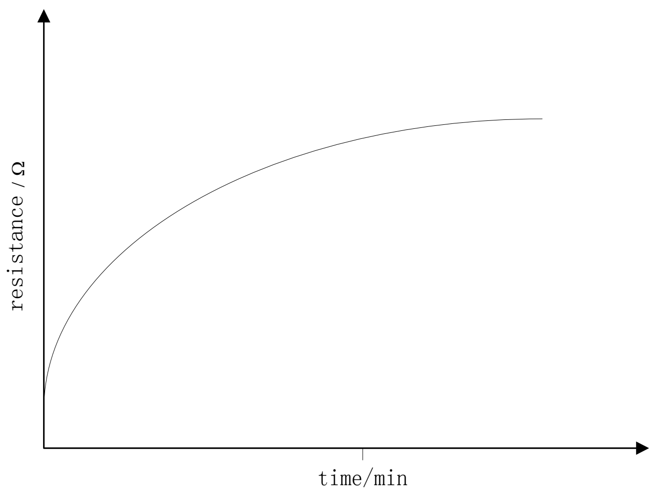


Fig.1 The curve of stator resistance value varying with time

## 3. Effects of the stator resistance value on the PMSM vector control system

### 3.1 Effects of the stator resistance value on the vector control quadrature-direct axis decoupling

Under the two phase rotating reference frame, the permanent magnet flux linkage equation of PMSM is:

$$\phi_f = \frac{L_m}{pT_r + 1} i_d \tag{1}$$

Wherein, $\phi_f$ is the permanent magnet flux linkage, $L_m$ is the stator winding and rotor winding inductance value, $T_r$ is the rotor time constant, namely $T_r = L / R$, and p is the differential operator. The rotor position angle of PMSM can be expressed as:

$$\theta = \int (\omega + \omega_s) dt = \int \left( \omega + \frac{L_m}{T_r \phi_f} i_q \right) dt \tag{2}$$

$\omega$ is the rotor angular velocity, and $\omega_s$ is the slip angular velocity.

The rotor position angle is used when PMSM is transformed from the three-phase static coordinates to the two-phase rotating coordinates, the above two equations suggest that the rotor position angle will be affected by the change of the rotor resistance value, which may reduce the accuracy of the motor quadrature-direct axis decoupling, affect the dynamic response of the motor, and even disable the motor to operate.

### 3.2 Effects of the stator resistance value on the system PI controller

PMSM vector control system normally comprises two closed-loop controls, in which the inner, outer loop is current loop and velocity loop respectively. Because the stator resistance value causes an error to the quadrature-direct axis decoupling affecting the current loop, it will also affect the design of the velocity loop PI controller. PMSM closed-loop control structure diagram is as shown in Fig.2.

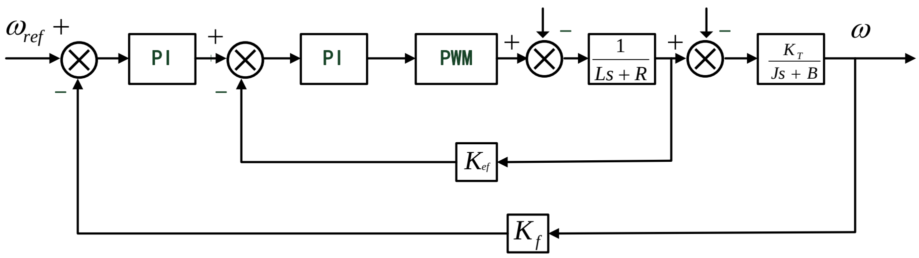


Fig.2 PMSM closed-loop control block diagram

In the actual system operation, the frequency of the current loop is much higher than that of the velocity loop, so the velocity is assumed to be constant for the convenience of analyzing the current loop. Therefore, the current loop is oriented at controlling:

$$G_i(s) = \frac{1}{Ls + R} \tag{3}$$

The transfer function for the current loop PI controller is:

$$G_c(s) = K_p\left(1 + \frac{K_i}{s}\right) \tag{4}$$

According to the above equation, the current loop of vector control system is indicated in Fig.3.

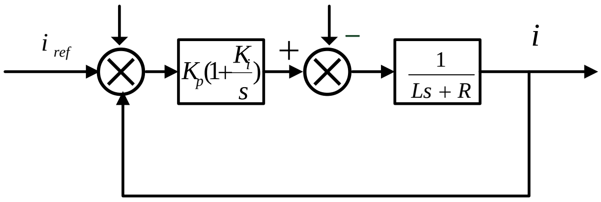


Fig.3 The block diagram of current loop control

Generally, the current loop PI controller is designed as $\frac{K_i}{K_p} = \frac{R}{L}$, according to the above contents, the design of PI controller is largely related to the stator resistance value. In any change of the motor parameters, if the design of PI controller maintains, it will inevitably affect the controlling effect, thus affecting the velocity loop. The system stability will be caused to be reduced.

### 3.3 Effects of the stator resistance value on the velocity estimation of sliding-mode observer

The counter electromotive force contains the information of rotor position, and the sliding-mode observer obtains the rotor position and the estimated velocity by observing the counter electromotive force, according to the relationship between the counter electromotive force and the position angle.

The mathematical model of the salient PMSM under $\alpha-\beta$ coordinates frame is:

$$\begin{cases} \frac{di_\alpha}{dt} = -\frac{R}{L} i_\alpha + \frac{u_\alpha}{L} - e_\alpha \\ \frac{di_\beta}{dt} = -\frac{R}{L} i_\beta + \frac{u_\beta}{L} - e_\beta \end{cases} \tag{5}$$

Wherein, $\begin{cases} e_\alpha = -\varphi_f \omega \sin\theta \\ e_\beta = -\varphi_f \omega \cos\theta \end{cases}$

$i_\alpha$, $i_\beta$ is the current component of stator current on $\alpha$ axis, $\beta$ axis, respectively;

$u_\alpha$, $u_\beta$ is the voltage component of stator voltage on $\alpha$ axis, $\beta$ axis, respectively;

$e_\alpha$, $e_\beta$ is the component of counter electromotive forceon $\alpha$ axis, $\beta$ axis, respectively;

$R$ is the stator resistance value, $L$ is the stator inductance value, and $\varphi_f$ is the permanent magnet flux linkage;

$\omega$ is the rotor angular velocity, and $\theta$ is the rotor angle.

So the rotor position angle is:

$$\theta = -\arctan\left(\frac{e_\alpha}{e_\beta}\right) = -\arctan\left(\frac{u_\alpha - Ri_\alpha - L\frac{di_\alpha}{dt}}{u_\beta - Ri_\beta - L\frac{di_\beta}{dt}}\right) \tag{6}$$

From the above equation, the stator resistance value must be used when the sliding-mode observer is operated to estimate the rotor position, so the accuracy of position estimation is related to the accuracy of the resistance value, thereby affecting the overall system performance.

According to the above analysis, it is necessary to estimate the rotor resistance value online to obtain the accurate resistance value in the occasion requiring higher accuracy of control, so as to enhance the system performance.

## 4. Design of Sliding-mode Observer

### 4.1 Design of sliding-mode current observer

According to the mathematical model of PMSM motor, the sliding-mode surface is designed as $S(x) = \hat{i}_s - i_s$ (s is valued as $\alpha$, $\beta$), $\hat{i}_s$ is the observed value of stator current, and $i_s$ is the actual value of stator current. The equation of sliding-mode observer is constructed as:

$$\begin{cases} \frac{d\hat{i}_\alpha}{dt} = -\frac{R}{L} i_\alpha + \frac{u_\alpha}{L} - \frac{K}{L} sign\left(\hat{i}_\alpha - i_\alpha\right) \\ \frac{d\hat{i}_\beta}{dt} = -\frac{R}{L} i_\beta + \frac{u_\beta}{L} - \frac{K}{L} sign\left(\hat{i}_\beta - i_\beta\right) \end{cases} \tag{7}$$

Wherein, $\hat{R}$ is the observed value of stator resistance, K is the switch gain coefficient of sliding-mode observer. sign function is the symbolic function, specifically expressed as:

$$sign(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases} \tag{8}$$

The equation of current error is attained by subtracting Equation (6) from Equation (8):

$$\begin{cases} \frac{d\left(\hat{i}_\alpha - i_\alpha\right)}{dt} = \left(-\frac{R}{L} - \frac{R}{L}\right)\hat{i}_\alpha - \frac{R}{L}\left(\hat{i}_\alpha - i_\alpha\right) - \frac{K}{L} sign\left(\hat{i}_\alpha - i_\alpha\right) + \frac{e_\alpha}{L} \\ \frac{d\left(\hat{i}_\beta - i_\beta\right)}{dt} = \left(-\frac{R}{L} - \frac{R}{L}\right)\hat{i}_\beta - \frac{R}{L}\left(\hat{i}_\beta - i_\beta\right) - \frac{K}{L} sign\left(\hat{i}_\beta - i_\beta\right) + \frac{e_\beta}{L} \end{cases} \tag{9}$$

According to the basic theory of sliding-mode observer, the sliding-mode observer must meet the requirement of stability, which means that it has good quality and asymptotic stability when the system enters the “sliding-mode”. Accordingly, Lyapunov function is constructed as:

$$V = \frac{1}{2} S^T S + \frac{1}{2}\left(R - R\right)^2 \tag{10}$$

According to the stability requirement of sliding-mode observer, then:

$$\dot{V} = \dot{S}^T S + \left(R - R\right)\frac{dR}{dt} < 0 \tag{11}$$

$$\dot{V} = \left[\hat{i}_\alpha - i_\alpha, \hat{i}_\beta - i_\beta\right]\begin{bmatrix} \left(-\frac{R}{L} - \frac{R}{L}\right)\hat{i}_\alpha - \frac{R}{L}\left(\hat{i}_\alpha - i_\alpha\right) - \frac{K}{L} sign\left(\hat{i}_\alpha - i_\alpha\right) + \frac{e_\alpha}{L} \\ \left(-\frac{R}{L} - \frac{R}{L}\right)\hat{i}_\beta - \frac{R}{L}\left(\hat{i}_\beta - i_\beta\right) - \frac{K}{L} sign\left(\hat{i}_\beta - i_\beta\right) + \frac{e_\beta}{L} \end{bmatrix}\left(R - R\right)\frac{dR}{dt} \tag{12}$$

Obviously, for $\dot{V} < 0$, it must have

$$\begin{cases} e_\alpha - K sign\left(\hat{i}_\alpha - i_\alpha\right) < 0 \\ e_\beta - K sign\left(\hat{i}_\beta - i_\beta\right) < 0 \end{cases} \tag{13}$$

Obtain the switch gain coefficient $K > \max\left(|e_\alpha|, |e_\beta|\right)$.

And it is easily known that:

$$\left[\hat{i}_\alpha - i_\alpha, \hat{i}_\beta - i_\beta\right] \begin{bmatrix} -\frac{R}{L}\left(\hat{i}_\alpha - i_\alpha\right) + \frac{1}{L}\left(e_\alpha - K sign\left(\hat{i}_\alpha - i_\alpha\right)\right) \\ -\frac{R}{L}\left(\hat{i}_\beta - i_\beta\right) + \frac{1}{L}\left(e_\beta - K sign\left(\hat{i}_\beta - i_\beta\right)\right) \end{bmatrix} < 0 \tag{14}$$

Let

$$\left[\hat{i}_\alpha - i_\alpha, \hat{i}_\beta - i_\beta\right] \begin{bmatrix} \left(-\frac{R}{L} - \frac{R}{L}\right)\hat{i}_\alpha \\ \left(-\frac{R}{L} - \frac{R}{L}\right)\hat{i}_\beta \end{bmatrix} + \left(R - R\right)\frac{dR}{dt} = 0 \tag{15}$$

The estimation equation for the stator resistance can be then obtained:

$$\frac{dR}{dt} = \frac{1}{L}\left[\left(\hat{i}_\alpha - i_\alpha\right)\hat{i}_\alpha + \left(\hat{i}_\beta - i_\beta\right)\hat{i}_\beta\right] \tag{16}$$

The said resistance estimation is input into the sliding-mode observer, which doesn't affect the robustness of sliding-mode observer, enhances its performance, and effectively improves the estimation accuracy of rotor position, thereby improving the overall system performance.

### 4.2 Estimation of the rotor position angle and velocity

As mentioned in Section 2.3, the counter electromotive force of stator winding is observed by the sliding-mode observer, and then the rotation angle is calculated according to the relationship between the rotor position angle and the counter electromotive force. The counter electromotive force obtained from the sliding-mode current observer contains more high-frequency components with bigger error, which is generally not used for the direct calculation of rotation angle, the high-frequency component is firstly removed by the low pass filter, and then the counter electromotive force through the low pass filter is:

$$\begin{cases} e_\alpha = -\varphi_f \omega \sin\theta \\ e_\beta = -\varphi_f \omega \cos\theta \end{cases}$$

So the rotation angle is:

$$\theta = -\arctan\left(\frac{e_\alpha}{e_\beta}\right) \tag{17}$$

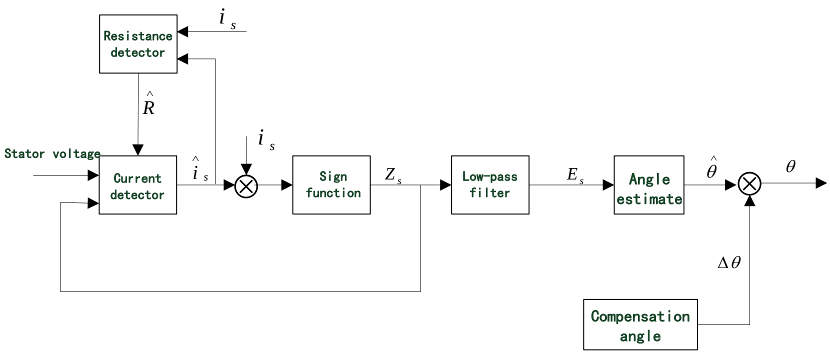


Fig.4 Design diagram of the modified sliding-mode observer

The velocity $\omega = \frac{d\theta}{dt}$.

As the phase shift will inevitably occur through the low pass filter, it is necessary to make the phase compensation accordingly. The phase shift degree is related to the frequency of low-pass filter, according to the phase response of low pass filter, the phase delay table is compiled in which the corresponding compensation angle at different velocities can be checked out. The compensation angle $\Delta\theta = -\arctan\left(\frac{\omega}{\omega_c}\right)$, wherein $\omega_c$ is the frequency of low pass filter.

The overall block diagram of the sliding-mode observer is as shown in Fig.4.

## 5. Simulated Results and Analysis

Based on the modified sliding-mode observer, the simulation model of PMSM control system is as shown below.

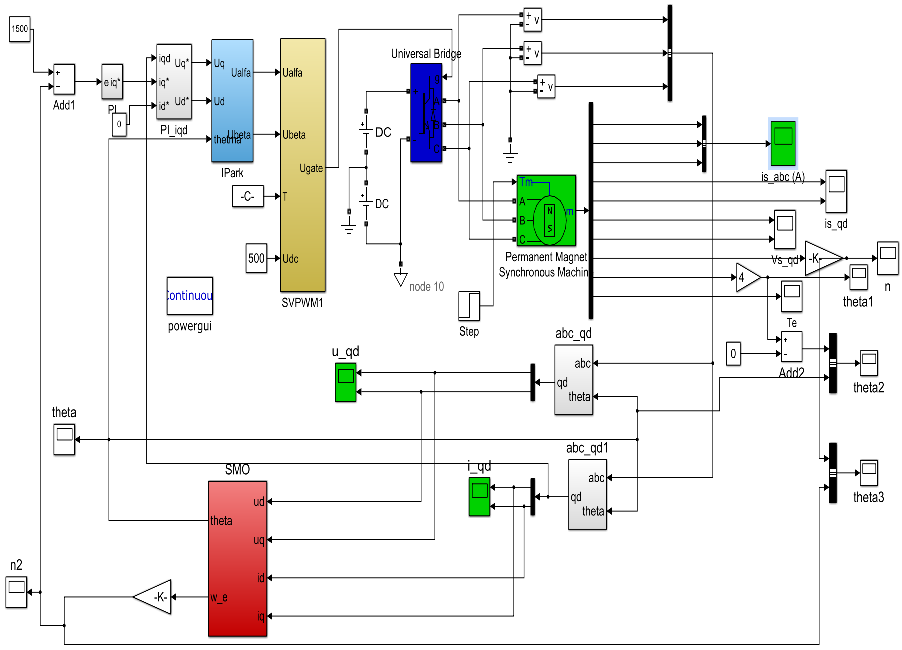


Fig.5 The simulation diagram of PMSM vector control based on the modified sliding-mode observer

The simulation system model uses vector control, and the velocity is set as 1,500r/min, which is controlled by two closed-loops: speed loop and current loop. In this work, the motor parameters of the designed control system are: Rated power 550W, rated voltage 220V, rated velocity 1,500 r/min, stator inductance 8.5mH, and stator initial resistance 2.875Ω. The simulation time is set as 0.4s. As the stator resistance of the motor will not change drastically within 0.4s, the stator resistance is not changed in the same simulation, and the system performance after the change of stator resistance is directly simulated and verified by comparing it with the vector control system based on the common sliding-mode observer.

### 5.1 R=3.25Ω The simulated analysis of the vector control system based on common sliding-mode observer

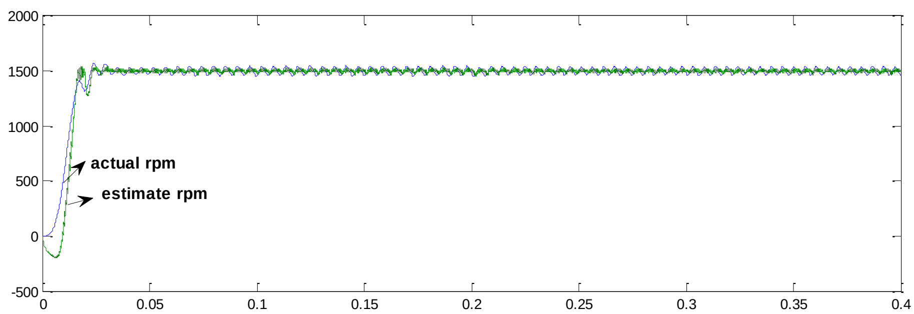


Fig.6 The comparison between actual velocity and estimated velocity of the vector control system based on the common sliding-mode observer

Fig.6 suggests that the actual velocity of the motor in vector control system with the common sliding-mode observer has a large fluctuation with the increased time for the tendency to stability, when the stator resistance of the motor increases by 13% to 3.25Ω, and gradually stabilized at about t=0.15s, which indicates that the sliding-mode observer is not responded quickly enough. Meanwhile, the following of estimated velocity with the actual velocity is not stable. Fig.7 suggests that there is a bigger error between estimated position and actual position of the motor rotor, which can be primarily attributed to no change of the sliding-mode observer parameters after the change of resistance. If such error is seriously accumulated, it may cause the system crash and disable the motor to operate normally. This indicates that the performance of common sliding-mode observer is reduced after the change of resistance, which cannot precisely estimate the rotor position of the motor. This simulation verifies that the overall performance of vector control system based on the common sliding-mode observer significantly is reduced after the change of stator resistance, which cannot meet the requirement of high-precision control.

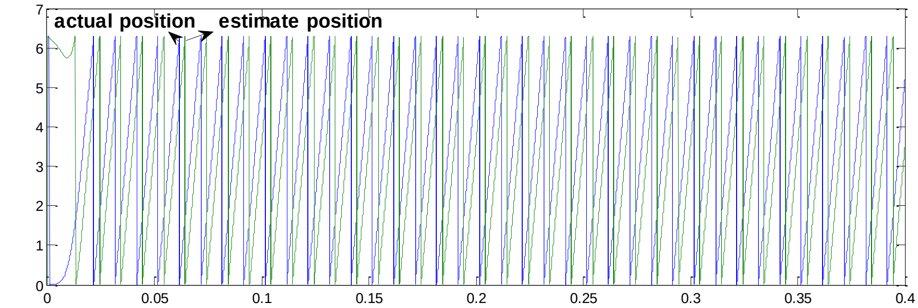


Fig.7 The comparison between actual position and estimated position of the vector control system based on the common sliding-mode observer

### 5.2 R=3.25Ω The simulated analysis of the vector control system based on modified sliding-mode observer

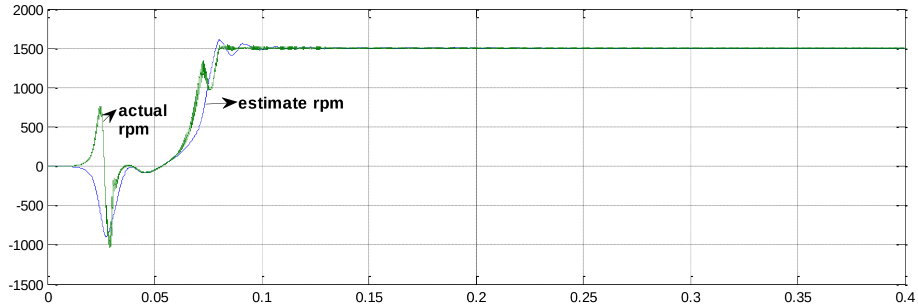


Fig.8 The comparison between actual velocity and estimated velocity of the vector control system based on the modified sliding-mode observer

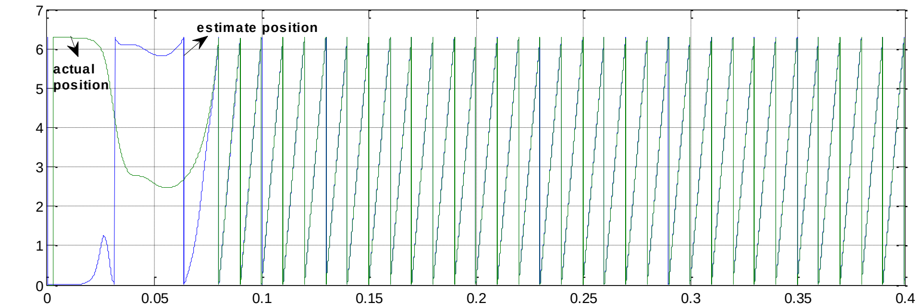


Fig.9 The comparison between actual position and estimated position of the vector control system based on the modified sliding-mode observer

Fig.8 suggests that the system reaches a stable state at 0.08s when the velocity is 1,500r/min, which proves that the system has a superior fast response. The estimated velocity enables the fast response for the actual velocity after 0.03s. However, the velocity obviously has a larger fluctuation at a lower velocity, which is also one problem of the sliding-mode observer, namely the performance of the motor is unsatisfied at a low velocity, primarily in that the value of counter electromotive force is too small. Fig.9 suggests that the estimated position and the actual position of the rotor completely overlap, which is primarily attributed to the fact that the parameters of the modified sliding-mode observer can vary with the stator resistance of the motor. This indicates that the system has a very high precision of position estimation with fast response. Before t=0.07s, the velocity doesn't reach 1,500r/min, the estimationerror of rotor position is too big as a result of the chattering problem of the sliding-mode variable structure, but the estimation error after the stabilization of velocity is almost 0, implying its superior performance at a high velocity. This simulation verifies that the vector control system based on the modified sliding-mode observer can better overcome the adverse effects from the change of stator resistance of the motor, and the overall system performance proves to be superior.

## 6. Conclusion

Through the quantitative analysis, this work draws a conclusion that the change of stator resistance will affect the vector control system, and the existing sliding-mode observer is modified based on this conclusion. The designed modified sliding-mode observer has a good performance at the estimated rotor position, the real-time feedback of stator resistance lessens the effects of the error of system parameters, which substantially improves the accuracy in the estimation of position and enhances the overall system performance to a great extent.

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