Research on LMS Class of Adaptive Filters

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Abstract

Adaptive filter occupies an extremely important position in the field of signal processing, which is widely used in communication, radar, navigation system and industrial control, etc.In the case of some unpredictable signal and the noise characteristics, it cannot use filters with fixed filter coefficient to achieve optimal filtering signal,whose only solution is to introduce the adaptive filter.Because linear mean square(LMS) algorithm has the characteristics of low computational complexity, good convergence in stable environments, the average unbiased to converge to the wiener solution and use of limited accuracy to achieve the stability of the algorithm, which makes LMS algorithm become the most widely used algorithm in the adaptive algorithm. In this paper, the LMS algorithm is adopted to adjust the parameters of the adaptive filter, in order to achieve the optimal filtering performance.

Keywords

Adaptive filter, linear mean square(LMS) algorithm, adaptive algorithm.

1. Introduction

Because LMS algorithm has low computational complexity, good convergence in stable environments, the average unbiased to converge to the wiener solution and the stability when using the limited accuracy to implement the algorithm such as features, which makes LMS algorithm be one of the most widely used algorithm in the adaptive algorithm. Due to the extensive use of LMS algorithm, and in the actual conditions, to solve practical problems, based on LMS algorithm the new LMS algorithms appear constantly. Therefore, the adaptive filtering with its prospect of development is LMS algorithms.

This paper firstly the basic principle of the adaptive LMS algorithm, then studies its improved form-normalized LMS algorithm, lastly analyzes its performance and simulates by MATLAB.

2. The basic principle of adaptive LMS algorithm

LMS algorithm is the minimum mean square algorithm, using the minimum mean square error (MMSE) criterion, which is the design criteria of the filters, that is, to make mean square error (MSE) between the actual output-y (n) and expected response-d (n) of filters be a minimum.

Fig.1 is the principle diagram of the adaptive realization of FIR filters. The so-called adaptive implementation means; the weight coefficient of tapping about M order FIR filters $w_0, w_1, \ldots, w_{M-1}$ can be self-regulation according to the size of the estimation error-e(n), as a direct result of a cost function is minimal.

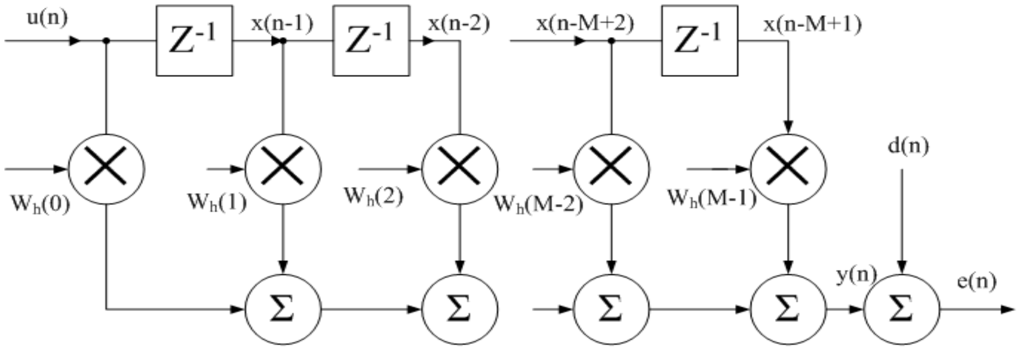

Fig.1 The adaptive realization of FIR filters

Define MSE-J (n) is the cost function, because the evaluated error of filters in the n time is

$$e(n) = d(n) - w^H x(n) \tag{1}$$

And the cost function can be expressed as

$$J(n) \overset{def}{=} E\left\{ |e(n)|^2 \right\} = E\left\{ |d(n) - w^H x(n)|^2 \right\} \tag{2}$$

The available gradient of J (n) is

$$\nabla J(n) = 2E\{x(n)H(n)\}w(n) - 2E\{x(n)d^*(n)\} \tag{3}$$

Windrow,etc. put forward to least mean square(LMS) algorithm at Stanford university of the United States in 1960, which is a method of using instantaneous value to estimate gradient vector, namely

$$\hat{\nabla}(n) = \frac{\partial[e^2(n)]}{\partial w(n)} = -2e(n)x(n) \tag{4}$$

It is thus clear that this method of instantaneous estimation is unbiased, because its expectations- $E\left[\hat{\nabla}(n)\right]$ is really equal to the vector- $\nabla(n)$. So, according to the relationship between the change of the adaptive filter's filter coefficient vector and the direction of the gradient vector estimation, first the formulation of LMS algorithm is written as follows:

$$\begin{aligned} \hat{w}(n+1) &= \hat{w}(n) + \frac{1}{2}\mu[-\hat{\nabla}(n)] \\ &= \hat{w}(n) + \mu e(n)x(n) \end{aligned} \tag{5a-b}$$

$\hat{w}(n+1) = \hat{w}(n) + \mu x(n)\,[d(n) - \hat{w}^{H}(n)x(n)]$ Substitute the formulation of e(n)=d(n)-y(n) and(1)into the above formulation to obtain

$$=[I-\mu x(n)x^{H}(n)]\hat{w}(n)+\mu x(n)d(n) \tag{6}$$

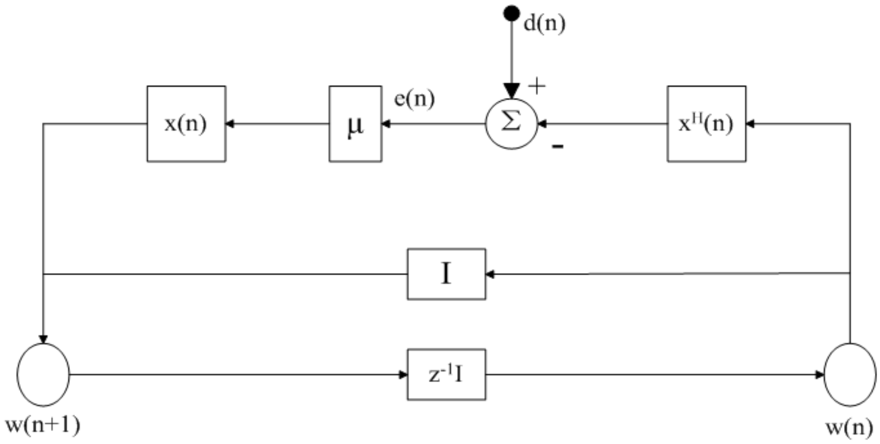

Fig.2 The signal flow diagram of adaptive LMS algorithm

Through the above formulation to obtain the signal flow diagram of adaptive LMS algorithm, this is a model of a feedback form, as shown in Fig. 2. As the steepest descent method, filter coefficient vector when the time n equals to zero is as the arbitrary starting value-w(0), and then begin to the calculation of LMS algorithm. Its steps are as follows:

Firstly, through the filter's filter coefficient vector in the time n of now to value of assessment- $\hat{w}(n)$, the input signal vector-x(n) and the desired signal-d(n), calculate the error signal:

$$e(n)=d(n)-x^{H}(n)\hat{w}(n) \tag{7}$$

Secondly, using recursive method to calculate the update valuation of the filter's filter coefficient vector:

$$\hat{w}(n+1)=\hat{w}(n)+\mu e(n)x(n) \tag{8}$$

Increase time index-n to one, go back to the first step, and repeat the calculation of the above steps, until being reached so far.

Thus, the adaptive LMS algorithm is simple, and neither calculates the correlation function of the input signal, nor resolves the inverse matrix, thus has got a wide application. However, because the LMS algorithm uses gradient vector of the instantaneous estimation, and has the big variance, so that it can not obtain the optimal filtering performance.

3. The simulation and analysis of its results

It can be seen from Fig.3 and Fig.4 that when the step size is small, the convergence process of LMS algorithm is far from particularly fast.

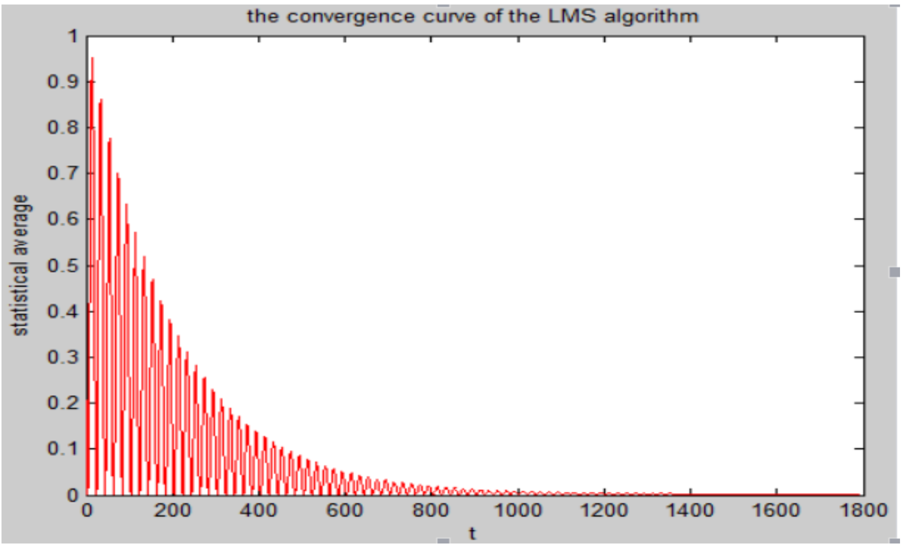

Fig.3 The convergence curve of LMS algorithm

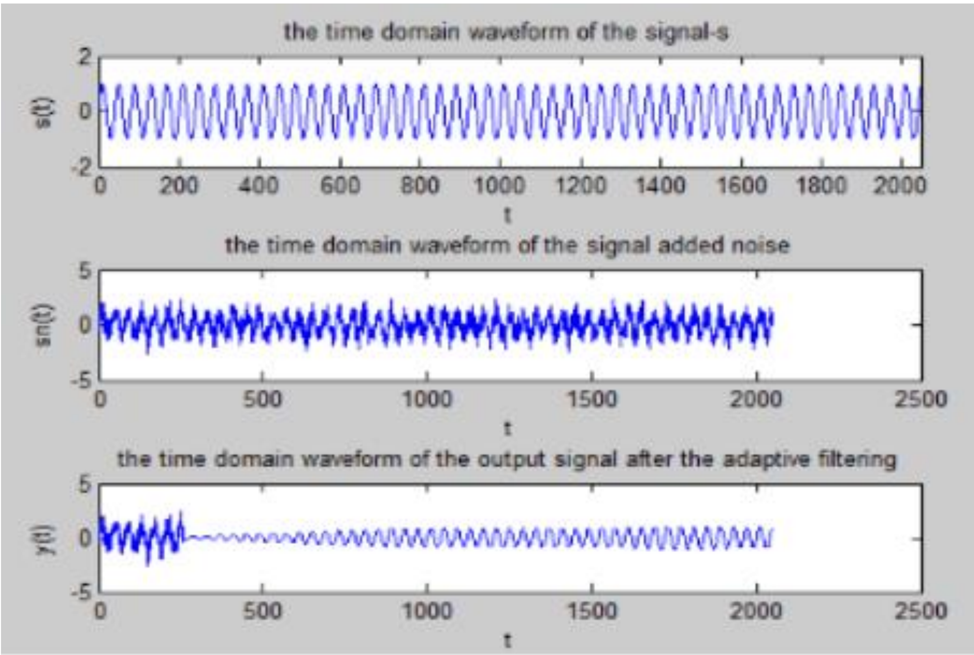

Fig.4 The waveform of signal-s in time domain

4. Conclusion

The adaptive filters has good filtering ability, which makes it become powerful equipment in the application of signal processing automatic control, and successfully apply in areas such as communications, radar, seismology and bio-medicine, whose concrete application in system identification, interference elimination, channel equalization and forecast analysis.

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