

Fractal geometry and its applications

Liping Zhao^{1, a}, Qiliang Shu² and Bingxiang Liu³

¹Department of Informatics and Engineering, Jingdezhen Ceramic Institute, Jingdezhen, Jiangxi, 333403, China

²College of Technology and Art Jingdezhen Ceramic Institute, Jingdezhen, Jiangxi, 333403, China

³Department of Informatics and Engineering, Jingdezhen Ceramic Institute, Jingdezhen, Jiangxi, 333403, China

^azlp815@sina.com

Abstract

Fractal theory is a new branch of mathematics; it is an extension of traditional geometry and has been widely used in every field. In this paper, we expounded the development of fractal theory and its application field, and give several fractal patterns use mathematical formula to generate, and the application of fractal figure. Through the analysis and study of the fractal algorithm, we can aware of the fractal image actually is not mysterious, and them widely exist in People's Daily life.

Keywords

The fractal graph; Dimension; Application.

1. Introduction

The object of ordinary geometry researched, usually with an integer dimensions. For example, the dimensional of zero point, the one-dimensional of a line, two-dimensional surface and three-dimensional solid, and even the four dimensional space-time. In recent years, the emerging of fractal geometry, its space is not necessarily an integer dimensions, but there is a fractional dimension, this is a new breakthrough of geometry that cause a great attention of the person that mathematicians and natural science. Objective nature, there are many things, with "levels" of self-similar structure, in an ideal world, even with infinite level. Enlarged or reduced to the appropriate geometry size, the whole structure is not changed. Behind many complicated physical phenomena, is reflecting the hierarchical structure of fractal geometry.

An objective thing has its own characteristic length, must use the proper dimension to measure. It is too short use a ruler to measure the Great Wall; it is too long use a ruler to measure e. coli. Thus produced the characteristic length. There are some things no characteristic scales, it must consider since many scale (or scale), which is called "scale-free" problem. Turbulence, such as physics, turbulence is a common phenomenon in the nature, from the static chamber of light smoke, the giant to Jupiter's atmosphere; eddy current, fluid motion is very disorder. Fluid macroscopic movement of energy, through the large, medium and small scales, such as micro many degrees, and finally into thermal motion and molecular scales involves a large number of different scales of motion at the same time, from the "scale-free" solve the problem, high turbulence vortex area, you need to use the fractal geometry.

2. Introduction of the fractal geometry

2.1 Fractal geometry is proposed

The French mathematician Mandelbrot has a significant role on the fractal geometry. In 1975, 1977 and 1975 he has published three books in French and English, especially 《the Fractal - shape, the opportunities and the dimension》 and 《the Fractal geometry of nature》, opened a new branch of mathematics---Fractal geometry. The word "fractal" is Mandelbrot built in 1975, the root is Latin fractus, is the meaning of "broken".

In the 1970 s, in Mandelbrot's book "how long is the coastline of British" is discussed in this problem. It depends on the use when measuring scale.

If we use the measuring unit, km from several meters to tens of meters of some twists and turns can be ignored. To switch to m to do unit, measured the total length will increase, but some cm magnitude cannot be reflected in the following. Due to the high tide ebb tide makes the coastline of the amphibious dividing line with various levels of irregularity. Coastline in two directions are natural to the size of the restrictions, take Britain several prominent point on the outer edge, with straight line put them together, get a lower bound coastline length. Use longer than this scale is meaningless. And the smallest scale of sea sand is atoms and molecules, using a much smaller scale, are of no significance. Exists between the two natural limits, can change a lot of order of magnitude "scale-free" area, the length is not quantitative characteristics of coastline, with fractal dimension.

Mathematicians Koch starting from the "island" of a square, always maintain area is the same, make its "coast" infinite curve, its length also increases, and tends to infinity. Later you can see, the fractal dimension is exactly the characteristics of "Koch island" coast the coastline fractal dimension are between 1 and 2.

These natural phenomena, especially physical phenomenon and has close relation with the fractal, if broken if continue distribution of stars in the Milky Way, the fractal dimension of the attractor. Fluid flow in porous media seepage model of movement and it produces, is the research object of fractal. These force further research mathematician, result in fractal geometry.

2.2 The content of the fractal geometry

Fractal geometry, the basic idea is: the hierarchical structure of the objective things is self-similar, partial and whole in the form, function, information, time, space, etc, the similarity between the statistical significance, become the self-similarity. For example, every part of a magnet as a whole with the north and south poles, split down continuously, each part has the same magnetic field and the whole magnet. This self-similar hierarchical structure, enlarged or reduced to the appropriate geometry size, the whole structure remains the same.

Dimension is an important feature of geometric objects; it is the position of a point in geometry object number of independent coordinates required. In Euclidean space, people used to place the space as a three-dimensional, flat or spherical as 2 d, the straight line or curve as one dimension. Can also be a little, that point is zero dimensional, can also bring in high dimensional space, for the more abstract or more complex object, as long as each local can and Euclidean space, and easy to determine the dimension. But usually people are accustomed to the dimensions of the integer.

Fractal dimension theory can also be a score; the dimension is a physicist in the research of chaotic attractor theories need to introduce the important concepts. In order to quantitatively describe the objective things "rules", in 1919, mathematician from the Angle of the measurement of dimension is introduced the enlarging dimension from integer to score, to break through the limit of general topology set dimension for the integer.

Dimension and measurement have close relations; we explain the concept of fractal dimension by the example below.

When we draw a straight line, the result is infinity if we use point to measure it because the line contains infinite number points; but the result is zero if we use a plane to measure it because the line does not contain in a plane. So, how to measure it we can receive limited value? We have to use line that has same dimension with it its' dimension is 1 (greater than zero, less than 2).

For Koch curve is an infinitely long line fold, obviously, the result is infinity by some small straight lines to measure and the result is 0 (the curve does not contain the plane) by plane. So we must find ruler have a same dimension with "Koch Island" to get limited value, and obviously, the dimension was greater than 1 and less than 2, so only a decimal. So there has been fractal dimension. The dimensions of the "Koch island" is 1.2618....

3. The definition of fractal

Mandelbrot has two methods to defined fractal:

The set A that satisfy

$$\text{Dim}(A) > \text{dim}(A)$$

Be called fractal sets. There, the $\text{Dim}(A)$ is Hausdorff dimension of set A (or fractal dimension) and $\text{dim}(A)$ is its topological dimension. Generally, $\text{Dim}(A)$ is not the integer, but the score.

The concept of fractal dimension we can set up from two aspects: first, we draw a line, square and cube, all the side length of them are 1. second, bisect the all side length, and now, the side length become 1/2 of original .and the original image can be divided into several similar graphics. Its line, square and cube were divided into 2^1 , 2^2 , and 2^3 similar sub graph respectively, index of 1, 2, 3, is equal to the corresponding experience dimension. Generally, if a graph is composed of b shrink the original graph for $1/a$, there are:

$a^D = b$, $D = \log b / \log a$ established, D is called similarity dimension, D can be integer, can also be a score.

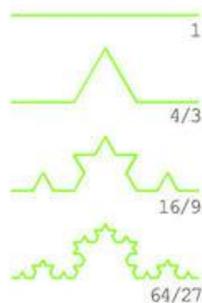


Figure 1 Von Koch curve

Each section of the Koch curve is composed by four with its ratio of 1:3 small curve of the same shape, then its hausdorff dimension (fractal dimension) for $d = \log(4) / \log(3) = 1.26185950714$.

It is a fractal if its part and whole are similar in some form.

However, after the inspection of theory and application it was found that the two definitions are difficult to include so rich content of fractal. In fact, for what is a fractal, there hasn't a precise definition up to now. Just as the definition of "life" is not strictly a clear. But people usually listed a series of characteristics of living organisms to illustrate. The definition of fractal can also be the same method.

Fractal generally has the following characteristics:

On any small scale can have fine structure; So irregular that it is difficult to descript in traditional Euclidean geometry language. Self-similar at least roughly or arbitrarily. Hausdorff dimension will be greater than the topological dimension (but the space filling curve and the Hilbert curve is exceptions).It has a simple recursive definition.

[1] The fractal sets have any details the proportion of small scales, or it has a fine structure.

- [2] The fractal sets can't use the traditional geometric language to describe, it is neither satisfy certain conditions of point trajectory nor some simple solution set of equation.
- [3] have some form of self-similar fractal sets, can be approximate self-similar or statistical self similarity.
- [4] In general, the fractal dimension of fractal set, strictly greater than its corresponding topological dimension.
- [5] In of the most interesting case, fractal sets defined by very simple way.

4. The application of fractal geometry

Now for the application of fractal graphics are:

- (1) The application of fractal theory and mathematical theory combining;
- (2) The fractal graphics applications in architectural design, for example, the walls decorated with fractal figure;
- (3) The application of fractal figure in textile pattern design;
- (4) The application of fractal theory in the information encryption security;
- (5) The fractal graph in the application of ceramic design, etc.

4.1 the IFS iterated function system fractal graph

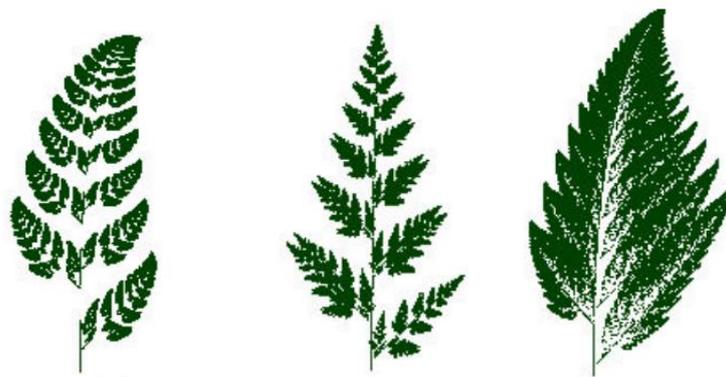


Figure 2 IFS plant morphological fractal graph

4.2 Newton iteration algorithm of fractal figure



FIG. 3 $F(z) = z^4 - 1$ according to the Newton iteration of fractal figure

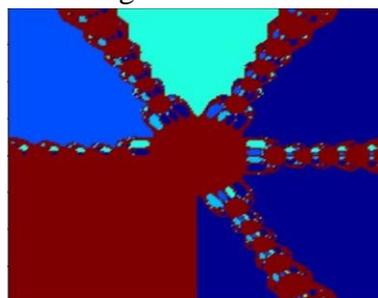


Figure 4 $F(z) = z^6 + 1$ according to the Newton iteration of fractal figure

4.3 The fractal swimsuit design



Figure 5 the fractal swimsuit design

5. Conclusions

Fractal theory opens up a door of the geometry that is brand-new and exciting. Nowadays, the pattern design is very popular and some people tend to pursue a certain artistic graphics. So, we use the fractal theory, by the specific algorithm and select a set of proper parameters, drawn out the graph on the basis of calculation results that will be able to produce a wide variety of fractal art graphics with symmetrical beauty.

Reference

- [1] Wu Yunbing Fractal theory research and application in computer graphic design master's thesis Xian university of science and technology,
- [2] Tian Xingyan, Deng Jiyuan with the improved Newton iterative method of fractal art graphics design computer system application of 2011.10
- [3] Sun Hui, jin-chang Chen application iteration function system to obtain the simple method of fractal graphics Journal of engineering graphics 2001.3
- [4] Hong-jun sun, li-hong zhao, fractal theory and its application [M]. Journal of liaoning institute of technology, 2005